Capacitor current

From the relation $Q = CV$, the following can be deduced.

$$\text{Since } Q = It \text{ then } It = CV$$

$$\text{or } I = C\frac{V}{t}$$

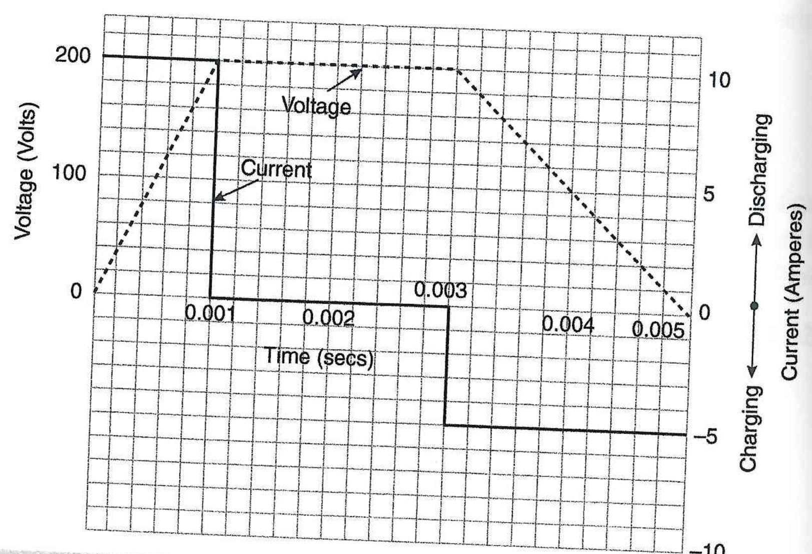
The expression shows that current only flows when the voltage across a capacitor changes, as $\frac{V}{t}$ represents a rate of change of voltage. The current at any instant can be found if the rate of change of the voltage is known at that instant. If however, the rate of change is uniform for a period of time, then a constant current will flow, as illustrated by the next example.

Example 8.3. The P.D. across the plates of a 50μF capacitor varies thus:

From time $t = 0$ to $t = 1$ms, V rises uniformly from 0–200V

From time $t = 1$ to $t = 3$ms, V is constant at 200V

From time $t = 3$ to $t = 5$ms, V falls uniformly from 200–0V



▲ Figure 8.8

Illustrate the voltage variations on a graph and deduce the shape of the current wave during the period of 5ms.

Since $Q = CV = It$ then $It = CV$ or $I = C\frac{V}{t}$

(a) $I = 50 \times 10^{-6} \times \frac{(200-0)}{0.001} = 10\text{A}$ (charging)

(b) $I = 50 \times 10^{-6} \times \frac{(200-200)}{0.002} = 0\text{A}$

(c) $I = 50 \times 10^{-6} \times \frac{(0-200)}{0.002} = -5\text{A}$ (discharging)

The required graphs are shown in figure 8.8.

Energy stored in an electric field or dielectric

Consider the voltage to rise uniformly across the capacitor plates, to a value of V volts, in a time t seconds. The average P.D. value will be $\frac{V}{2}$ volts and the charging current constant equal to I amperes. The average power supplied during the charging period will be $\frac{V}{2} \times I$ watts and the energy fed in will be $\frac{V}{2} \times I \times t$ joules.

This energy is not converted into heat, as a capacitor has no resistance, but it does work in establishing the electric field. It is this energy which is stored, and then recovered when the field collapses as the capacitor discharges. Thus:

$$\text{Energy stored} = \frac{V}{2}It \text{ joules or } = \frac{V}{2}Q \text{ joules}$$

$$\text{or alternatively, } W = \frac{1}{2}CV^2 \text{ joules.}$$

Example 8.4. Consider the capacitor arrangement of Example 8.2 and calculate the total energy stored for a steady applied voltage of 1000V for (a) series (2 decimal places) and (b) parallel (2 significant figures) connections.

(a) Series

Energy stored is given by $W = \frac{1}{2}CV^2$ joules

$$= \frac{1}{2} \times 33.33 \times 10^{-6} \times 1000^2$$

$$= 16.67\text{J}.$$

(b) Parallel

Energy stored or $W = \frac{1}{2} \times 150 \times 10^{-6} \times 1000^2$

$$= 75\text{J}.$$

Note. The equivalent capacitance for each arrangement is used for *C* in the energy expression.

Relative permittivity

If a parallel-plate capacitor is made so one plate is connected to earth and the other to an electroscope, the effect of altering the dielectric can be noted. With air as the insulating medium between the plates, a capacitor is charged to a given value as indicated by the amount of divergence of an electroscope's leaves. If a sheet of insulating material, for example, a slab of paraffin wax, is placed in the air gap, a *converging* effect is produced, indicating that charge *appears* to have reduced. However, if the insulation is removed the leaves again diverge to the original extent. A correct assumption is that the capacitance has increased, i.e. the capacitor now accepts a greater charge for the same leaf divergence. This is confirmed if a capacitor is charged to an amount giving the original leaves divergence and the wax insulation removed. The leaves will then diverge to a greater extent, showing a larger charge has been imparted. The experiment shows that capacitance can vary with the type of dielectric, a property termed its *permittivity*. Permittivity is likened to the permeability of a magnetic material. Relative permittivity can be defined as the ratio of the capacitor's capacitance with the material taken as the dielectric, compared with the capacitance of the same capacitor with air as the

dielectric. Another term for relative permittivity is the *dielectric constant*. The symbol used is $\in_r$ (the Greek letter – small Epsilon).

Typical values for relative permittivity are as follows: air 1.0006, paraffin wax 2.2, mica 4.5 to 8 and glass 4 to 10.

Absolute permittivity

Absolute permittivity can also be considered as $\in_r$ was introduced relative to air. As for a magnetic circuit, we can write:

Absolute permittivity = relative permittivity × permittivity of free space

$$\text{So } \in = \in_r \times \in_0$$

Permittivity of free space

As comparisons have been made with the magnetic circuit, permittivity is best understood by comparing it with permeability, its magnetic equivalent.

$$\text{Permeability is defined as the ratio} = \frac{\text{Flux density}}{\text{Magnetising force}}$$

$$\text{or } \mu = \frac{B}{H}$$

$$\text{Similarly permittivity} = \frac{\text{Electric flux density}}{\text{Electric force}} \quad \text{or } \in = \frac{D}{E}$$

For air, permittivity is measured to be

$$\frac{1}{4\pi \times 9 \times 10^9} \text{ farads per metre}$$

i.e. $\in_0 = 8.85 \times 10^{-12}$ farads per metre.

Although other alternatives are derived to estimate $\in_0$, the following deduction gives the required answer. If unity is taken for the area dimensions *A* and *d* the plate spacing:

$$\text{Then since } \epsilon_0 = \frac{D}{E} = \frac{Q/A}{V/d} = \frac{Q}{V} \times \frac{d}{A}$$

$$\text{or } \epsilon_0 = \frac{CV}{V} \times \frac{d}{A} = \frac{Cd}{A}$$

When d and A are equal to 1, ϵ_0 = the capacitance value of the arrangement. For a vacuum, the capacitance value of the standard capacitor, using unity for A and d, is measured to be 8.85×10^{-12} SI units.

$$\text{or } \epsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \text{ also expressed as,}$$

8.85×10^{-12} farads per metre.

Note. Although a vacuum is mentioned for the above capacitor arrangement, air can be taken as the dielectric, as the variation is small enough to be neglected.

Capacitance of a parallel-plate capacitor

Consider the area of the plates to be A square metres and their spacing d metres, i.e. the thickness of the dielectric. Applied voltage is taken as V volts, resulting in a charge of Q coulombs. As charge Q is assumed to be uniformly distributed over the whole area of the plates, electric flux density D will be $\frac{Q}{A}$.

The electric force or potential gradient E in the dielectric is $\frac{V}{d}$ volts per metre and permittivity ϵ (by definition) = $\frac{D}{E}$.

$$\text{Thus } \epsilon = \frac{D}{E} \text{ or } \epsilon = \frac{Q/A}{V/d} = \frac{Qd}{VA}.$$

$$\text{When } \epsilon = \frac{CVd}{VA} = \frac{Cd}{A} \text{ or } C = \frac{\epsilon A}{d} \text{ but } \epsilon = \epsilon_0 \epsilon_r$$

$$\text{So } C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ farads.}$$

Example 8.5. A capacitor consists of 2 parallel metal plates, each 300mm × 300mm, separated by a sheet of polythene 2.5mm thick, with relative permittivity 2.3. Calculate the energy stored in the capacitor when connected to a D.C. supply of 150V (3 significant figures).

$$C = \frac{8.85 \times 10^{-12} \times 2.3 \times (300 \times 10^{-3})^2}{2.5 \times 10^{-3}} \text{ farads}$$

$$C = 733 \times 10^{-12}\text{F}$$

$$C = 733\text{pF}$$

$$\text{Energy stored} = \frac{1}{2} CV^2 \text{ joules}$$

$$= \tfrac{1}{2} \times 733 \times 10^{-12} \times 150^2$$

$$= 8.25 \times 10^{-6} \text{ joules}$$

$$= 8.25\mu\text{J}$$

Example 8.6. A capacitor of 5μF charged to a P.D. of 100V is connected in *parallel* with an identical uncharged capacitor. What quantity of electricity flows into the second capacitor and to what voltage will it be charged?

Consider the first capacitor designated A, then as $Q = C_A V$, $Q = 5 \times 10^{-6} \times 100 = 5 \times 10^{-4}$ coulombs.

When capacitor B is connected across A, charge passes from A to B until the potential of each is the same. The capacitor arrangement is considered as a parallel connection or the joint capacitance is the same as that of 1 unit of 10μF.

Applying the formula $Q = CV$

$$\text{Then } V = \frac{Q}{C} = \frac{5 \times 10^{-4}}{10 \times 10^{-6}}$$

or $V = 50$ volts, the final voltage.

This may be deduced from the fact that the capacitors are similar, so charge passes from A to B until th

Transient effects in D.C. circuits

We have considered the factors affecting capacitance and charge on capacitor plates, without reference to real circuits. We will return to the effects of capacitance on A.C. circuits later but here it is appropriate to consider the application of a D.C. voltage to a capacitive circuit.

In a resistive D.C. circuit current rises to its final steady state and the P.D. across it settles to a steady value almost immediately. Similarly on opening a circuit, current and P.D. fall almost immediately to zero. However, when a capacitor and resistor, or an inductor and resistor, are connected in series to a D.C. supply, the final steady state of current and P.D. is *not* achieved immediately. The change takes a short period of time depending on the circuit component values. Such changes of current and P.D. up to the final steady state conditions are called *transient* values, which grow or decay in an *exponential* way.

Capacitor in a D.C. circuit

Consider figure 8.9, an unchanged capacitor in series with a resistor R connected to a D.C. supply V.

$$V = P.D. \text{ across } R + P.D. \text{ across } C$$

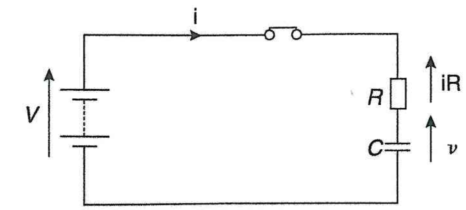
$$V = iR + v$$

$$\therefore i = \frac{V - v}{R}$$

At the instant of switch-on, the instantaneous voltage value across the capacitor (v) is zero, and the current (i) is maximum, limited only by resistance R.

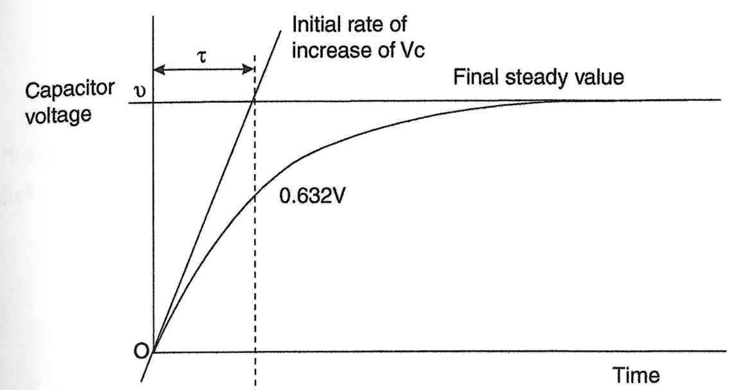
At switch-on $i = \frac{V}{R}$, but this current is the charging capacitor current and, as the capacitor charges, v increases which in turn decreases i. The rate of charge therefore decreases until, when the capacitor is fully charged, current is zero. However zero current is achieved at infinite time but, for practical purposes, it is assumed this occurs in a time equal to 5 times the initial rate of charge. If the initial rate of charge is maintained, the capacitor will have fully charged in a definite time depending upon the circuit components. This time is the *time constant* and given the symbol τ (Greek letter TAU)

Thus $\tau = CR$ seconds



▲ Figure 8.9

The rate of charge decreases, a graph of capacitor voltage against time is in a non-linear manner, following an exponential law (figure 8.10).



▲ Figure 8.10

The equation for this line is:

$$v = V(1 - e^{-t/\tau}) \text{ volts}$$

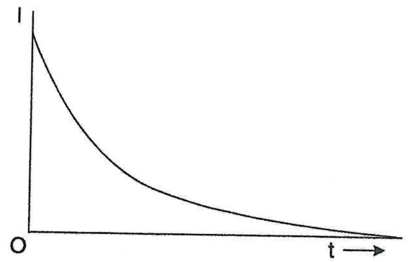
Where v = Instantaneous P.D. across capacitor

V = Applied circuit voltage

t = Time from switch-on

$\tau = CR$ = Time constant

It is noted the capacitor voltage will be 0.632 of the final steady value after a time equal to one time constant. Similarly during a second time constant, voltage only increases by 63.2% of the remaining voltage, and after each subsequent time interval, voltage increases similarly. As current i is a maximum value when capacitor voltage v is zero, and voltage increases exponentially, current will decrease exponentially as shown in figure 8.11.

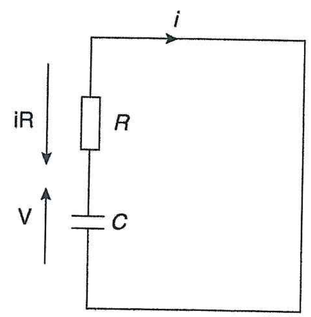


▲ Figure 8.11

The equation for this curve is:

$$i = Ie^{-t/\tau} \quad \text{or} \quad i = \frac{V}{R}e^{-t/\tau}$$

Consider now a fully charged capacitor C in figure 8.12 discharged through resistor R. Capacitor C acts as a supply source and as it discharges the P.D. across it falls and again the curve is exponential.



▲ Figure 8.12

As the current is limited by the resistance $\left(i = \frac{V}{R}\right)$, at the start of discharge the current is maximum $\left(i = \frac{V}{R}\right)$, so the current curve also follows a falling exponential curve.

Hence on discharge:

$$v = Ve^{-t/\tau}$$

$$\text{and } i = Ie^{-t/\tau}$$

Example 8.7. A 40kΩ resistor and a 20μF capacitor are connected in series to a 200V D.C. supply. Find the circuit current (3 significant figures) and the P.D. across the capacitor after 0.2 seconds (2 decimal places) from switch-on.

Time constant $\tau = CR = 20 \times 10^{-6} \times 40 \times 10^3$

$$= 0.8s.$$

$$i = \frac{V}{R}e^{-t/\tau} = \frac{200}{40 \times 10^3}e^{-0.2/0.8} = 5 \times 10^{-3}\, e^{-0.25}$$

$$\therefore \text{ After } 0.2s \quad i = 3.89mA$$

$$v = V(1 - e^{-t/\tau}) = 200\,(1 - e^{-0.2/0.8})$$

$$\therefore \text{ After } 0.2s \quad v = 44.24V$$

Example 8.8. A 20μF capacitor, fully charged to a voltage of 300V, is discharged through a 1MΩ resistor. Find the time taken for the capacitor voltage to fall to 60V (1 decimal place).

$$\tau = CR = 20 \times 10^{-6} \times 1 \times 10^6$$

$$= 20s$$

$$= 20s$$

$$v = Ve^{-t/\tau}$$

$$60 = 300e^{-t/20}$$

$$0.2 = e^{-t/20}$$

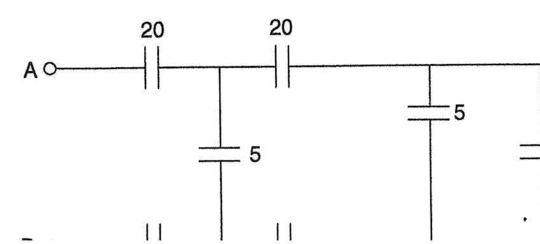
$$\frac{-t}{20} = -1.61$$

$$t = 32.2s.$$

Practice Examples

8.1. Two capacitors of 0.02μF and 0.04μF are connected in series across a 100V D.C. supply. Find the voltage drop across each unit (1 decimal place).

8.2. For the circuit shown, calculate the effective capacitance between A and B. The capacitance values shown are in microfarads (1 significant figure).



8.3. A variable capacitor having a capacitance of 1000μF is charged to a P.D. of 100V. The plates of the capacitor are then separated by means of an insulated rod, so that the capacitance is reduced to 300μF. Find, by calculation, by how much the capacitance changes (1 decimal place).

8.4. A plate capacitor consists of a total of 19 metal-foil plates each 2580mm^2 and separated by mica 0.1mm thick. Find the capacitance of the assembly if the relative permittivity of mica is 7 (3 significant figures).

8.5. A P.D. of 10kV is applied to the terminals of a capacitor consisting of 2 circular plates, each having an area of 10 000mm^2, separated by a dielectric 1mm thick. If the capacitance is 3×10^{-4}μF, calculate the electric flux density (1 significant figure) and the permittivity of the dielectric (2 decimal places).

8.6. A capacitor consists of 2 parallel metal plates, each 200mm by 300mm, separated by a sheet of polythene 3.5mm thick, having a relative permittivity of 3.0. Calculate the energy stored in the capacitor when connected to a D.C. supply of 300V (4 significant figures).

8.7. Calculate the capacitance value of a capacitor which has 10 parallel plates separated by insulating material 0.3mm thick. The area of one side of each plate is 1500mm^2 and the relative permittivity of the dielectric is 4 (2 significant figures).

8.8. Two capacitors A and B having capacitances of 20μF and 30μF respectively are connected in series to a 600V D.C. supply. Determine the P.D. across each capacitor (3 significant figures). If a third capacitor C is connected in *parallel* with A and it is then found that the P.D. across B is 400V, calculate the value of C (2 significant figures) and the energy stored in it (1 decimal place).

8.9. A D.C. voltage of 500V is applied to a 40μF capacitor. Find the value of the charging current at the instants when the voltage is varying as follows:

Time $\left(\frac{1}{1000}\text{ sec}\right)$	0–1	1–2	2–3	3–4	4–5
Voltage values	0–100	100–150	160 constant	150–50	50–0

8.10. The capacitance of a single-phase concentric cable has a value of C = 0.289μF. The diameter of the inner core is 12mm and the insulation has a radial thickness of 10mm. Calculate the permittivity of the insulating material (2 decimal places).

9 BASIC ALTERNATING CURRENT (A.C.) THEORY

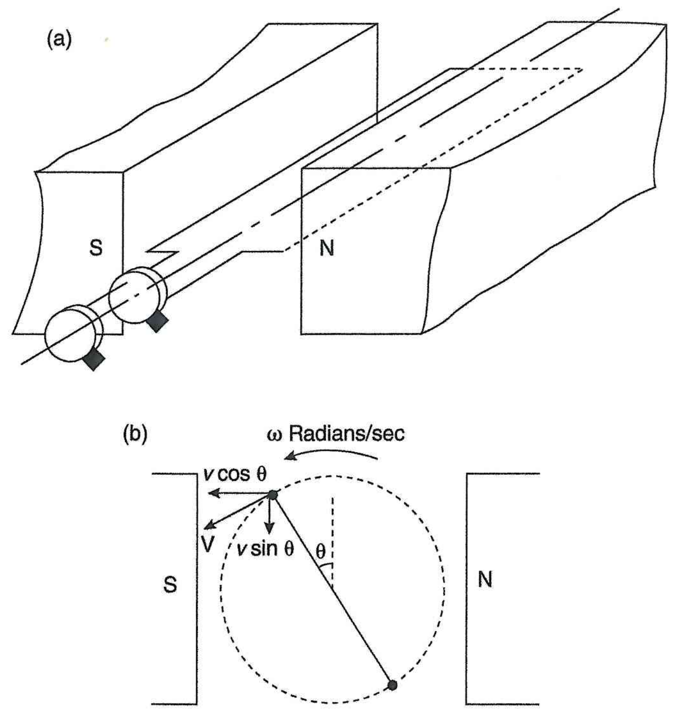
The invention of the wheel was perhaps rather obvious; but the invention of an invisible wheel, made of nothing but a magnetic field, was far from obvious, and that is what we owe to Nikola Tesla.

Reginald Kapp

Let us revise briefly the fundamental principles of Chapter 7. Figures 9.1a and 9.1b show a simple A.C. generator where a coil rotates in a uniform magnetic field. The sides of the coil, i.e. the conductors, cut the magnetic flux and an e.m.f. is induced which from first principles is $e = B\ell v$ volts. The letter e, for the induced e.m.f., is introduced because this value varies from instant to instant. Even though the coil rotates at a uniform velocity v, the cutting rate is not constant, but depends upon the *angle* at which the conductors cut the magnetic flux. The velocity is resolved into a cutting component ($v \sin \theta$) and a non-cutting component ($v \cos \theta$). Only the cutting velocity component is responsible for e.m.f. and creates a general expression for the e.m.f. at any instant: $e = B\ell v \sin \theta$ volts.

The A.C. waveform

In the expression $e = B\ell v \sin \theta$, as for an alternator, B, ℓ and v are assumed constant and equal to K. The expression thus becomes $e = K \sin \theta$ and a value for K is obtained



▲ Figure 9.1

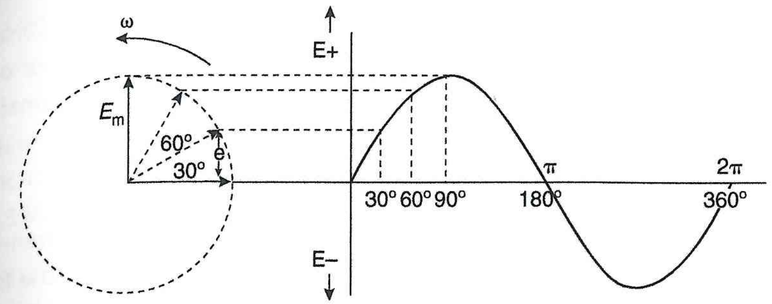
component $v \sin \theta$ generates a maximum e.m.f. designated E_m. At this instant $e = E_m$ and we write $E_m = K \sin \theta$. But as $\sin \theta = \sin 90 = 1$. $\therefore E_m = K$.

Substituting back in the original expression:

$$e = E_m \sin \theta$$

This is not just clever manipulation of an equation but is important as it shows that the generated e.m.f. varies *sinusoidally*, where e is the *instantaneous* value and E_m the *maximum* value.

If we examine the waveform plotted to a time or angle base, it must be remembered from vector treatment that a sine wave is deduced from a rotating vector's vertical component – for electrical work a rotating vector is called a *phasor*. If the phasor's length represents E_m, then for any angle θ, the instantaneous value is the vertical projection used as an ordinate for the waveform, when plotted to an angle or time base. Figure 9.2 illustrates this process and the method is summarised: Draw a circle of radius equal to the maximum wave's value. Start from the horizontal, and move the phasor through a known angle and project the vertical value onto an angle or time scale. Choose suitable



▲ Figure 9.2

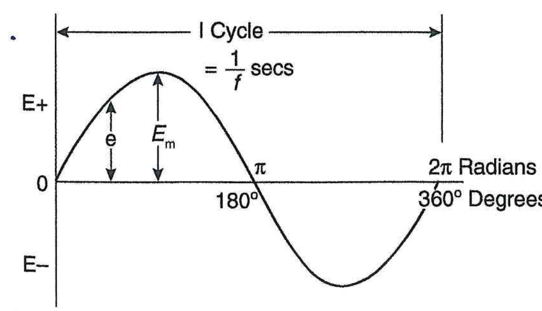
scales to avoid distorting the wave's sinusoidal shape. The connection between the construction and representation of a sinusoidally induced e.m.f. is seen in the various triangles:

$$\frac{e}{E_m} = \sin \theta \text{ so } e = E_m \sin \theta.$$

The expression is the same as that deduced previously but can be modified for phasor representation.

In accordance with accepted procedure, assume the phasor E_m rotates from the zero or horizontal position *anticlockwise* with an angular velocity of ω radians/second (ω is a Greek letter – small omega). So $\theta = \omega t$ where t is time in seconds and the equation written as $e = E_m \sin \omega t$.

Figure 9.3 shows some of the terms used in A.C. theory. *Periodic time* = the time for 1 complete cycle. The *frequency f* of a wave = the number of complete cycles in a time interval of 1 second. In accordance with SI recommendations, the *hertz* (Hz) is used



▲ Figure 9.3

for frequency measurement. Current marine practice usually have either 50Hz or 60Hz A.C. systems. The maximum value reached by a wave is the *peak* value or *amplitude* and at any instant is termed the *instantaneous* value and denoted by a small letter *e*. We observe that sinusoidal current conditions also occur and that the expression for a current following a sine wave law can also be written as $i = I_m \sin \omega t$.

Following introduction of the sine wave, derived from a phasor, and the generation of a sinusoidal e.m.f. by a rotating coil, the treatment can be combined by the following. The phasor is assumed to rotate at a constant angular velocity of ω radians/second and the waveform, if at a frequency of *f* hertz, will cover in 1 second, an angle of 360*f* degrees or 2π*f* radians. The phasor meanwhile will pass through ω radians in 1 second so ω can be equated to 2π*f*° or 360*f*°. The earlier expressions can now be written in their most useful form, namely:

$$e = E_m \sin 2\pi ft.$$

This is the first fundamental A.C. theory formula. It is important to note that if $\frac{22}{7}$ or 3.14 is substituted for π, the angle will be in radians but can be converted into degrees by multiplying by 57.3. A simpler method is to substitute 180° for π, converting into degrees directly.

Example 9.1. Find the *instantaneous* value of a 50Hz sinusoidal e.m.f. wave, maximum value 100V, at a time 0.003s after the zero value (1 decimal place).

Substituting in $e = E_m \sin 2\pi ft$ we have:

$$e = 100 \sin (2 \times 180 \times 50 \times 0.003)$$

$$\text{or } e = 100 \sin 54°$$

$$\text{and } e = 80.9\text{V}.$$

Important Note. A problem can occur if the instantaneous value is given and the *time* is required. Attention must be given to the following example, which illustrates this point.

Example 9.2. Find the *first* time after zero when the instantaneous value of a sinusoidal current wave is 6.8A. The maximum value is 12A and the frequency 50Hz. Find also the *second* time after zero.

$$\text{Here } i = I_m \sin 2\pi ft$$

$$\text{or } 6.8 = 12 \sin (2 \times 180 \times 50 \times t)$$

$$\text{Thus } \frac{6.8}{12} = \sin (180 \times 100 \times t)$$

and 0.566 = sin (18 × 10³ × *t*). Solution of this equation can only be made by use of electronic calculator or sine tables, from which an angle can be found whose sine equals 0.566.

Thus let θ = this angle, then sin θ = 0.566. So θ = $\sin^{-1}(0.566)$, hence θ = 34°30′ = 34.5°

$$\therefore \sin \theta = \sin (18 \times 10^3 \times t) \text{ or } 18 \times 10^3 \times t = 34.5$$

$$\text{and } t = \frac{34.5}{18 \times 10^3} = 1.9\text{ms}$$

The second time value is also required, and occurs $\frac{1}{2}$ cycle later. Subtracting the interval, from the zero value, is needed to attain a second instantaneous height of 6.8A.

Thus time for $\frac{1}{2}$ cycle $= \frac{1}{100}$ seconds = 0.01s (as T = 1/f)

So second time required = 0.01 − 0.0019 = 0.0081s or 8.1ms.

Representation of Sinusoidal Alternating Quantities

Earlier it was shown that an alternating voltage or current can be respectively represented by:

$$e = E_m \sin 2\pi ft \quad \text{or} \quad i = I_m \sin 2\pi ft.$$

This notation method conveys all that is required about the quantity, i.e. it follows a form whose amplitude, frequency and instantaneous value at any particular time can be found. This method of notation is called a trigonometrical representation.

Trigonometrical representation

This is useful for 2 alternating quantities, but not necessarily simultaneous quantities. A 50Hz alternating voltage can create an alternating current in a circuit to alternate at

50Hz. The current need not however, be *in-phase* with the voltage, which may reach its maximum value a little time before the current reaches its maximum value. The voltage is said to *lead* the current or the current to *lag* behind the voltage. There is a *phase difference* between the 2 quantities or waveforms. Such a phase difference is shown by inclusion of a phase angle in radians. Thus if 2 current waveforms are represented by:

$$i_1 = I_{1m} \sin 2\pi ft \quad \text{and} \quad i_2 = I_{2m} \sin (2\pi ft + \frac{\pi}{3}).$$

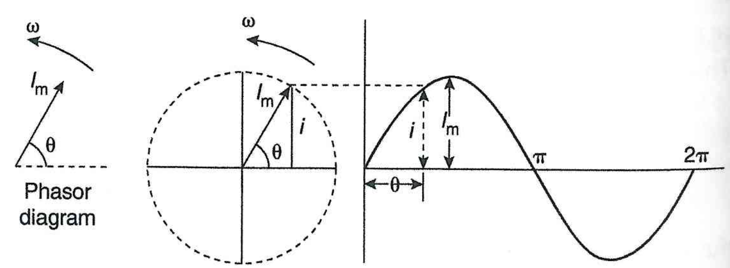
The second waveform will lead the first by $\frac{\pi}{3}$ radians or $\frac{180}{3} = 60°$. A third waveform written $i_3 = I_{3m} \sin (2\pi ft - \frac{\pi}{6})$ will lag the first or reference waveform by $\frac{180}{6} = 30°$.

This mathematical trigonometrical form of representation is used for trigonometrical operations, such as multiplication, division, expansion, etc. and these will be illustrated in A.C. theory later.

Phasor representation

This is used for A.C. quantities such as current, voltage, flux, etc. and was introduced with vectors. Since voltages or currents are quantities whose magnitudes and directions are known, they can be described by rotating vectors. As 'phase' is often involved it is customary to represent these with phasors. Thus a voltage phasor drawn to scale represents the magnitude of a voltage and the direction in which it acts is shown by an arrow. This technique was introduced earlier in this chapter, and we will now consider the standard phasor operation methods.

The relation between the graphical deduction of a waveform from a phasor has been covered. For most practical A.C. work waveforms and instantaneous values are of little



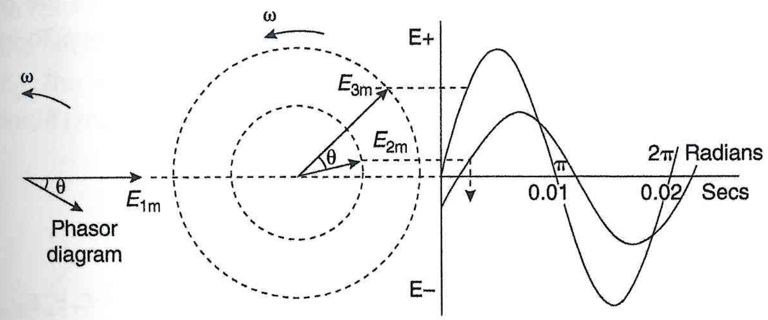
▲ Figure 9.4

importance compared with its magnitude and phase, so using only a phasor as shown makes representation simpler (figure 9.4). Further simplification can result from a correct use of *phasor diagrams* to illustrate A.C. circuit relationships especially if more than one current and/or voltage is considered at one time.

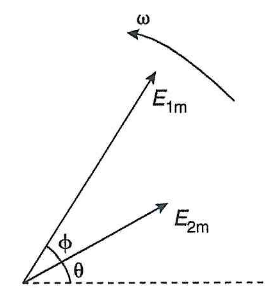
Note. For most practical electrical engineering work *r.m.s.* (root mean square) values are used. The full meaning of this term will be considered later in this chapter, but here it is convenient to make a phasor equal to this value rather than the maximum value with phase difference shown by phasors. Consider two 50Hz sinusoidal voltages represented E_{1m} and E_{2m}. The phase angle φ is known, voltages being of the same frequency but out-of-phase. Voltages are written $e_1 = E_{1m} \sin \omega t$ and $e_2 = E_{2m} \sin (\omega - \phi)$ where the angle ϕ is say 60° or $\left(\frac{\pi}{3}\right)$ radians and,

$$e_2 = E_{2m} \sin\left(\omega - \frac{\pi}{3}\right).$$

Waveforms are drawn graphically with the first leading the second. If the instant when the first goes through its zero value is the start of the angle or time scale, and the first wave considered the reference, then the phasor diagram can be drawn (figure 9.5).



▲ Figure 9.5



Here we depict the 2 phasors and their relation to each other where the first phasor is the reference and the second lags it by an angle ϕ.

Direction of phasor rotation is anticlockwise so E_{2m} is *behind* E_{1m} by angle ϕ. If an instant θ degrees later in time is considered the diagram can be redrawn (figure 9.6). The horizontal is taken as the zero time or reference axis.

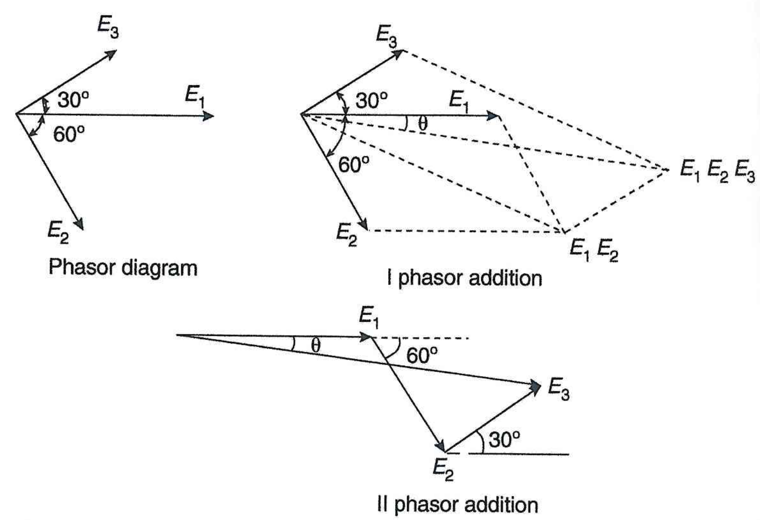
Addition and Subtraction of Alternating Quantities

When 2, or more, sinusoidal voltages or currents act in a circuit the resultant is obtained by either of the following: (1) trigonometrical methods or (2) phasor methods.

(1) TRIGONOMETRICAL METHODS. These methods require knowledge of trigonometrical identities and follows established procedures. Examples of this approach will be given.

(2) PHASOR METHODS. The resultant of 2 or more phasors may be obtained (a) *graphically* or (b) *mathematically*.

(a) The *graphical method* is performed by setting out phasors to scale at the given angles, completing the parallelogram or polygon and measuring the resultant. Figure 9.7 shows the method employed with phasor addition shown. To subtract a phasor, reverse its direction and proceed as before.



▲ Figure 9.7

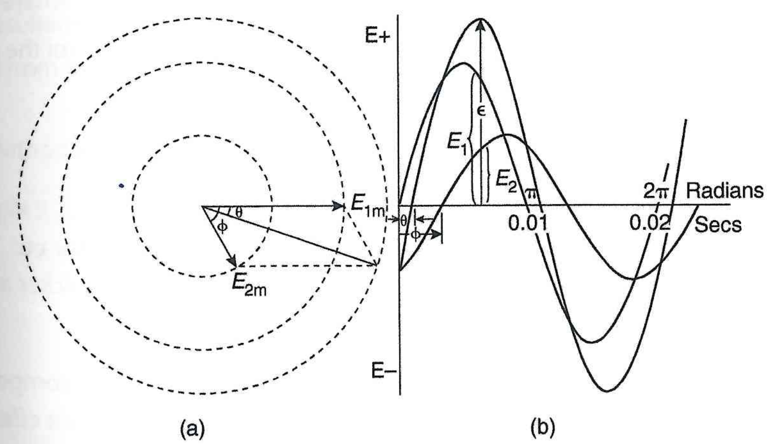
In the diagram phasor addition is shown: (I) by completing the parallelogram to obtain the resultant of 2 phasors and using this resultant with the third phasor to obtain the final resultant, (II) by completing the polygon as shown. Both methods are tedious and unfortunately errors accumulate.

If the resultant of 2 waveforms is needed, either of 2 procedures can be taken.

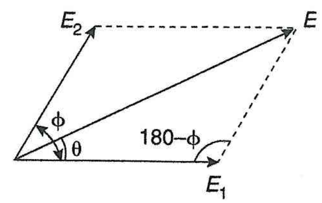
The first method uses the fact that the sum of 2 sine waves with the same frequency is itself a sine wave. Thus any instantaneous value of the resultant wave is the *sum* of the individual instantaneous values taken from the 2 waves. Each wave is drawn graphically in accordance with the method already outlined, care being taken to displace one from the other by a given phase angle. By adding instantaneous values as shown (figure 9.8b) the *resultant* instantaneous value will give a point on the resultant wave, for example, $e = e_1 + e_2$. Other points are similarly obtained and a smooth curve drawn between points.

Note. One waveform is chosen as the reference, the other and the resultant are drawn to its base and zero value.

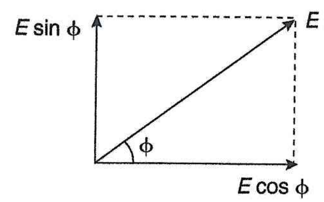
An alternative procedure for obtaining the resultant waveform is as follows. As individual voltages or currents are treated as phasors, the resultant of any 2 (or more) values is obtained as described. If a parallelogram is completed, as in figure 9.8b, the resultant E_m gives the maximum value of the resultant wave. Using E_m as the radius of the largest circle, the resultant waveform is deduced as before. For example, if E_{1m} = 10V, E_{2m} = 6V and the phase difference is 60°, the resultant E_m will be 14V, E_m will also lag 21°45′ behind E_{1m}. The same procedure for magnitude and phase angle will give the resultant r.m.s. value, if r.m.s. values are used for component values instead of maximum values.



▲ Figure 9.8



▲ **Figure 9.9**



▲ **Figure 9.10**

The second method of obtaining a resultant waveform is a quicker method if component waveforms are not required. The resultant phasor is obtained graphically or by one of the following mathematical methods.

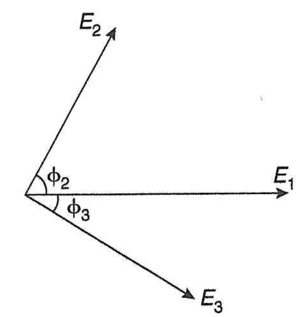
(b) The *mathematical method* is performed in 2 ways: (i) by use of the Cosine Rule or (ii) by resolving into horizontal and vertical components.

(i) The Cosine Rule can be used if the resultant of only 2 phasors is needed. Consider figure 9.9. The resultant E is obtained from $E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\phi}$ where E_1 and E_2 are the given phasors and ϕ the angle between them. The phase angle θ of the resultant is obtained from the Sine Rule:

$$\frac{E}{\sin(180-\phi)} = \frac{E_2}{\sin\theta}$$

For more than 2 phasors, the resultant is combined with a third phasor etc. The next method is suggested for summation of more than 2 phasors as it is quicker and is the method used with series and parallel A.C. circuits.

(ii) Horizontal and vertical components. Any phasor can be split into 2 components at right angles to each other which combined *together* produces the same effect as the original phasor. Thus in figure 9.10 the e.m.f. phasor E is split into a horizontal and a vertical component. If E lies at an angle ϕ to the horizontal, a horizontal component E



▲ **Figure 9.11**

If all the phasors to be added are as shown (figure 9.11) the resultant is obtained.

The sum of the horizontal components is:

$$E_H = E_1\cos\phi_1 + E_2\cos\phi_2 + E_3\cos\phi_3.$$

Similarly the sum of the vertical components will be:

$$E_V = E_1\sin\phi_1 + E_2\sin\phi_2 - E_3\sin\phi_3.$$

Note. Due allowance must be made for the *signs*. If vertical components are considered to be +ve when acting up, then $E_3 \sin\phi_3$ must be *subtracted* from the sum as it acts in a downward or –ve direction.

The resultant E is obtained from $E = \sqrt{E_H^2 + E_V^2}$ and ϕ, the angle at which it acts is found from the sine, cosine or tangent values. So $\cos\phi = \frac{E_H}{E}$.

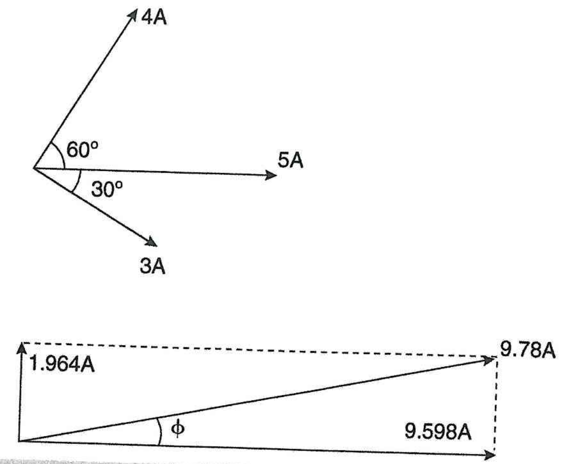
The method is illustrated by the following example.

Example 9.3. Find the resultant of the following currents where:

$$i_1 = 5\sin\omega t$$

$$i_2 = 4\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$i_3 = 3\sin\left(\omega t - \frac{\pi}{6}\right)$$



▲ **Figure 9.12**

Express the resultant in a trigonometrical form, i.e. in the same form as the individual quantities.

Figure 9.12 should be considered for this solution.

$I_H = 5\cos 0 + 4\cos 60 + 3\cos 30$

$$= (5 \times 1) + (4 \times \tfrac{1}{2}) + (3 + 0.866)$$
$$= 9.598\text{A}$$

And $I_v = 5\sin 0 + 4\sin 60 - 3\sin 30$

$$= (5 \times 0) + (4 \times 0.866) - \left(3 \times \tfrac{1}{2}\right)$$
$$= 1.964\text{A}$$

From which $I = \sqrt{I_H^2 + I_V^2} = \sqrt{9.598^2 + 1.964^2}$

$$= 9.78\text{A}$$

$$\cos\phi = \frac{I_H}{I} = \frac{9.598}{9.78} = 0.98 \quad \phi = \sin^{-1}(0.98)$$

$$\phi = 12° \text{ (approx.)} = \frac{\pi}{15} \text{ radians}$$

Note. The following points are again made in this example. (1) In line with mathematical practice, phasors drawn to the right and drawn up are given +ve signs, while those drawn to the left and drawn down are –ve. In the example all the I_H components are +ve. Consider the phasor diagram. For I_V 3 sin 30° acts down and is subtracted from 4 sin 60° which acts up. (2) The resulting sign of I_V indicates whether the resultant I is in the first or fourth quadrant, i.e. whether it lags or leads the reference (which here is the horizontal). In the solution ϕ is about 12° so $\left(\frac{180}{12} = 15\right) = \frac{\pi}{15}$ radians and the resultant is written as shown.

In the treatment introducing the mathematical method and in these examples (figures 9.9–9.12) the suffix m is omitted from the e.m.f. symbol E, illustrating that the method is equally applicable to maximum values *and* r.m.s. values. The meaning of r.m.s. values will be considered next. As these are commonly used in A.C. work, it is vital to appreciate that phasor representations, applications and solutions will follow the r.m.s. convention.

Root Mean Square and Average Values

R.m.s. or effective value

The magnitude of an A.C. current varies from instant to instant and the power dissipated in a resistor varies accordingly. Energy dissipated over time will manifest itself as heat. A resulting steady temperature rise will occur due to constant power dissipation, i.e. due to passage of a constant current will give the same heating effect in the same time. From a heating aspect, an A.C. current value will have an equivalent value of D.C. current if the heating effect is proportional to 'current squared'. Since $P = I^2R$, the magnitude of this equivalent value can be found.

Let I amperes be the equivalent D.C. current with the same *average* heating effect as the A.C. current of *varying* instantaneous value i amperes.

For the D.C. condition:

$$\text{Energy expended} = \left(\text{current}^2 \times \text{resistance}\right) \times \text{time}$$

For the A.C. condition:

$$\text{Energy expended} = \left(\text{mean or average of } i^2R\right) \times \text{time}$$

$$= \left(\text{mean or average } i^2\right) \times R \times t$$

As both energy conditions are considered equal then I^2Rt = (mean or average of i^2) Rt

$$\text{so } I = \sqrt{\text{mean or average of } i^2}$$

The effective or r.m.s. value of an alternating current is the square root of the mean or average of the squares of the instantaneous values. This is true for the shape of *any* half cycle.

Note. The above deduction shows where the term 'root mean square' comes from, as the alternative to 'effective'. In practical electrical engineering the term r.m.s. is used.

The following definition is useful.

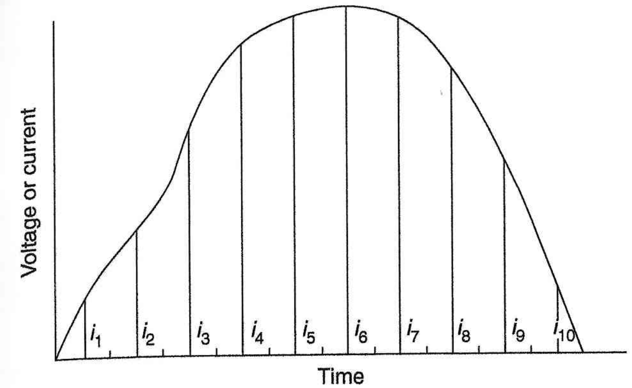
The r.m.s. or effective value of an alternating current or voltage is the value of direct current or voltage which, when passed through or applied to a given resistor for a time of 1 cycle, produces the same amount of heat as the alternating current or voltage.

It is noted in this definition, voltage is mentioned although the r.m.s. value of a voltage wave wasn't *specifically* mentioned. The heating effect on a resistor of value R ohms is taken as the basis of discussion, but an alternative to $P = I^2R$ is $P = \dfrac{V^2}{R}$, so an r.m.s. value of alternating voltage wave of instantaneous value v can be deduced similarly.

The waveform's r.m.s. value can be graphically obtained (figure 9.13). The current or voltage instantaneous values are plotted to a time or angle base with suitable scales. The base of 1 half cycle is subdivided into equal divisions with mid-ordinates i_1, i_2, i_3, etc. up to i_n. These are measured to scale and substituted in the expression:

$$I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2}{n}}$$

Only a half cycle is considered, as the next half cycle is similar to the first, but –ve. As the *square* of the current ordinates is required, +ve values result and the r.m.s. average taken over a complete cycle will be the same as for a half cycle.



▲ Figure 9.13

It is noted that the mid-ordinate rule is applied to the *ordinates squared* and *not* to the ordinates directly.

Example 9.4. The following alternating voltage measurements result at intervals over 1 half cycle:

Time (t ms)	0	0.45	0.95	1.5	2.1	2.5	3.1	3.9	4.5	5.0
Voltage (V volts)	0	20	36	40	37.5	33	32	31	20	0

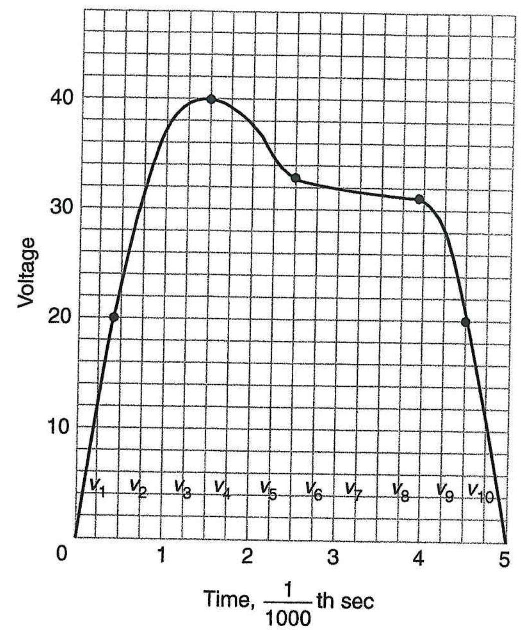
Calculate the voltage's r.m.s. value (1 decimal place) and the wave's frequency.

The solution is shown (figure 9.14), where if the waveform is plotted to scale and dividing the base into 10 equal parts, mid-ordinates can be drawn and measured.

$$\text{Mean or average of } v_2 = \frac{9427}{10} = 942.7$$

$$\text{R.m.s. value} = \sqrt{942.7} = 30.7\text{V}$$

Time for a half cycle $= \dfrac{5}{1000}$th seconds, so time for a whole cycle $= \dfrac{2\times5}{1000} = \dfrac{1}{100}$ second or 100Hz .



▲ **Figure 9.14**

For a sine wave the r.m.s. value is shown mathematically = 0.707 of the maximum value. The most direct approach involves calculus but a graphical method illustrates the relationship.

Consider an A.C. current of sinusoidal waveform with a maximum or peak value of 4 amperes. The current squared is found to be half the maximum height, i.e. half of 16 = 8 as shown (figure 9.14). The number of amperes of continuous current that gives the same heating effect is √8 = 2.828 amperes and is the square root of the mean of the squares of the current, i.e. the true r.m.s. value.

The ratio of r.m.s. to maximum value $= \frac{2.828}{4}$ or $\frac{\text{r.m.s value}}{\text{Maximum value}} = \frac{2.828}{4} = \frac{0.707}{1}$, this ratio is true for any A.C. current or sinusoidal voltage.

The r.m.s. value is always used in electrical engineering. Sine-wave working is assumed and any departure from it will be stated. If a supply voltage is stated as 220V, this will be the r.m.s. value and the voltage varies over a cycle between zero and a peak of $\frac{220}{0.707} = 311.2\text{V}$.

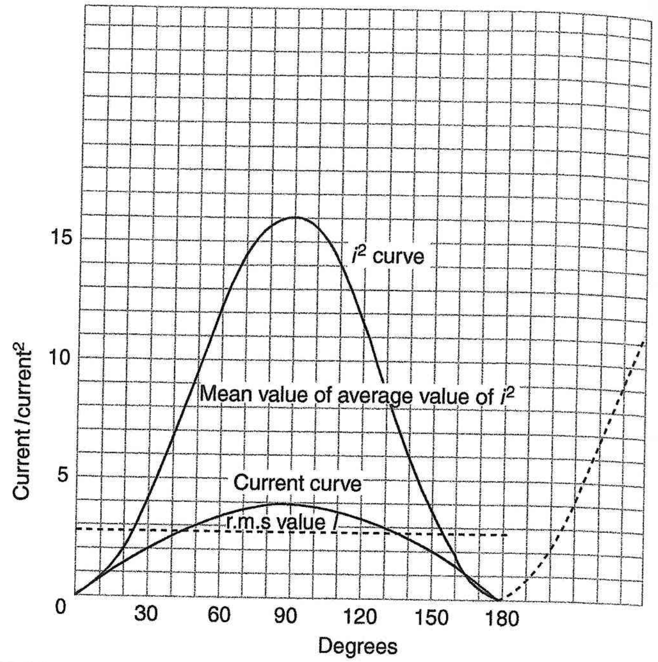
Table 9.1

$v_1 = 12$	$v_1^2 = 144$
$v_2 = 32$	$v_2^2 = 1024$
$v_3 = 39.5$	$v_3^2 = 1560$
$v_4 = 39.5$	$v_4^2 = 1560$
$v_5 = 35$	$v_5^2 = 1225$
$v_6 = 32.5$	$v_6^2 = 1056$
$v_7 = 31.5$	$v_7^2 = 992$
$v_8 = 31$	$v_8^2 = 961$
$v_9 = 28$	$v_9^2 = 784$
$v_{10} = 11$	$v_{10}^2 = 121$
	Total 9427

Average value

This is the true average value as understood mathematically of a half cycle, as that of a full A.C. cycle is zero. A waveform's average value is of interest for devices which don't depend on the effect of current squared, such as a rectifier. Rectifiers were developed to convert A.C. into D.C. current, without rotating machinery, and are found in various forms. It is common for semiconductor devices to meet the necessary rectification requirements, but in certain industrial and military devices, for example, valve radio sets, with greater immunity to EMP than semiconductor devices, gas or vapour-filled vacuum valves, convert A.C. mains voltage to a direct voltage and may operate other valves.

In the marine industry it is still common to find gas or vapour type rectifiers, the most common of which is the 'mercury-arc' rectifier seen in ship dockyards and which provide



▲ **Figure 9.15**

a 'shore supply' to D.C. ships. Metal and semiconductor rectifiers are however, used for A.C. in ships, large enough to supply D.C. current for 'degaussing' of induced magnetic signature, yet small enough to build into moving-coil indicating instruments, enabling them to be used in A.C. circuits.

D.C. current or voltage has a value equal to the *average* value of the A.C. waveform rectified and for a non-sinusoidal wave is obtained graphically, as follows. Referring to the waveform in figure 9.13 let I_{av} equal the average value, so

$$I_{av} = \frac{i_1 + i_2 + i_3 \cdots i_n}{n}$$

Example 9.5. Consider the same waveform and ordinates which are the subject of Example 9.4. Find the average value.

$$\text{Here } I_{av} = \frac{12+32+39.5+39.5+35+32.5+31.5+31+28+11}{10}$$

$$= \frac{292}{10} = 29.2\text{A}$$

For a sine wave the average value is 0.6365 of the maximum value. This is proven mathematically or graphically. As for the r.m.s. value, the most direct calculation method requires calculus but a graphical method can be checked by plotting a sine wave to a time or angle base. Subdivide the base into equal divisions, erect the mid-ordinates and again obtain the average value using the mid-ordinate rule or substitute in the expression:

$$I_{av} = \frac{i_1 + i_2 + i_3 \cdots i_n}{n}$$

The ratio of average to maximum value is $\frac{2}{\pi}$ or $\frac{\text{Average value}}{\text{Maximum value}} = \frac{2}{3.14} = 0.6365$.

This ratio is true for any A.C. current or sinusoidal voltage.

Form factor

This factor, when given a numerical value, states how near a waveform approaches the theoretical ideal sine wave. For any waveform it is defined as the ratio of the r.m.s. to the average value.

$$\text{Thus form factor} = \frac{\text{r.m.s. value}}{\text{Average value}}$$

For a sine wave, the form factor is 1.11, obtained from:

$$\text{Form factor} = \frac{0.707 \text{ Maximum value}}{0.6365 \text{ Maximum value}} = \frac{0.707}{0.6365} = 1.11.$$

Example 9.6. For the Example 9.5 already considered, in obtaining the r.m.s. and average values, find the form factor. The form factor will be $\frac{30.7}{29.2} = 1.05$.

Peak factor

The term 'peak factor' is encountered when dealing with A.C. waveforms, and defined as the ratio of the maximum value to the r.m.s. value. Thus:

$$\text{Peak factor} = \frac{\text{Maximum value}}{}$$

For a sine wave the peak factor will be

$$\frac{1}{0.707} = 1.4.$$

Practice Examples

9.1. Three circuits carrying currents: I_1, I_2 and I_3 are joined in parallel. $I_1 = 4A$, $I_2 = 6A$ *lagging* I_1 by 30° and $I_3 = 2A$ leading I_1 by 90°. Find by a phasor construction drawn to scale the resultant current and its phase angle with reference to current I_1 (2 decimal places).

9.2. A sinusoidal, 25Hz A.C. voltage has a maximum value of 282.8V. Find the time interval, after the zero value, when the voltage wave reaches (a) its first (3 decimal places) and (b) its second instantaneous value of 200V (2 decimal places).

9.3. A sinusoidal e.m.f. of 100V maximum value is connected in series with an e.m.f. of 80V maximum value, lagging 60° behind the 100V e.m.f. Determine the maximum value of the resultant voltage (1 decimal place) and its phase angle with respect to the 100V e.m.f. (4 decimal places).

9.4. The table below gives instantaneous A.C. current values which vary smoothly over 1 half cycle.

Time (milliseconds)	0	1	2	3	4	5	6	7
Current (amperes)	0	0.4	0.75	1.1	1.4	1.7	1.9	2.0
Time (milliseconds)	8	9	10					
Current (amperes)	1.8	1.3	0					

Plot the curve of current and find its r.m.s. value (3 decimal places). Calculate the power dissipated when the above current flows through a resistance of 8Ω (2 decimal places).

9.5. Three currents of peak values 10A, 17.32A and 20A respectively flow in a common conductor. The 17.32A current lags the 10A current by 90° and leads the 20A current by 60°. Draw a phasor diagram and find the resultant current value (2 decimal places) giving its phase relation with respect to the 10A current.

9.6. An alternating sine-wave voltage, having a peak value of 340V, is applied to the ends of a 24Ω resistor. Calculate the r.m.s. value of the current in the resistor (2

9.7. Represent by phasors, the following e.m.f.s:

$$e_1 = 100 \sin \omega t,$$

$$e_2 = 50 \cos \omega t,\ e_3 = 75 \sin\left(\omega t + \frac{\pi}{3}\right), \text{ and}$$

$$e_4 = 125 \cos\left(\omega t - \frac{2\pi}{3}\right).$$

Determine by calculation the values of E and θ if:

$$e_1 + e_2 + e_3 + e_4 = E \sin(\omega + \theta).$$

9.8. Two alternators are coupled together to allow the phase angle between their generated e.m.f.s to vary. If the machines are connected in *series* and generate 100V and 200V respectively, find the total output voltage when the phase difference is: (a) zero, (b) 60°, (c) 90°, (d) 120° and (e) 180° (all 1 decimal place).

9.9. A stepped A.C. current wave has the following values over equal intervals of time.

Value (amperes)	4	6	6	4	2	0	0	−2	−4
Time interval (seconds)	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9 etc.

Plot the waveform and find what D.C. current value gives the same heating effect (2 decimal places).

9.10. The 50Hz induced e.m.f.s. in 4 separate coils of an A.C. generator are each of maximum value 4V and successively 10° out-of-phase. If these coils are connected in *series*, find by calculation and phasor construction, the resultant maximum value, expressed in the form $e = E_m \sin(\omega t + \theta)$, where θ is the angle of phase difference with respect to the first coil.

10

THE SERIES A.C. CIRCUIT

We now know a thousand ways not to build a light bulb.

Thomas Edison

The approach we will take in teaching the subject of series A.C. circuits will be to introduce the essential fundamentals first and then to add details as required. At this stage several new terms are mentioned and their relationships stated. It will be helpful to memorise these terms.

Impedance

For the A.C. circuit, conditions are comparable with those for Ohm's law discussed with the D.C. circuit. We will modify Ohm's law as it applies to the A.C. circuit; where current is directly proportional to applied voltage and *inversely* proportional to the 'opposition' or resistance of the circuit to the flow of current. This A.C. opposition is termed the circuit *Impedance* (symbol – Z; unit – Ohm) but is due to more than a circuit's simple D.C. ohmic resistance R. The difference between Z and R is now considered.

For an A.C. circuit, the current flowing is given by:

$$\text{Current } I = \frac{\text{Applied voltage}}{\text{Impedance}}$$

V (volts)

Note the 3 variations of the relationship. Thus:

$$I = \frac{V}{Z} \quad \text{or} \quad V = IZ \quad \text{or} \quad Z = \frac{V}{I}$$

For the D.C. circuit, it is known that $I = \frac{V}{R}$ where R is the circuit's ohmic resistance. If a straight conducting wire is connected to a D.C. supply of V volts, the measured current I will be $\frac{V}{R}$ amperes. However, if the same wire is then connected to an A.C. voltage supply, the measured current I will be $\frac{V}{Z}$ amperes and of the same magnitude as for the D.C. supply. In this case Z and R are equal as the impedance is due to resistance only. The circuit is said to be 'resistive' or 'non-inductive'.

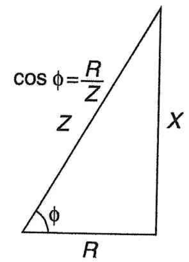
If the wire is then wound into a coil or solenoid and the same voltage V applied, the current will be *smaller*, i.e. the new impedance is greater than the ohmic resistance. Likewise if an iron core is inserted into the solenoid, the impedance will increase and it should be clear that the impedance is made up of ohmic resistance and an extra current-limiting quantity. This extra quantity is the *Reactance* (symbol – X; unit – Ohm). It is noted that the impedance is *not* given by a straightforward arithmetical summation of resistance R and reactance X but by a right-angled relationship where:

$$\text{Impedance} = \sqrt{\text{Resistance}^2 + \text{Reactance}^2}$$

$$\text{or } Z \text{ (ohms)} = \sqrt{R^2\text{(ohms)} + X^2\text{(ohms)}}$$

The relationship between R, X and Z is represented by an 'Impedance Triangle' shown in figure 10.1.

The angle ϕ is the 'phase angle' and cos ϕ is a measure of a circuit's *Power Factor*. The reactance X for a wound coil is of only one particular form, namely *inductive reactance*. If a coil with its associated magnetic field is used in an A.C. circuit, then its inductive reactance must be known, which requires knowledge of the *inductance* and the supply frequency. The term 'inductive reactance' is associated with an *inductor* coil – usually iron-cored. Alternative terms for an inductor are: *reactor*, *choke* or *coil*, where all of these devices store energy in their magnetic fields. The term reactor is often used for a large coil passing heavy currents strengthened to withstand the greater than usual electromagnetic forces.



▲ Figure 10.1

In contrast to inductive reactance there is also a *capacitive reactance*(symbol – X; unit – Ohm), a term associated with a capacitor. For comparison a capacitor stores energy in its electric field, while a resistor does not store energy but instead dissipates it as heat. The capacitor and capacitive reactance will be considered later, but immediate focus is now given to inductance and inductive reactance.

Example 10.1. An inductor has an ohmic resistance of 3Ω and an inductive reactance of 4Ω. If it is connected to a 20V A.C. supply, find the current which flows (1 significant figure) and the power factor (1 decimal place) at which the coil operates. Note the diagram (figure 10.2).

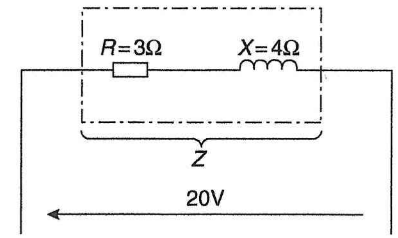
Here $R = 3\Omega$ and $X = 4\Omega$. Since $Z = \sqrt{R^2 + X^2}$

Then $Z = \sqrt{3^2 + 4^2 = 5\Omega}$

Current $I = \frac{V}{Z} = \frac{20}{5} = 4A$

The circuit power factor is given by $\cos\phi = \frac{R}{Z}$ (from the impedance triangle). Thus $\cos\phi = \frac{3}{5} = 0.6$ (lagging). The term 'lagging' is associated with an A.C. circuit containing inductive reactance.

Figure 10.2 represents the circuit. All the ohmic resistance is considered to be concentrated in a resistor R and the reactance in an inductor X, even though together they constitute the impedance Z of the inductor. The dotted rectangle represents the inductor and is omitted in future diagrams.



▲ Figure 10.2

Inductance

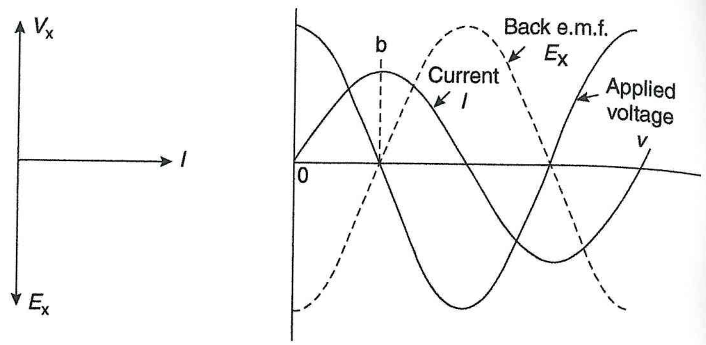
Basic electromagnetic induction theory covered in Chapter 7 showed that if magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit. Faraday's law shows that the induced e.m.f. is proportional to the rate of change of flux-linkages and this e.m.f. only exists while this flux-linkage change occurs. In an A.C. circuit, current changes continually, so the associated flux of a coil carrying an A.C. current also changes continually; flux-linkages change and an induced e.m.f. is continually generated. By Lenz's law this is a 'back e.m.f.' tending to *oppose* the change causing it.

When the current in an inductive circuit is made to change, due to the inductance, the current value will, at the instant of change, be controlled by more than just voltage and resistance. During the change or *transient* conditions, a back e.m.f. is generated and new conditions of voltage balance occur. For the A.C. circuit, current varies sinusoidally and changes constantly, so inductance provides a vital and continuous effect.

Inductive reactance

Imagine an inductor with no resistance and only inductance of value L Henries with an alternating voltage of V volts applied giving a current of I amperes. Figure 10.3 represents the current and voltage conditions.

Assume the current of I amperes (r.m.s. value) to be sinusoidal. As induced e.m.f. $= \frac{LI}{t}$ or $= L \times$ (rate of change of current), then at point 'a' the current value is zero, but increasing at its maximum rate, as the slope or gradient of the waveform is steepest here. Maximum induced e.m.f. occurs at this instant 'a' and as this is a 'back e.m.f.', by



▲ Figure 10.3

and the corresponding phasor diagrams are shown with I as the reference phasor. At point 'b' on the current waveform, no e.m.f. is induced as current is maximum and *not* changing at this instant. Thus there is a 90° phase difference between current and induced voltage (E_x) and a further 90° difference between the current and the applied voltage (V_x). *Note.* This condition applies to a circuit with inductance only.

Referring to figure 10.3, it is seen that, as current rises to its maximum value I_m in the first quarter cycle, flux-linkages LI_m are created $\left(\text{since } L = \frac{N\Phi}{I} \text{ or } N\Phi = LI\right)$. As current falls to zero in the second quarter cycle, linkages are destroyed. For the next half cycle the same number of linkages are created and destroyed. The change of flux-linkages in 1 cycle = $4LI_m$ and the change of flux-linkages in 1 second = $4fLI_m$ (where f is the frequency). The average value of induced e.m.f. = rate of change of flux-linkages.

$$\text{So average e.m.f.} = \frac{\text{Flux-linkages}}{\text{time}} = \frac{4fLI_m}{1} \text{ volts}$$

$$\text{Hence back e.m.f. } E_{Xav} = 4fLI_m \text{ volts.}$$

The supply voltage is equal yet opposite, its value being V_x (r.m.s.) or V_{Xav} (average). As r.m.s. values are preferred the following conversion is needed.

$$\text{Since } V_{Xav} = \frac{2}{\pi} V_{Xm} \quad \text{and} \quad V_{Xav} = 4fLI_m$$

$$\text{then } \frac{2}{\pi} V_{Xm} = 4fLI_m \quad \text{or} \quad V_{Xm} = 2\pi fLI_m$$

giving $0.707\,V_m = 2\pi fL \times 0.707I$ or $V = 2\pi fL\,I$

So the voltage drop in an inductor is proportional to the current and a constant which involves the circuit inductance and the supply frequency. This constant is given the name 'reactance' and as it is for an inductive circuit, we represent it with the symbol X and the suffix L.

Thus $X_L = 2\pi fL$ ohms which for a purely inductive circuit $V_x = IX_L$ where $X_L = 2\pi fL$.

Inductive reactance is measured in ohms and is proportional to both frequency and inductance. As resistance has been omitted, the phase relationship between reactive voltage drop IX_L and current is fixed at 90° so these quantities are in *quadrature* with respect to each other.

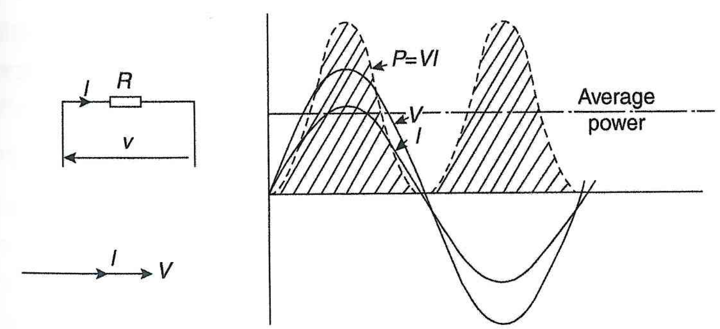
1. Circuit with pure resistance

The circuit conditions are illustrated (figure 10.4).

Assume a sinusoidal voltage of value $v = V_m \sin \omega t$ applied to a purely non-inductive resistor of R ohms. The applied voltage overcomes the ohmic voltage drop at each instant. We can write $i = \frac{v}{R}$ and since maximum current occurs when the voltage is a maximum: $I_m = \frac{V_m}{R}$.

$$\text{Since } \frac{v}{R} = \frac{V_m}{R} \sin \omega t \quad \therefore i = I_m \sin \omega t.$$

The circuit current is also sinusoidal and in-phase with the applied voltage. The phasor diagram is drawn as shown.



▲ Figure 10.4

R.m.s. values are deduced. Power at any instant is given by $p = vi$ or $p = V_m \sin \omega t \times I_m \sin \omega t = V_m I_m \sin^2 \omega t$ and using standard trigonometric identities:

$$= V_m I_m \frac{(1 - \cos 2\omega t)}{2} \quad \ldots *$$

$$\text{Average power } P = \text{Average value of } \left\{ \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2} \right\}$$

$$= \frac{V_m I_m}{2} - 0.$$

As the average value of a cosine wave across its cycle is zero. So $P = V_m/2I_m$.

$$\text{Hence } P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = VI \text{ or } P = VI \text{ (watts)}.$$

From the expression marked thus*, the power wave is a *periodic* quantity, always +ve and at **twice** the supply frequency. This is confirmed if the power wave is plotted by obtaining values of v and i for various instants in time and multiplying these together to give p, the power value at that instant. The resulting power wave is fully displaced above the horizontal and its maximum value is equal to $V_m I_m$. Being symmetrical, its average value is obtained from the distance its axis is displaced from the horizontal. This will be $\frac{V_m I_m}{2}$ and is a measure of average power.

$$\text{Thus: } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \text{ or } P = VI \text{ (watts)}.$$

For a resistive circuit power equals the product of voltage and current, but this is only true for non-inductive circuits. Generally, if an attempt is made to correlate power with voltage and current (or volt amperes of a circuit), the product of V and I must be multiplied by a so-called *power factor*.

For the condition of a purely resistive circuit, if we write $P = VI \times$ power factor, it is clear that the power factor will be 1, since the power factor is related to the ratio of resistance and impedance of a circuit and is obtained from $\frac{R}{Z}$. From a general circuit's impedance triangle: $\frac{R}{Z}$ is the cosine of the phase angle ϕ between circuit voltage and current with $\cos \phi = \frac{R}{Z} =$ the power factor.

The assumption made is that $P = VI \times$ power factor and can be rewritten as $P = VI \cos \phi$.

Further as this circuit is resistive $Z = R$ so $\cos \phi = \frac{R}{Z}$ and $\frac{R}{Z} = 1$, giving the condition of unity power factor as stated.

Note. The following deduction is also of value and identical to the D.C. circuit.

As $P = VI \cos \phi$ we can write $P = VI\frac{R}{Z} = \frac{V}{Z}IR = I \times I \times R$ or $P = I^2R$ (watts).

Example 10.2. An electric fire rated at 2kW power is connected to a 220V supply. Find the current which will flow and the resistance value of the fire element (1 decimal place).

An electric fire consists of a heating element which is purely resistive, so the circuit operates with unity power factor. Thus $\cos \phi = 1$ or the general expression $P = VI \cos \phi$ becomes $P = VI$

$$\text{Therefore } I = \frac{P}{220} = \frac{2000}{220} = 9.1\text{A}$$

$$\text{Again } Z = \frac{V}{I} = \frac{220}{9.1} = 24.2\Omega$$

$$\text{Here } Z = R \quad \therefore R = 24.2\Omega$$

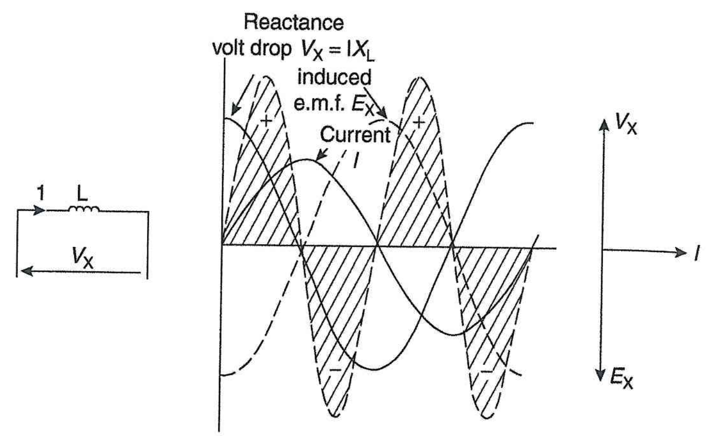
$$\text{Alternatively } P = I^2R \;\therefore R = \frac{P}{I^2} = \frac{2000}{9.1^2}$$

$$\text{Hence } R = \frac{2000}{82.81} = 24.2\Omega.$$

2. Circuit with pure inductance

This condition was introduced under inductance and reactance, but as further deductions are necessary, the circuit is illustrated (figure 10.5) of a coil with no resistance and an inductance of L Henries.

Assume a sinusoidal current given by $i = I_m \sin \omega t$ flows through a coil. As i varies sinusoidally, the magnetic field also varies and a sinusoidal self-induced back e.m.f. is set up *opposing* the applied voltage at each instant. Treatment of the A.C. circuit with inductance shows this 'back e.m.f.' is regarded as equivalent to a voltage drop caused by the current – a property termed inductive reactance (symbol – X_L; unit – Ohm). Thus we have $E_X = V_X = I \times X_L$. X_l is shown equal to $2\pi fL$. The associated phasor diagram is now



▲ Figure 10.5

considered with the waveform. E_X is the e.m.f. of self-inductance, displaced 90° behind the current I. V_X is the supply voltage and, being 90° ahead of the current, is 180° out-of-phase with E_X. V_X is thus always equal and opposite to E_X, illustrated by the waveforms and shown thus:

The self-induced e.m.f. is written mathematically as $e = -L\frac{di}{dt}$. By Lenz's law, as it at all times *opposes* the supply voltage, so we can write $v = L\frac{di}{dt}$. Thus $e = -L\frac{di}{dt} = -\frac{Ld(I_m \sin\omega t)}{dt} = \omega L I_m \cos\omega t$ or $e = \omega L I_m\left(\sin\omega t - \frac{\pi}{2}\right)$. Similarly v is deduced as $v = \omega L I_m\left(\sin\omega t + \frac{\pi}{2}\right)$.

Note. v is 180° ahead or in anti-phase with e.

As ωL is the reactance X_L, where $\omega = 2\pi f$, $e = E_m \sin\left(\omega t - \frac{\pi}{2}\right)$ and $v = V_m \sin\left(\omega t + \frac{\pi}{2}\right)$ giving the 90° phase displacement between current and voltage waves as shown. *Note.* $V_m = X_L I_m$. The relationship $X_L = 2\pi f L$ is a fundamental one and deduced earlier.

The power condition is deduced thus: Power at any instant is given by $p = vi$

$$\text{or } p = V_m \sin\left(\omega t + \frac{\pi}{2}\right) I_m \sin\omega t$$

$$= V_m I_m \sin\omega t \cos\omega t$$

$$= V_m I_m \frac{\sin 2\omega t}{2}$$

Thus instantaneous power $p = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \sin 2\omega t$

$$= VI \sin 2\omega t \cdots *$$

Average power P = Average of value of $VI \sin 2\omega t = 0$. As the average value of a sine wave is zero, a different result to a purely resistive circuit.

From the expression marked *, it is seen that the power wave is a sine wave of *twice* the supply frequency. The waveform is symmetrically disposed about the horizontal and the average value is zero. This is confirmed if the power wave is plotted from values obtained from the voltage and current waves. As the axis of the power wave lies along the horizontal, the average power used must be zero as the +ve halves of the power wave are exactly equal to the −ve halves. This indicates that if power is taken from the supply to establish a magnetic field associated with a coil, it is subsequently returned to the supply when the magnetic field collapses.

If the general expression $P = VI \times$ power factor is adopted for a circuit, the power factor here must equal zero as $P = 0$. If used in the form $P = VI\cos\phi$ then $\cos\phi = 0$ since $\cos\phi = \frac{R}{Z}$ if $R = 0$ so $\cos\phi = \frac{0}{Z} = 0$.

In summary a circuit with inductance only and no resistance is purely imaginary and has a zero power-factor working condition.

Example 10.3. A 220V, 50Hz supply is applied to an inductor of negligible resistance and the circuit current measured to be 2.5A. Find the coil inductance and the power dissipated (2 decimal places).

$$\text{As } Z = \frac{V}{I} \text{ then } Z = \frac{220}{2.5} = 88\Omega$$

$$\text{Now } R = 0. \therefore X_L = Z \text{ or } X_L = 88\Omega$$

$$\text{In addition as } X_L = 2\pi f L$$

$$\text{so } L = \frac{X_L}{2\pi f} = \frac{88}{2 \times 3.14 \times 50}$$

$$\text{or } L = 0.28\text{H}$$

Also as

$$R = 0 \text{ so } \cos\phi = \frac{0}{88} = 0$$

$$\therefore P = 220 \times 2.5 \times 0 = 0$$

Alternatively since $P = I^2R$

then $P = 2.5^2 \times 0 = 0$

3. Circuit with resistance and inductance in series

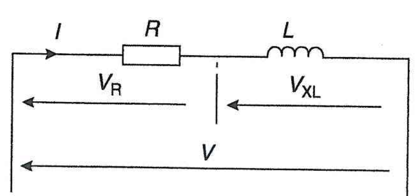
Consider a pure resistance and a pure inductance in series as shown (figure 10.6). It is noted that for a practical inductor, resistance and inductance are physically inseparable, and for illustrative purposes they are shown as 2 separate components R and L.

As the circuit conditions for both resistance and inductance have been introduced, if a current I flows, 2 voltage drops $V_R = IR$ and $V_{XL} = IX_l$ exist. Both of these form the applied circuit voltage and we assume that the applied voltage V consists of 2 components. One component V_R is the voltage needed to overcome the resistance voltage drop of the circuit and the other component V_{XL} is the voltage needed to overcome the reactance voltage drop or oppose the self-induced back e.m.f. As these components are at right angles to each other, as shown by consideration of circuit conditions 1 and 2, then the applied voltage is the resultant of the 2 components. The relationships being discussed is illustrated (figure 10.7) showing the relevant waveforms and appropriate phasors.

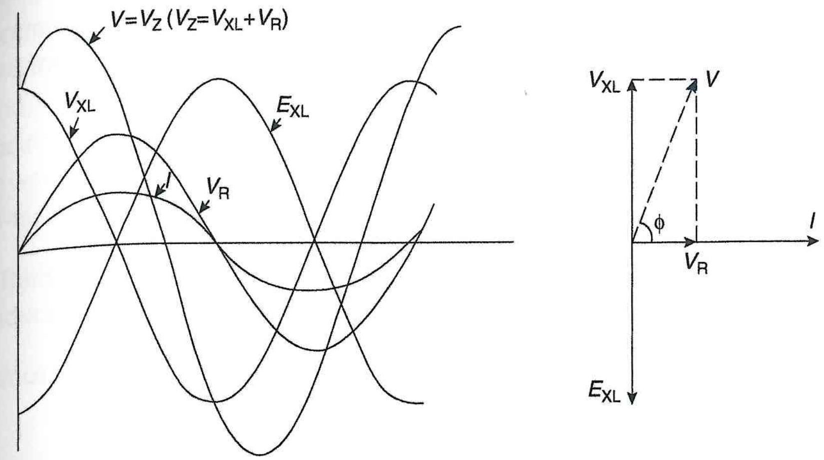
For the phasor diagram, current is common to both components (being a series circuit) and is used as the reference phasor. The resistance voltage drop $V_R = IR$ is in-phase with the current and drawn horizontally. The reactance voltage drop $V_{XL} = IX_L$ is at right angles to the current and is drawn vertically. The e.m.f. of self-inductance is also shown but is omitted as it serves no useful purpose on the phasor diagram.

The resultant of V_R and V_{XL} is V the applied voltage, and the current will lag V an angle φ which is the circuit's phase angle.

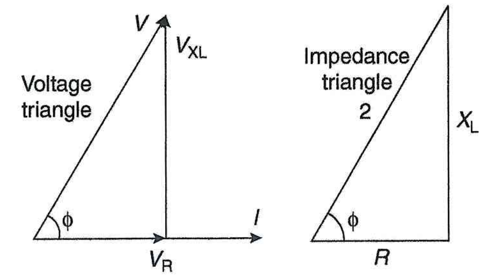
Simplification of a phasor diagram allows an appropriate 'voltage triangle' to be found, which if modified gives the 'impedance triangle' (figure 10.8).



▲ Figure 10.6



▲ Figure 10.7



▲ Figure 10.8

From the voltage triangle $V = \sqrt{V_R^2 + V_{XL}^2}$ and $\cos\phi = \frac{V_R}{V}$ Since $V_R = IR$ and $V_{XL} = IX_L$ this can be written:

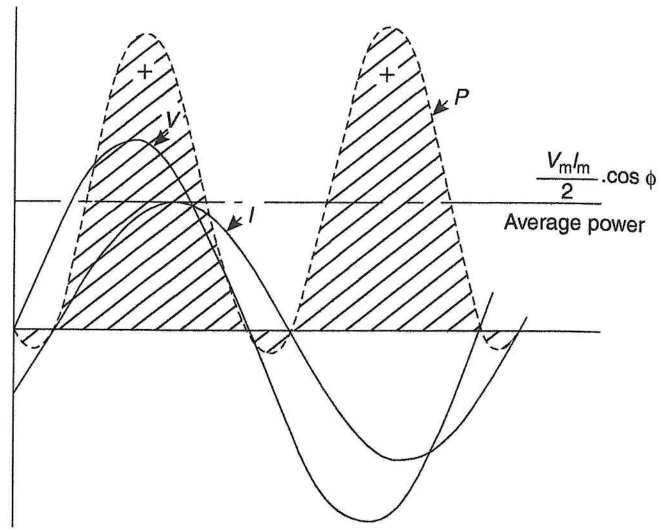
$$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

If Z is the circuit's equivalent impedance then $V = IZ$ or $Z = \frac{V}{I}$. So $IZ = I\sqrt{R^2 + X_L^2}$ and we have the impedance triangle relationship of:

$$Z = \sqrt{R^2 + X_L^2} \quad \text{and} \quad \cos\phi = \frac{R}{Z}$$

The power condition for the R, L series arrangement is as follows:

Figure 10.9 shows the basic waveforms of v and i, redrawn to allow the power wave to be obtained



▲ Figure 10.9

Let $v = V_m \sin \omega t$ be the applied voltage, and

$i = I_m \sin(\omega t - \phi)$ the circuit current lagging the voltage by an angle ϕ.

Then the instantaneous power $p = vi$

$= V_m \sin \omega t \times I_m \sin(\omega t - \phi)$

or $p = V_m I_m \sin \omega t \sin(\omega t - \phi)$

$= V_m I_m \left\{\frac{\cos\phi - \cos(2\omega t - \phi)}{2}\right\}$. Again using standard trigonometric identities:

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \{\cos\phi - \cos(2\omega t - \phi)\} \cdots *$$

Thus $p = VI\cos\phi - VI\cos(2\omega t - \phi)$.

The Average power P = Average of $VI\cos\phi$ – Average of $VI\cos(2\omega t - \phi)$

Whence $P = VI\cos\phi - 0$ as the average of a cosine wave is 0.

Note. $VI\cos\phi$ is a constant quantity, thus its *average* value is apparent.

In the expression $P = VI\cos\phi$, the power factor is already present, varying between limits of 0 and 1, to enable the extreme conditions of pure R or L to be satisfied. If we examine the power wave we see how power factor or $\cos\phi$ is involved with actual

displacement of a power wave axis above the horizontal. The expression marked * shows the power wave is periodic and at twice the supply frequency. It consists of +ve and –ve areas, showing that some power is returned to the supply, the amount depending on the operating circuit's power factor. The greater +ve net result of the power wave area, the greater the circuit's power consumption and the nearer unity the power-factor condition.

Note. The only component responsible for power consumption is the resistance and the deductions made previously are repeated here.

Thus since $P = I^2R$ and $I = \frac{V}{Z}$ we write:

$$P = I \times I \times R = \frac{V}{Z} IR \quad \text{or} \quad P = VI\frac{R}{Z} = VI\cos\phi.$$

Power factor is considered further later, but it is pointed out that the product VI is often referred to as the circuit's 'volt amperes' and suggests 'apparent power'. P, we know, is the 'true or active component of power', so we have the relation:

True power = 'Apparent power' × cos ϕ hence the name 'power factor' for cos ϕ.

Example 10.4. A circuit has a resistance of 25Ω and an inductance of 0.3H. If it is connected to a 230V, 50Hz supply, find the circuit current (2 decimal places), the power factor (3 decimal places) and the power dissipation (3 significant figures).

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.3 = 94.2\Omega$$

$$Z = \sqrt{25^2 + 94.2^2} = 97.5\Omega$$

$$I = \frac{230}{97.5} = 2.36\text{A}$$

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{25}{97.5} = 0.256 \text{ (lagging)}$$

$$P = VI\cos\phi = 230 \times 2.36 \times 0.256 = 139\text{W}$$

$$\text{or } P = I^2R = 2.36^2 \times 25 = 139\text{W}.$$

Note the word 'lagging' introduced after the power-factor figure, indicating a current 'lags' or 'leads' the voltage. The latter is possible, but it is assumed that inductive circuits always operate with a **lagging** power factor. The term is used for a circuit current with respect to the applied voltage, i.e. current lags voltage.

Example 10.5. A coil of wire dissipates 256W when a D.C. current of 8A flows. If the coil is connected to an alternating applied voltage of 120V, the same current flows. Find the resistance (1 significant figure) and impedance of the coil (2 significant figures) and the power dissipated on A.C. (3 significant figures).

D.C. condition. As $I = 8$A and $P = 256$W, applied voltage $= \frac{256}{8} = 32$V. Coil resistance must be $\frac{32}{8} = 4\Omega$

A.C. condition. As $I = 8$A and applied voltage is 120V then coil impedance $= \frac{120}{8} = 15\Omega$

Resistance is, as for the D.C. case $= 4\Omega$

Power dissipated $= I^2R$ or $8^2 \times 4 = 256$W, as for the D.C. condition.

The last part is solved by $P = VI \cos\phi = 120 \times 8 \times \cos\phi$ and $\cos\phi$ obtained from $\frac{R}{Z}$

So $\cos\phi = \frac{4}{15} = 0.266$ (lagging)and $P = 960 \times 0.266 = 255.4$W

Capacitance

A complete treatment of the capacitor and its property of capacitance was discussed fully in Chapter 8. However, the capacitor is associated with A.C. circuits so it is necessary to revise capacitance briefly, before proceeding with A.C. theory.

If 2 plate conductors, arranged as plates, are separated by insulation and connected to a D.C. voltage, then at the instant of connection, a current flows. This current is of maximum value at the instant of switching on but dies away to zero. This is termed a 'charging' current and explained by considering the insulation to be in a state of electrical stress likened to a 'back e.m.f.' which builds up in a capacitor to oppose the supply voltage. Once a capacitor is charged and the voltage built up, its presence is apparent if the supply voltage is lowered, where the back e.m.f. causes a current to flow in the reverse direction, i.e. it is a 'discharging' current.

As an alternating voltage varies all the time (rising or falling), it follows that, if it is applied to a capacitor or a capacitive circuit, an *exchange* of A.C. current will result. As the voltage across the capacitor plates rises, a charging current results and as the voltage falls, a discharging current results. Current magnitude depends on the circuit's

capacitive reactance (symbol – X_C; unit – Ohm) a term corresponding to the inductive reactance with units as discussed previously.

Note. Current only flows if the voltage across the plates is changing (see the expression $Q = C/V$ where Q is the charge, V the voltage and C the capacitance). The change in P.D. across the plates from 0 to V volts occurs when a switch is closed. If the P.D. increases by v volts in t seconds and i is the average charging current, we can write:

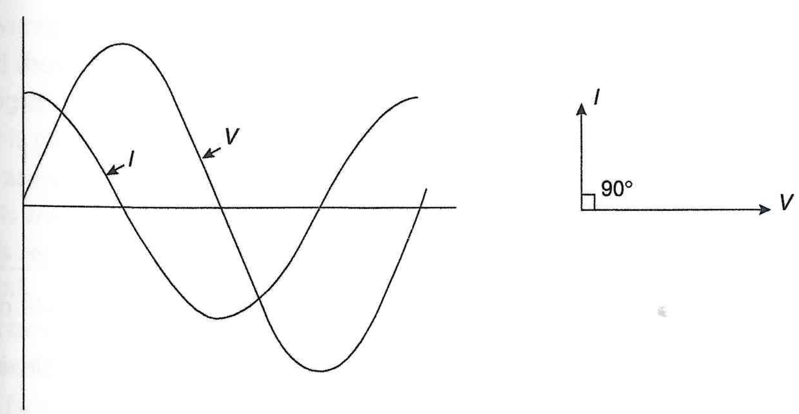
$$q = Cv \text{ or it} = Cv \text{ or } i = \frac{Cv}{t}$$

For very short instants in time, the above will become:

$$i = \frac{Cdv}{dt}$$

Capacitive reactance

The action of a capacitor when an alternating voltage is applied across its plates is now considered (figure 10.10). As voltage rises from 0 to V_m in a quarter cycle, the charge on the plates rises to CV_m, as $Q = CV$. During the next quarter cycle, the charge falls to zero. For the next quarter cycle, the charge falls further to $-CV_m$ and returns to zero for the last quarter of a cycle. The total change of charge for a complete cycle is thus $4CV_m$ and this occurs f times a second. The average current during this time is $\frac{Q}{t}$



▲ Figure 10.10

$$\text{or } I_{av} = \frac{4fCV_m}{t} = 4fCV_m \text{ since } t \text{ is 1 second.}$$

For a sine wave $I_m = \frac{\pi}{2} \times I_{av}$. Hence $I_m = \frac{\pi}{2} \times 4fCV_m$

$= 2\pi fCV_m$ or in r.m.s. values $0.707\, I_m = 2\pi fC \times 0.707\, V_m$ and $I = 2\pi fCV$.

Hence $\frac{V}{I} = \frac{1}{2\pi fC} = X_C$. X_C is the capacitive reactance and the expression is in step with that developed for inductive reactance X_L.

Thus $X_C = \frac{1}{2\pi fC}$ mega ohms or $\frac{10^6}{2\pi fC}$ ohms with C in microfarads.

X_c is usually measured in ohms. It is noted that current leads voltage by 90° as maximum current occurs at the instant of maximum rate of charge of voltage. If a phasor diagram is drawn, the current phasor I is 90° ahead of the applied voltage phasor V.

These conclusions be shown thus:

Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across a capacitor's plates.

Since $i = C\frac{dv}{dt}$ then $i = \frac{Cd(V_m \sin \omega t)}{dt}$ or $i = C\omega V_m \cos \omega t$

$= C\omega V_m \sin\left(\omega t + \frac{\pi}{2}\right)$. Capacitor current is in quadrature (in step) with the voltage, and is also sinusoidal. If $\frac{1}{\omega C}$ is the capacitance reactance and equal to X_c then $i = \frac{V_m}{X_c}\sin\left(\omega t + \frac{\pi}{2}\right)$ and i becomes a maximum when the wave becomes a maximum or $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$.

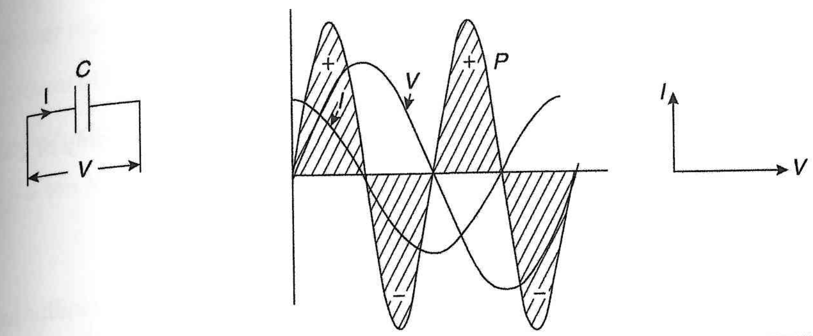
Thus $I_m = \frac{V_m}{X_c}$ or $0.707\, I_m = \frac{0.707V_m}{X_c}$ giving $I = \frac{V}{X_c}$

Summarising $V = IX_C$ (as for the inductive circuit), except that $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$.

The current leads voltage by 90° and unlike the inductive circuit, where it was said that an inductor without resistance was not possible, a capacitor with negligible resistance and thus circuit condition 4, as set out below, can exist and is practical.

4. Circuit with pure capacitance

The circuit diagram is shown (figure 10.11) and the waveforms and phasor diagram repeated, so the power condition may be considered.



▲ Figure 10.11

The power at any instant $p = vi$

$$\text{or } p = V_m \sin \omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$V_m I_m \frac{\sin 2\omega t}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \sin 2\omega t \text{ again using standard trigonometric relationships.}$$

$$= VI \sin 2\omega t.^*$$

Thus average power $P = 0$ since the average of a sine wave is zero. The expression marked thus * shows the power wave is periodic and at twice the signal frequency. The diagram also shows power to be +ve at the times when voltage is increasing and energy is put into a capacitor's electrostatic field. When voltage decreases power is shown as −ve, i.e. energy is recovered from the field as the capacitor discharges so no power is wasted. The power wave is symmetrical about the axis and the circuit power factor is zero. Thus if the expression $P = VI \cos \phi$ or $P = VI \times$ power factor is applied to this condition, $\cos \phi = 0$, as from $\cos \phi = \frac{R}{Z} = \frac{0}{Z} = 0$, there being no resistance.

Example 10.6. A capacitor of value 200µF is connected across a 220V, 50Hz supply mains. Find the current (1 decimal place) which would be recorded and the circuit impedance (1 decimal place).

$$\text{Here } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}}\,\Omega$$

$$X_C = 15.92\Omega$$

$$\text{Current is given by } \frac{V}{X_C} \text{ or } I = \frac{220}{15.92} = 13.8\text{A}$$

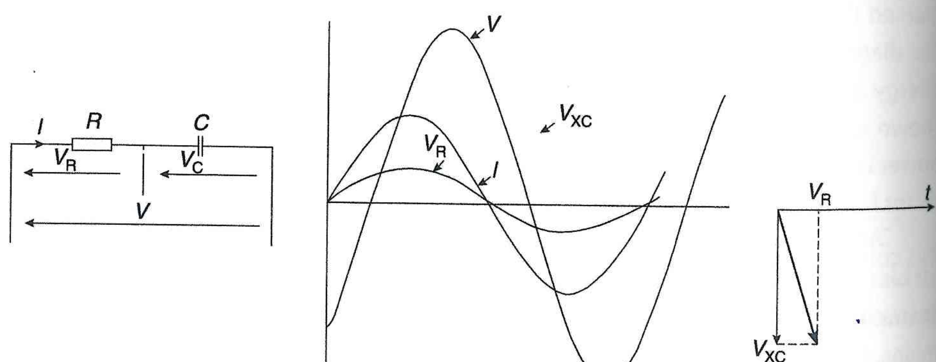
Since there is no circuit resistance impedance is made up of reactance only or $Z = X_C = 15.92\Omega$.

5. Circuit with resistance and capacitance

Figure 10.12 illustrates the circuit conditions and the technique used is similar to that used for the inductive circuit of condition 3.

The applied voltage V is resolved into 2 components V_R and V_X. One component V_R overcomes the resistance voltage drop due to the passage of current I, and the other component V_{XC} maintains the charging current of the capacitor and is at all times equalled and sustained by the internal stress voltage. As seen from condition 4, there is a 90° phase displacement between V_{XC} and I. If current is used as the reference for the waveform and phasor diagram, as it is common to R and C (being a series circuit), then the conditions shown can be deduced. If the voltage triangle (shown heavy) is extracted, the impedance triangle and relationships can be found:

$$V = \sqrt{V_R^2 + V_{XC}^2} = \sqrt{(IR)^2 + (IX_c)^2} = I\sqrt{R^2 + X_C^2}$$



▲ Figure 10.12

If Z is the circuit impedance then $\frac{V}{I} = Z = \sqrt{R^2 + Xc^2}$

or $Z = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$ As before $\cos\phi = \frac{R}{Z}$

The power relation follows the form already used several times.

Thus power at instant $p = vi$

where $i = I_m \sin \omega t$ and $v = V_m \sin(\omega t - \phi)$

then $p = Vm\, Im \sin \omega t \sin(\omega t - \phi)$

$$= V_m I_m \left\{\frac{\cos\phi - \cos(2\omega t - \phi)}{2}\right\}$$

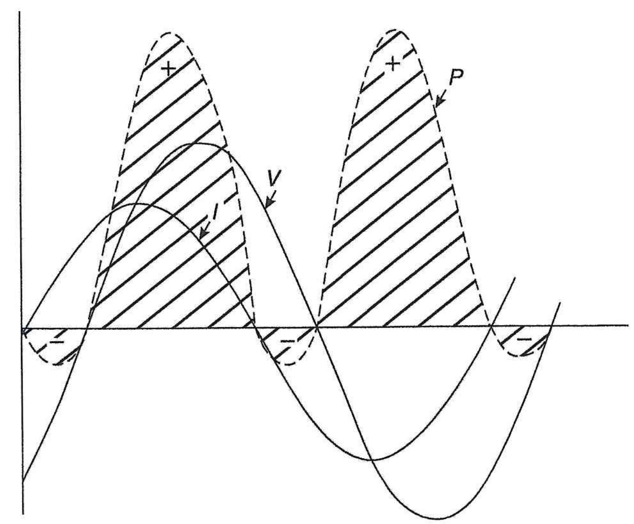
or $p = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \{\cos\phi - \cos(2\omega t - \phi)\}$ using standard trigonometric relationships.

$$= VI\cos\phi - VI\cos(2\omega t - \phi)$$

Average power P = Average of $VI\cos\phi$ – Average of $VI\cos(2\omega t - \phi)$ or $P = VI\cos\phi - 0$. Since the average of a cosine wave is zero.

Thus $P = VI\cos\phi$ – an expression encountered previously. If the power wave is plotted as before it will be as shown in figure 10.13.

The power wave is seen as before to be periodic at twice the signal frequency and consist of +ve and –ve sections. The average value is found from the amount by which



▲ Figure 10.13

the axis is displaced above the horizontal and this displacement varies with a circuit's power factor. Conditions 1 and 4 are also covered. If $X_C = 0$ and the circuit is purely resistive $\cos\phi = 1$ and the wave is fully displaced above the horizontal. If $R = 0$ and the circuit is purely capacitive then $\cos\phi = 0$ and the wave is symmetrical about the horizontal giving $P = 0$. As for the inductive circuit, the only component responsible for dissipation of power is resistance.

As before $P = I^2 R$ or $P = I \times I \times R = VI \dfrac{R}{Z}$

and $P = VI \dfrac{R}{Z}$ or $P = VI\cos\phi$ as deduced.

Example 10.7. A 500W, 100V bulb is connected across 250V, 50Hz mains A.C. supply. Find the value of the capacitor required to be connected in *series* (3 significant figures).

Current taken by bulb is $\dfrac{500}{100} = 5\text{A}$.

Resistance of lamp $= \dfrac{100}{5} = 20\Omega$

On 250V, impedance of the circuit will be $\dfrac{250}{5} = 50\Omega$

$$\text{Thus } X_C = \sqrt{50^2 - 20^2}$$

$$= 45.8\Omega$$

$$\text{Again } X_C = \frac{1}{2\pi fC}$$

$$\text{or } C = \frac{1}{2\pi f X_c}$$

$$\therefore C = \frac{1}{2\pi \times 50 \times 45.8} F$$

giving $C = 69.5\mu\text{F}$.

The Series Circuit

From the various circuit conditions considered, cases 1 to 5, we see the general techniques used for a series circuit. The phasor diagram is easily drawn with current

used as the reference phasor. From this diagram circuit relationships and expressions are deduced.

Inductive impedances in series

Figure 10.14 shows the circuit arrangement and the relevant phasor diagram.

Impedances A and B consist of both resistances and reactances: R_A, R_B, X_A and X_B ohms respectively, connected in series. From the phasor diagram we deduce an expression for the total circuit impedance Z noting that it is **not** equal to $Z_A + Z_B$.

Using the diagram we have:

$$V = \sqrt{V_R^2 + V_X^2} = \sqrt{(V_{RA} + V_{RB})^2 + (V_{XA} + V_{XB})^2}$$

$$= \sqrt{(IR_A + IR_B)^2 + (IX_A + IX_B)^2}$$

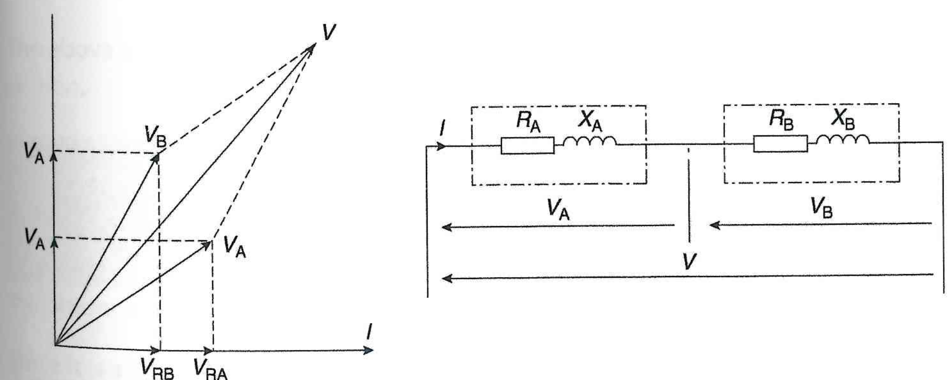
$$\text{or } V = I\sqrt{(R_A + R_b)^2 + (X_A + X_B)^2}$$

If Z is the equivalent circuit impedance then:

$$Z = \frac{V}{I} = \sqrt{(R_A + R_B)^2 + (X_A + X_B)^2}$$

or summarising, for more than 2 inductive impedances:

$$Z = \sqrt{(R_A + R_B + R_c \cdots)^2 + (X_A + X_B + X_c \cdots)^2}$$



▲ Figure 10.14

Also the power factor is given by:

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} = \frac{(R_A + R_B + \cdots)}{Z}$$

The example below shows how simply this expression can be adapted for practical use.

Example 10.8. Two coils A and B are connected in series to 50Hz mains A.C. supply. The current is 1A and the voltage across each coil is measured to be 45V and 70V respectively. When the coils are connected to a D.C. supply, the current is also 1A, but the voltages across the coils are now 20V and 40V respectively. Find the impedance, reactance and resistance of each coil, the total circuit impedance, the applied A.C. voltage and the power factor of the complete circuit.

On D.C.

$$R_A = \frac{20}{1} = 20\Omega$$

$$R_B = \frac{40}{1} = 40\Omega$$

On A.C.

$$Z_A = \frac{45}{1} = 45\Omega$$

$$Z_B = \frac{70}{1} = 70\Omega$$

Then $X_A = \sqrt{45^2 - 20^2} = 40.3\Omega$

Also $X_B = \sqrt{70^2 - 40^2} = 57.4\Omega$

Total $R = 20 + 40 = 60\Omega$

Total $X = 40.3 + 57.4 = 97.7\Omega$

Total impedance $Z = \sqrt{60^2 + 97.7^2} = 114\Omega$

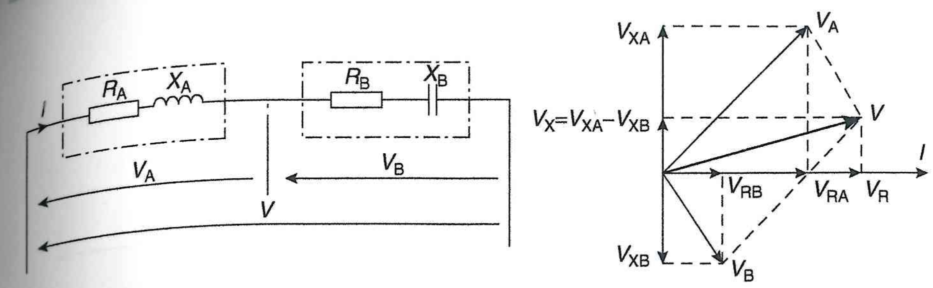
Applied voltage $= 114 \times 1 = 114V$

Circuit power factor $= \frac{R}{Z} = \frac{60}{114} = 0.53$ (lagging)

Inductive and capacitive impedances in series

Figure 10.15 shows the following arrangement:

From the phasor diagram we deduce the expression for the total circuit impedance Z. It is noted that although V_{RA} and V_{RB} are in-phase and can be added, V_{XA} and V_{XB} are in



▲ Figure 10.15

anti-phase and the resultant of the vertical phasors are obtained by *subtraction*. Thus from the resulting final diagram (shown heavy):

$$V = \sqrt{V_R^2 + V_X^2} = \sqrt{(V_{RA} + V_{RB})^2 + (V_{XA} - V_{XB})^2}$$

$$= \sqrt{(IR_A + IR_B)^2 + (IX_A - IX_B)^2}$$

$$= I\sqrt{(R_A + R_B)^2 + (X_A - X_B)^2}$$

If Z is the equivalent circuit impedance then:

$$Z = \frac{V}{I} = \sqrt{(R_A + R_B)^2 + (X_A - X_B)^2}$$

$$\text{Summarising } Z = \sqrt{(R_A + R_B)^2 + (X_A - X_B)^2}$$

Also for the circuit, the power factor

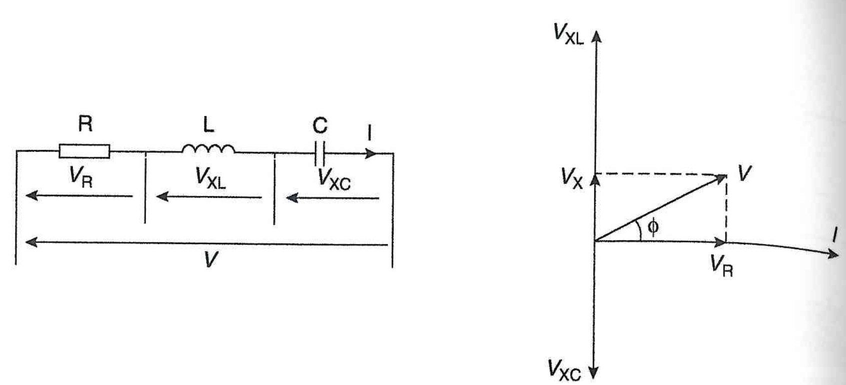
$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \text{ or, } \cos\phi = \frac{R_A + R_B}{Z}$$

The above 2 circuit conditions give rise to the general series circuit set out in the next section.

The general series circuit

From work already done on circuit theory, a fundamental expression is deduced from the phasor diagram (figure 10.16).

Since it is a series circuit, current is common and is used as the reference phasor. Note this circuit condition is similar to that of inductive and capacitive impedances in series,



▲ Figure 10.16

except that all the circuit resistance is considered within one resistor *R*. Then for the phasor diagram.

$V_R = IR$ and is in-phase with the current

$V_{XL} = IX_L$ and is 90° ahead of I

$V_{XC} = IX_C$ and is 90° behind I

V_{XL} and V_{XC} are 180° out-of-phase or anti-phase and a phasor difference can be obtained where $V_x = V_{XL} - V_{XC}$. Here V_{XL} is assumed greater than V_{XC}.

Further deduction from the diagram is possible, thus:

$$V = \sqrt{V_R^2 + V_X^2} = \sqrt{(V_R^2 + (V_{XL} - V_{XC})^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

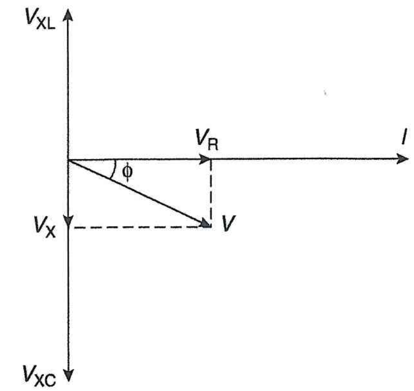
$$= I\sqrt{(R)^2 + (X_L - X_C)^2} \text{ or } \frac{V}{I} = \sqrt{(R)^2 + (X_L - X_C)^2}$$

If Z is taken as the equivalent impedance of the circuit then:

$$Z = \frac{V}{I}$$

$$\text{or } \frac{V}{I} = Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\text{Thus } Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$



▲ Figure 10.17

Example 10.9. A series circuit is made up of an inductor of resistance 20Ω and inductance 0.08H, connected in series with a 100μF capacitor. If the circuit is connected across 200V, 50Hz mains A.C. supply, find (a) the circuit current (1 decimal place) and (b) its power factor (2 decimal places).

$$\text{Here } X_L = 2\pi fL = 2\pi \times 50 \times 0.08 = 25.2\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.75\Omega$$

Resultant reactance $= X = X_L - X_C = 25.2 - 31.75$

$$= -6.55\Omega$$

The –ve sign denotes that the capacitive reactance predominates and the phasor diagram will be as shown (figure 10.17).

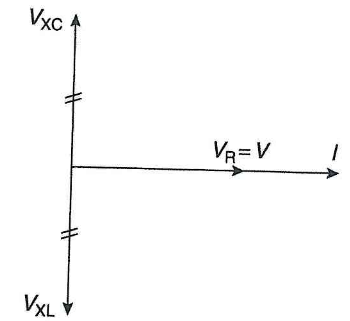
From the diagram as before $V = \sqrt{V_R^2 + V_X^2}$

$$\text{or } Z = \sqrt{R^2 + X^2}$$

$$\therefore Z = \sqrt{20^2 + 6.55^2} = 21\Omega$$

$$\text{The circuit current} = \frac{200}{21} = 9.5\text{A}.$$

The power factor is given by $\cos\phi = \frac{R}{Z} = \frac{20}{21} = 0.95$ (leading), i.e. the current leads the voltage as the circuit capacitive reactance dominates.



▲ Figure 10.18

SERIES RESONANCE. From the phasor diagram (figure. 10.18) a state can occur when V_{XL} and V_{XC} are equal, and also V_R is equal to the supply voltage V. An examination of the general series circuit expression

$$Z = \sqrt{R^2 + \left\{2\pi fL - \frac{1}{2\pi fC}\right\}^2}$$

shows that an unusual condition arises when $2\pi fL = \frac{1}{2\pi fC}$ in magnitude, i.e. the capacitive reactance equals the inductive reactance. Under this condition $Z = R$ and the circuit is said to be in a 'state of resonance'. The current passed will be limited by the value of R only and although large voltages may be present across components L and C, their effect on the supply voltage V is not evident. Series resonance is used to advantage for practical purposes, especially in radio.

Since inductive and capacitive reactances vary with frequency, a point exists where the reactances are equal at a particular frequency known as the resonant frequency.

$$\text{Since } X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$\therefore \text{Resonant frequency } f = \frac{1}{2\pi\sqrt{LC}}$$

Example 10.10. A 4μF capacitor is connected in series with a coil of inductance 39.6mH and resistance 40Ω to a 200V mains A.C. supply. Calculate (a) the frequency when the current is a maximum value (1 significant figure) and (b) the P.D. across the capacitor at this frequency (1 decimal place).

When current is maximum Z is minimum.

i.e. $Z = R$ and circuit is in its resonant condition

$$\therefore \text{Maximum current } I = \frac{V}{R} = \frac{200}{40}$$

$I = 5$A at resonance

(a) Resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{39.6\times10^{-3}\times4\times10^{-6}}}$$

$$F = 399.89 \text{ Hz (say 400Hz)}$$

$$X_c = \frac{1}{2\ fC} = \frac{10^6}{2\ \times400\times4} \quad X_c = 99.5\Omega$$

(b) P.D. across capacitor $V_{XC} = IX_C = 5\times99.5$

$$= 497.5 \text{ volts}$$

Example 10.11. A coil of unknown inductance and resistance is connected in series with a 25Ω, non-inductive resistor across 250V, 50Hz mains A.C. supply. The P.D. across the resistor is 150V and across the coil 180V. Calculate the resistance and inductance of the coil and also find its power factor.

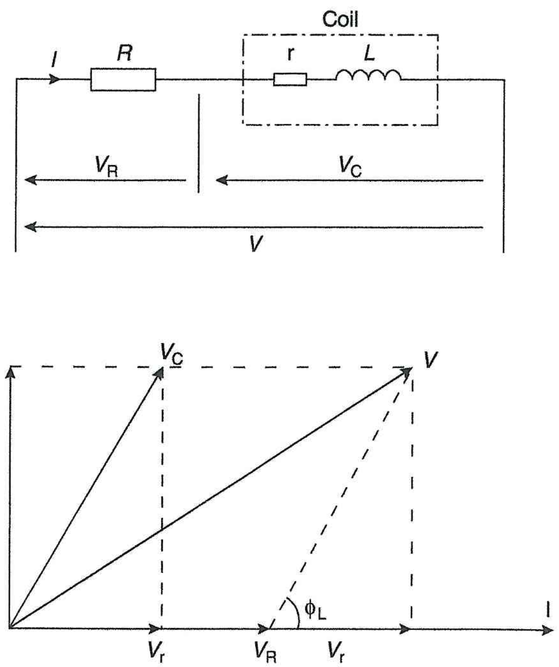
The circuit and phasor diagrams are shown (figure 10.19).

This example is important as it involves basic fundamentals but has a simple solution. The phasor diagram is first explained with the various voltage drops considered separately. V_R is the voltage drop across resistor $R = IR$. V_C is the voltage drop across the coil and is the resultant of 2 voltage drops: V_R across the resistance of the coil $= IR$ and V_L across the reactance of the coil $= IX_L$. V_R is in-phase with current and V_L is 90° *ahead* of the current. From the phasor diagram it is clear that V is the resultant of V_C and V_R and that the expression given for simple phasor summation can be applied here.

$$\text{Thus } V = \sqrt{V_R^2 + V_C^2 + 2V_RV_C\cos\phi_L}$$

$$\text{or } 250^2 = 150^2 + 180^2 + 2\times150\times180\times\cos\phi_L$$

$$\therefore 62\,500 = 22\,500 + 32\,400 + 54\,000\cos\phi_L$$



▲ **Figure 10.19**

$$\text{or } 54000\cos\phi_L = 62500 - 54900$$

$$\cos\phi_L = \frac{7600}{54\,000} = 0.141(\text{lagging})$$

$$\text{The current flowing } = \frac{150}{25} = 6\text{A}$$

$$\text{The impedance of the coil} = \frac{180}{6} = 30\Omega$$

$$\text{Resistance of coil} = Z\cos\phi_L = 30 \times 0.141 = 4.23\Omega$$

$$\text{Reactance of coil} = \sqrt{30^2 - 4.23^2} = 29.7\Omega$$

$$\text{Inductance of coil} = \frac{29.7}{2\times\pi\times 50} = 0.0945\text{H}$$

$$\text{Power factor of coil} = \cos\phi_L = 0.141 \text{ (lagging).}$$

Example 10.12. A coil of resistance 10Ω and inductance 0.1H is connected in series with a capacitor of capacitance 150μF, across a 200V 50Hz mains A.C. supply. Calculate (a) the inductive reactance, (b) the capacitive reactance, (c) the circuit impedance, (d) the circuit current, (e) the circuit power factor, (f) the voltage drop across the coil, (g) the voltage drop across the capacitor (all 1 decimal place).

(a) Inductive reactance $= 2\pi fL = 2 \times \pi \times 50 \times 0.1 = 31.4\Omega$

(b) Capacitive reactance $= \frac{1}{2\pi fC} = \frac{10^6}{2\times\pi\times 50\times 150} = 21.2\Omega$

(c) Resultant reactance $= 31.4 - 21.2 = 10.2\Omega$ (inductive)

Impedance $= \sqrt{R^2 + X^2} = \sqrt{10^2 + 10.2^2} = 14.3\Omega$

(d) Circuit current $= \frac{200}{14.28} = 14.0\text{A}$

(e) Power factor $= \frac{10}{14.28} = 0.7$ (lagging) – As the circuit reactance is overall inductive

(f) Impedance of coil $= \sqrt{10^2 + 31.4^2} = 33.0\Omega$

Voltage drop across coil $= 14 \times 33 = 462\text{V}$

(g) Voltage drop across capacitor $= 14 \times 21.2 = 296.8 = 297.0\text{V}$.

Note. Although resonance is *not* occurring here, the condition is very close to this and large voltages can build up across circuit components. Thus the fact that the voltages across the coil and capacitor are larger than the supply voltage agrees with theory.

Practice Examples

10.1. A circuit has a resistance of 3Ω and an inductance of 0.01H. The voltage across its ends is 60V and the A.C. frequency is 50Hz. Calculate (a) the impedance (2 decimal places), (b) the power factor (2 decimal places) and (c) the power absorbed (1 decimal place).

10.2. A 100W lamp for a 100V supply is placed across a 220V, 50Hz supply. What value of resistance must be placed in series with it so that it will work under its proper conditions (3 significant figures)? If a coil is used instead of the resistor and if the coil resistance is small compared to its reactance, what is the coil inductance (3 decimal places)? What is the total power absorbed in each case (2 significant figures)?

10.3. An inductive load takes a current of 15A from a 240V, 50Hz supply and the power absorbed is 2.5kW. Calculate (a) the load's power factor (3 decimal places), (b) the load's resistance (1 decimal place), reactance (2 significant figures) and impedance (1 decimal place). Draw a phasor diagram showing the voltage drops and the current components.

10.4. Two inductive circuits A and B are connected in series across 230V, 50Hz mains. The resistance values are A 120Ω and B 100Ω. The inductance values are A 250mH and B 400mH. Calculate (a) the current, (b) the phase difference between the supply voltage and current, (c) the voltages across A and B and (d) the phase difference between these voltages (all 3 decimal places).

10.5. Two coils are connected in series. When 2A D.C. is passed through a circuit, the voltage drop across the coils is 20V and 30V respectively. When passing 2A A.C. at 40Hz, the voltage drop across the coils is 140V and 100V respectively. If the 2 coils in series are connected to a 230V, 50Hz mains A.C. supply, find the current flowing (2 decimal places).

10.6. A simple transmission line has a resistance of 1Ω and a reactance at normal frequency of 2.5Ω. It supplies a factory with 750kW, 0.8pf (lagging) at a voltage of 3.3kV. Determine the voltage at the generator (3 significant figures) and its power factor (3 decimal places). Find also the generator output (3 significant figures) and draw the phasor diagram.

10.7. A non-inductive resistor of 8Ω is connected in series with an inductive load and the combination placed across a 100V supply. A voltmeter (drawing negligible current) is connected across the load and then across the resistor and indicates 48V and 64V respectively. Calculate (a) the power absorbed by the load (1 decimal place), (b) the power absorbed by the resistor (3 significant figures), (c) the total power taken from the supply (1 decimal place) and (d) the power factors of the load and whole circuit (2 decimal places).

10.8. A circuit, consisting of a resistor and a capacitor connected in series across a 200V, 40Hz mains A.C. supply, takes a current of 6.66A. When the frequency is increased to 50Hz and the voltage maintained at 200V, the current becomes 8A. Calculate the values of resistance (2 decimal places) and capacitance (3 significant figures) and sketch the phasor diagram (not to scale) for either frequency.

10.9. A coil, having an inductance of 0.5H and a resistance of 60Ω, is connected in series with a 10μF capacitor. This combination is now connected across a sinusoidal supply and it is found that at resonance, the P.D. across the capacitor is 100V. Calculate the circuit current flowing under this condition (3 decimal places). Sketch the phasor diagram (not to scale).

10.10. A coil has a resistance of 400Ω and, when connected to a 60Hz main A.C. supply, an impedance of 438Ω. If the coil is then connected in series with a 40μF capacitor and a P.D. of 200V, 50Hz is applied to the circuit, find the current (3 decimal places), the P.D. across the capacitor (1 decimal place) and the P.D. across the coil (3 significant figures).

11

A.C. PARALLEL CIRCUITS AND SYSTEMS

More than the diamond Koh-i-noor, which glitters among their crown jewels, they prize the dull pebble which is wiser than a man, whose poles turn themselves to the poles of the world, and whose axis is parallel to the axis of the world.

Ralph Waldo Emerson

A.C. Circuits

Power in the A.C. circuit

From the various circuit conditions considered in Chapter 10, for various series combinations of resistors, and inductive and capacitative reactance, it was seen that the current flowing was sinusoidal and displaced from the applied sinusoidal voltage by an angle φ, termed the *phase angle* of the circuit. The general expressions were:

For voltage $v = V_m \sin \omega t$ and for current $i = I_m \sin(\omega t - \phi)$ with a *lagging* phase angle assumed for convenience.

The instantaneous power $p = vi = V_m I_m \sin \omega t \sin(\omega t - \phi)$

or $p = V_m I_m \left\{\dfrac{\cos\phi - \cos(2\omega t - \phi)}{2}\right\} = VI\cos\phi - VI\cos(2\omega t - \phi)$

and Average power $P = VI\cos\phi - 0$ or $P = VI\cos\phi$

VI is often called a circuit's *Apparent Power* and P the *Active Power.*

So active power = apparent power × power factor

Cos ϕ is the 'power factor' as it is the factor by which the apparent power must be multiplied to obtain the active power value expended in a circuit.

$$\text{So power factor} = \frac{\text{active power}}{\text{apparent power}} \text{ or } \cos\phi = \frac{P}{VI}\frac{I^2R}{I|Z|} = \frac{R}{Z} \text{ as deduced earlier.}$$

Furthermore from the active power equation: P (watts) = VI (volt amperes) × cos ϕ

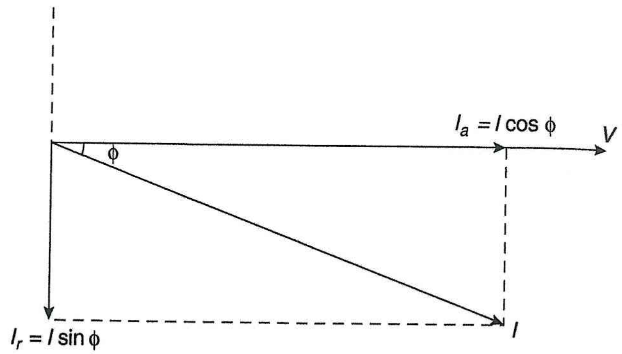
Wattage is given by volt amperes × power factor and expressed by $W = \text{VA}\cos\phi$ or kW = kVA cos ϕ.

Note. The term kVA is an accepted rating of A.C. generators, motors or transformers but it *does not* indicate the power rating. More information is required before this can be deduced and the power factor must be specified. The volt amperes or VA of a circuit term was retained from the earliest electrical engineering days, before standardised terms and symbols were introduced. VA or kV ratings are used internationally for A.C. circuits and machines.

Active and reactive components

These terms are used in connection with current but under certain conditions may apply to voltage and power. Consider the phasor diagram (figure 11.1) for a simple A.C. circuit with current lagging voltage by an angle ϕ. The current I splits into its 2 quadrature components I_a and I_r as shown so $I_a = I\cos\phi$ and $I_r = I\sin\phi$.

As $I\cos\phi$ is a current *in-phase* with the voltage V and as $VI\cos\phi$ is the measure of power expended in a circuit, then $I\cos\phi$ is the current component responsible for power dissipation and $I\cos\phi$ is called the *active* power, and is the working component of current. $I_r = I\sin\phi$, being always at right angles to the voltage, is responsible for no power and is called the *reactive*, wattless or idle current component. Example 11.1 illustrates these terms.



▲ **Figure 11.1**

Example 11.1. A single-phase A.C. motor of 15kW and 90% efficiency runs from a 400V single-phase supply. Find the current taken from the mains supply if the motor operates at 0.8 power factor (lagging). What is the value of the active current, the reactive current and the motor rating in volt amperes (1 decimal place).

Motor power output = 15kW = 15 × 1000 watts

$$\text{Motor power input} = \frac{15 \times 1000}{90} \times 100 \text{ watts}$$

$$= 16\,667\text{W or } 16.7\text{kW}$$

$$\text{Volt ampere rating} = \frac{16.67}{0.8} = 20.84\text{kVA}$$

The line current is obtained by dividing the volt ampere value by the supply voltage.

$$\text{Thus } I = \frac{20.84 \times 1000}{400} = 52.1\text{A}$$

Active component of current $I_a = I \cos \phi = 52.1 \times 0.8 = 41.7$A.

Reactive component of current $= I_r = I \sin \phi = 52.1 \times 0.6 = 31.3$A.

It is pointed out that the relation for $\sin \phi = 0.6$ when $\cos \phi = 0.8$ refers to a right-angled triangle of sides: 10, 8 and 6. For example, $\cos \phi$ is frequently given as 0.707 or sometimes 0.7, referring to a right-angled isosceles triangle and $\sin \phi$ in this case is also 0.707 or 0.7 (approx.).

The parallel circuit

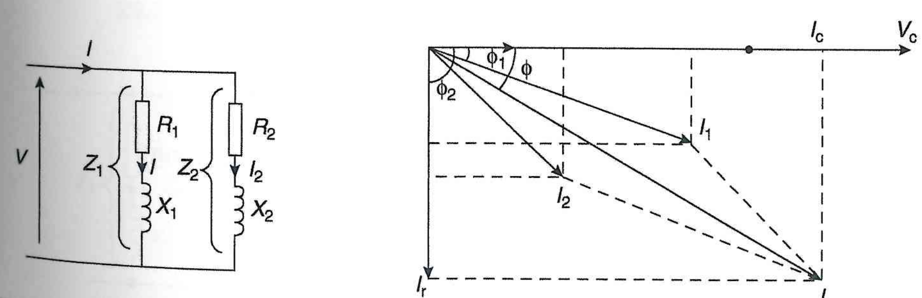
The parallel circuit is treated separately to show students that the procedure is different to that for the series circuit. Nevertheless the method employed follows the technique of phasor summation, i.e. resolving into horizontal and vertical components or in terms of the new terminology here: active and reactive components. The branches of a parallel circuit are made up of simple R, X_L or X_C series values, and all calculation work done in this regard remains altered. For a parallel circuit the same voltage is applied to all branches and it is usual to work with V as the reference for the phasor diagram. The current condition may be written as $\overline{I} = \overline{I_1} + \overline{I_2} + \overline{I_3}$ etc. The dash above the I indicating it is a phasor summation and not an arithmetical one. Thus all correct operations for a phasor summation must be performed.

Inductive impedances in parallel

Assume 2 inductive impedances connected in parallel (figure 11.2). Impedance Z_1 is made up of a resistance R_1 and inductive reactance X_1 while Z_2 is made up of resistance R_2 and inductive reactance X_2. The phasor diagram and circuit relationships are shown. As V is common to both branches it is used as the reference phasor. The problem is to find I where $\overline{I} = \overline{I_1} + \overline{I_2}$.

$$\text{Here } I_1 = \frac{V}{Z_1} \text{ and } I_2 = \frac{V}{Z_2}$$

Resolving into active and reactive components and using arbitrary signs: $I_a = I_1 \cos \phi_1 + I_2 \cos \phi_2$



▲ Figure 11.2

And, $I_r = -I_1 \sin\phi_1 - I_2 \cos\phi_2$. It must be remembered that –ve phasors are drawn vertically down.

Then $I = \sqrt{I_a^2 + I_r^2}$ and $\cos\phi = \frac{I_a}{I}$ where $\cos\phi$ is the power factor of the whole circuit.

Example 11.2. In the circuit above, let $R_1 = 3\Omega$ and $X_1 = 4\Omega$ while $R_2 = 8\Omega$ and $X_2 = 6\Omega$. If the applied voltage is 20V, find the total current supplied (2 decimal places) and the power factor (2 decimal places) of the complete circuit. Find also the total power expended (2 significant figures).

$$Z_1 = \sqrt{R_2^2 + X_1^2} = \sqrt{3^2 + 4^2} = 5\Omega$$

$$\text{Then } I_1 = \frac{20}{5} = 4\text{A}$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{8^2 + 6^2} = 10\Omega$$

$$\text{and } I_2 = \frac{20}{10} = 2\text{A}$$

$$\cos\phi_1 = \frac{3}{5} = 0.6 \text{ (lagging) and, } \sin\phi_1 = \frac{4}{5} = 0.8$$

$$\cos\phi_2 = \frac{8}{10} = 0.8 \text{ (lagging) and, } \sin\phi_2 = \frac{6}{10} = 0.6$$

$$\text{Also } I_a = (4 \times 0.6) + (2 \times 0.8) = 4\text{A}$$

$$I_r = -(4 \times 0.8) - (2 \times 0.6) = -4.4\text{A}$$

$$\text{Such that } I = \sqrt{4^2 + 4.4^2} = 5.95\text{A}$$

$$\text{Circuit power factor } \cos\phi = \frac{4}{5.95} = 0.67 \text{ (lagging)}$$

Power expended = 20 × 5.95 × 0.67 = 80W

The power expended can be checked thus:

$$\text{Power in branch } 1 = I_1^2 R_1 = 4^2 \times 3 = 48\text{W}$$

$$\text{Power in branch } 2 = I_2^2 R_2 = 2^2 \times 8 = 32\text{W}$$

Total 80W.

Inductive and capacitive impedances in parallel

The procedure for solving problems associated with this circuit type follows that outlined above, except that allowance is made for the directions and signs when adding reactive components. Thus in figure 11.3, impedance Z_2 is made up of resistance R_2 and capacitive reactance X_2 in *series*. The phasor for the reactive component of current is drawn vertically up and allocated a +ve sign, while the reactive component of current for branch 1 is allocated a –ve sign. The total reactive component is thus a *difference*. Voltage is again used as the reference for the phasor diagram.

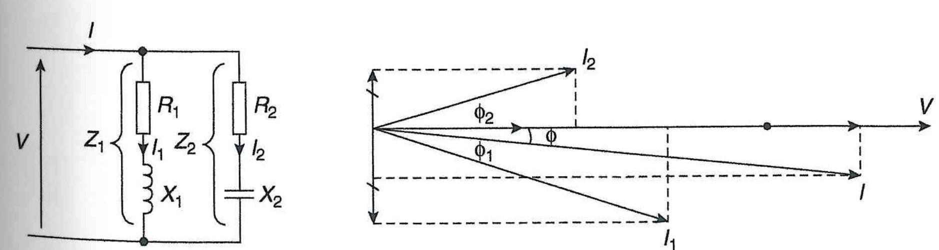
As before $I_a = I_1 \cos\phi_2 + I_2 \cos\phi_2$ and $I_r = -I_1 \sin\phi_1 + I_2 \sin\phi_2$. The sign of I_r is either +ve or –ve, decided by the relative values of $I_1 \sin\phi_1$ and $I_2 \sin\phi_2$. Thus the resulting reactive component will act either up or down and the resultant circuit current may lag or lead. As before $I = \sqrt{(I_a^2 + I_r^2)}$ and $\cos\phi = \frac{I_a}{I}$. The term lagging or leading is decided by the sign of I_r.

Example 11.3. A circuit consists of 2 parallel branches. Branch A consists of a 20Ω resistor in series with a 0.07H inductor, while branch B consists of a 60μF capacitor in series with a 50Ω resistor. Calculate the mains current and the circuit power factor if the voltage is 200V at 50Hz (1 decimal place all).

Branch A. $X_A = 2\pi fL = 2 \times 3.14 \times 50 \times 0.07 = 22\Omega$ $R_A = 20\Omega$

$$\therefore Z_A = \sqrt{20^2 + 22^2} = 29.7\Omega$$

$$\text{Thus } I_A = \frac{200}{29.7} = 6.74\text{A and } \phi_A = \frac{R_A}{Z_A} = \frac{20}{29.7} = 0.674 \text{ (lagging)}$$



▲ Figure 11.3

$$\sin\phi_A = \frac{X_A}{Z_A} = \frac{22}{29.7} = 0.74$$

Branch B. $X_B = \frac{1}{2\pi fC} = \frac{10^6}{2\times3.14\times50\times60} = 53'$

$R_B = 50\Omega$

$$\therefore Z_B = \sqrt{50^2 + 53^2} = 72.8\Omega$$

$$\text{Thus } I_B = \frac{200}{72.8} = 2.75\text{A}$$

$$\cos\phi_B = \frac{R_B}{Z_B} = \frac{50}{72.8} = 0.686 \text{ (leading)}$$

$$\sin\phi_B = \frac{X_B}{Z_B} = \frac{53}{72.8} = 0.728$$

Then $I_a = (6.74 \times 0.674) + (2.75 \times 0.68)$

$= 4.55 + 1.885 = 6.43\text{A}$

$I_r = -(6.74 \times 0.74) + (2.75 \times 0.728)$

$= -5 + 2.005 = -2.995\text{A}.$

Note. The mains current will lag as the inductive branch dominates.

$$I = \sqrt{I_a^2 + I_r^2} = \sqrt{6.43^2 + 2.995^2}$$

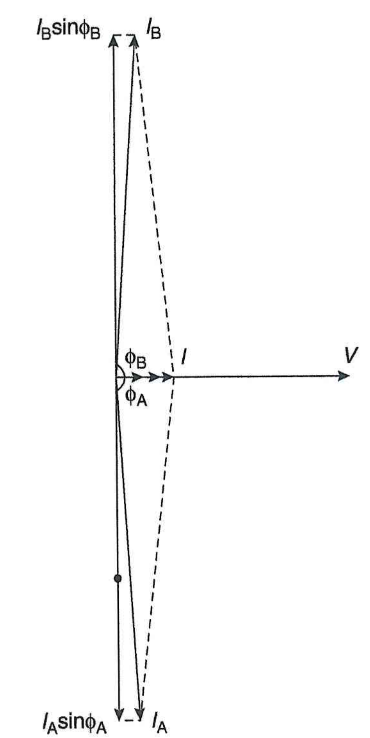
$I = 7.1\text{A}$

$$\cos\phi = \frac{I_a}{I} = \frac{6.43}{7.1} = 0.9 \text{ (lagging).}$$

Parallel resonance

It is worth pointing out that a resonance condition can also occur in a parallel circuit and is termed 'current resonance' to distinguish it from 'voltage resonance' as dealt with for the series circuit. From Example 11.3 a condition arises when $I_A \sin\phi_A = I_B \sin\phi_B$ and as these are the reactive components of currents in inductive and capacitive branches, they oppose each other producing a total reactive component of zero value. The remaining active components then add to give the line current, as $I = \sqrt{I_a^2 + 0} = I_a$ and the combined circuit will operate at unity power factor. This is illustrated by the phasor diagram (figure 11.4). Since the power factors of both branches are low, the phase angles ϕ_A and ϕ_B are large *yet* $I_A \cos\phi_A$ and $I_B \cos\phi_B$ are small compared to the reactive components.

Large currents can flow in inductive and capacitor branches, which are much greater than the main supply current and are not supplied from the line. On examining the power waves for an inductor and capacitor, it is seen they are directly opposite in-phase, as are the current waves. It is assumed that as the capacitor discharges with



▲ Figure 11.4

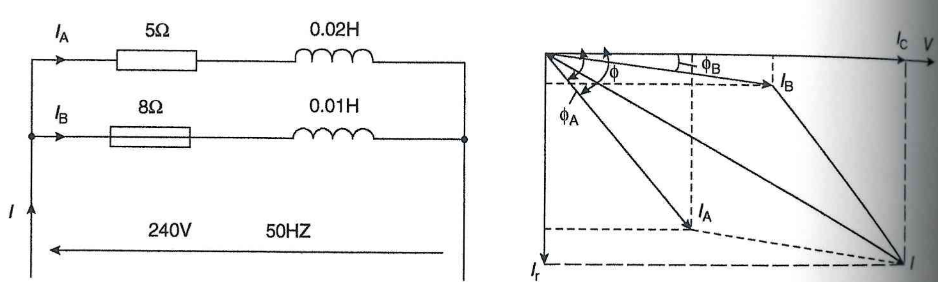
power given out absorbed by the inductor building up its field. When the inductive field collapses, power released charges the capacitor and there is a current due to oscillation of power between inductor and capacitor. This resonance creates oscillator circuits and has many applications in radio and electrical filter circuit design. If no supply is available current is not maintained due to energy loss in the circuit resistance, which although small cannot be neglected. To maintain an oscillatory current, the resistance loss must be supplied at the correct frequency from the external supply force.

Power-Factor Improvement

Power-factor improvement is often used in practical electrical engineering work and is best illustrated by the following example.

Example 11.4. (a) Two inductive coils of resistance values 5Ω and 8Ω and inductance values of 0.02H and 0.01H respectively are connected in parallel across a 240V, 50Hz mains supply. Find the coil currents (1 decimal place), the circuit current (1 decimal place) and its power factor (3 decimal places).

The arrangement is shown in figure 11.5. A phasor diagram is also drawn.



▲ Figure 11.5

Branch A. $X_A = 2\pi fL = 2 \times \pi \times 50 \times 0.02 = 6.28\Omega$

$$Z_A = \sqrt{5^2 + 6.28^2} = 8.02\Omega$$

$$I_A = \frac{240}{8.02} = 29.8\text{A} \quad \cos\phi_A = \frac{5}{80.2} = 0.622 \text{ lagging}$$

$$\sin\phi_A = \frac{6.28}{8.02} = 0.78$$

Branch B. $X_B = \frac{1}{2}$ that of branch A, since L is halved = 3.14Ω

$$Z_B = \sqrt{8^2 + 3.14^2} = 8.6\Omega$$

$$I_B = \frac{240}{8.6} = 27.9\text{A} \quad \cos\phi_B = \frac{8}{8.6} = 0.93 \text{ lagging}$$

$$\sin\phi_B = \frac{3.14}{8.6} = 0.366$$

Then $I_a = (29.8 \times 0.622) + (27.9 \times 0.93) = 44.6\text{A}$

$I_r = (29.8 \times 0.78) + (27.9 \times 0.366) = 33.5\text{A}$

It is noted that the arbitrary −ve sign is not used as both branches are inductive and there is no doubt as to, the resultant current being lagging.

Then $I = \sqrt{(44.6^2 + 33.5^2)}$ $\quad I = 55.6\text{A}$

$$\text{P.F.} = \cos\phi = \frac{44.6}{55.6} = 0.801 \text{ (lagging)}$$

Example 11.4. (b) Find the effect on the main circuit current (1 decimal place) and power factor (1 decimal place), if a capacitor of 400μF is connected across the supply in *parallel* with the coils.

The phasor diagram (figure 11.6), shows the new conditions.

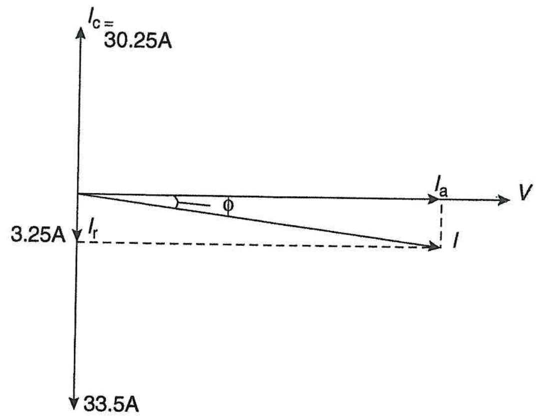
Branch C. Reactance of capacitor $X_C = \dfrac{1}{2\pi fC}$

Thus $X_C = \dfrac{10^6}{2 \times \pi \times 50 \times 400} = 7.95\Omega$

$$\therefore I_C = \frac{240}{7.95} = 30.25\text{A}$$

As there is no resistance in branch C, only capacitive reactance, then $\cos\phi_C = 0$ and $\sin\phi_C = 1$.

Again I_C acts at 90° to the voltage and is wholly reactive, there being no active component. Then I_a as before = 44.6A and $I_r = -23.3 - 10.2 + 30.25 = -3.25\text{A}$.



▲ Figure 11.6

It is seen that the arbitrary signs have been introduced because the reactive current of branch C acts in the *opposite* direction to that of branches A and B.

The circuit current is now:

$$I = \sqrt{I_a^2 + I_r^2} = \sqrt{44.6^2 + 3.25^2}$$

$$\therefore I = 44.6\text{A}$$

and $\cos\phi = \dfrac{44.6}{44.6} = 1.0$ i.e. unity.

From this example it is seen that, by connecting a capacitor in parallel with inductive loads, the total line current is reduced from 55.6 to 44.6A and overall circuit power factor is improved from 0.8 (lagging) to unity. The advantages of this arrangement are now considered in detail.

Advantages of power-factor improvement

For most commercial loads, current lags voltage, due to the system's inductance or the operating characteristics of motors and control gear. Typical power-factor values are as follows:

System supplying lighting loads only: power factor (lagging) = 0.95.

System supplying lighting and power loads: power factor (lagging) = 0.75 to 0.85.

System supplying power loads: power factor (lagging) = 0.5 to 0.7.

The lower the power factor, the *greater* the line current must be for a given load kW or output power rating and following are the disadvantages of this:

(1) The transmission losses in supply cables or power lines increase in accordance with I^2R, where R is the cable or line resistance. For a given transmitted power, the current at 0.7 power factor is $\dfrac{1}{0.7} \times$ current at unity power factor $=$ 1.43 $\times$ current at unity power factor. Also the transmission loss at 0.7 power factor is $(1.43)^2 \times$ loss at unity power factor = 2 × loss at unity power factor.

(2) Because of the larger currents resulting from a low power factor, there is a greater voltage drop in the supply lines resulting in a lower voltage at the load, thus conductor size must be increased to keep the voltage drop to an acceptable value!

(3) Since the larger current results from a low power factor, the size of the current-carrying conductors in transformers, control gear and alternators must be larger than the minimum possible. This means the physical dimensions of equipment must be larger – an inferior design. Equipment is also more costly.

(4) 'Regulation' – a term used for the stabilising or 'sitting-down' of the generating and transmitting plant voltage is adversely affected by a low power factor. The lower the power factor the greater the internal voltage drop in this equipment, i.e. armature reaction and its effects will be worse.

Example 11.5. A 40kW load, operating at 0.707 power factor (lagging), is supplied from 500V, 50Hz mains supply. Calculate (a) the capacitor value required to raise the line power factor to unity (3 significant figures) and (b) the capacitance required to raise the power factor to 0.95 (lagging) (3 significant figures).

(a) Load $\text{kVA} = \dfrac{\text{kW}}{\cos\phi} = \dfrac{40}{0.707}$

$$\text{Load current} = \frac{40\times10^3}{0.707\times500} = 113.15\text{A}$$

Active component of load current $I_1 = I_1\cos\phi_1$

$$= 113.15 \times 0.707$$

$$= 79.997\text{A} = 80\text{A}$$

Reactive component of load current $I_1 = I_1 \sin \phi_1$

$$= 113.15 \times 0.707$$

$$= 80\text{A}.$$

To cancel the reactive current, a capacitor is fitted to operate in *parallel* with the load, which must pass a similar reactive current value as shown by the phasor diagram (figure 11.7).

Thus I_C must be 80A. Reactance X_C of capacitor must be: $\frac{500}{80} = 6.25\Omega$

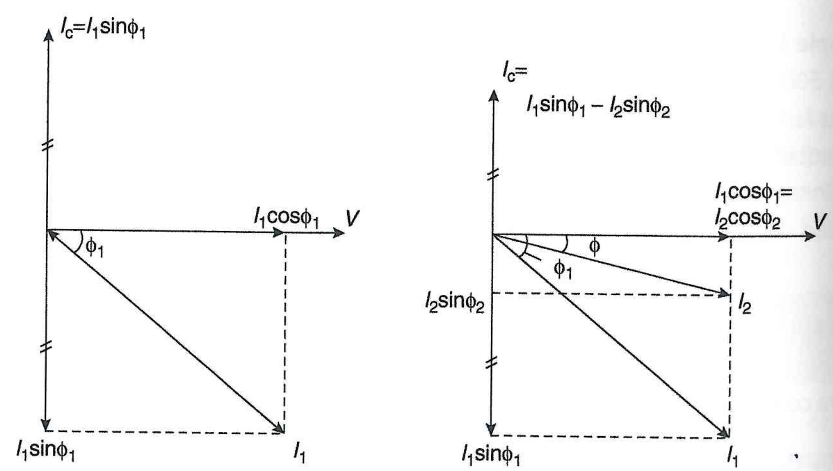
Since $X_C = \frac{1}{2\pi fC}$ then $\frac{10^6}{2\pi fC} = 6.25$ (where C is in microfarads)

Thus $C = \frac{10^6}{2 \times 3.14 \times 50 \times 6.25}$ $C = 510\mu\text{F}$.

(b) *Note.* $I_1 \sin \phi_1$ is not to be cancelled completely as the line phase angle is *reduced* from ϕ_1 to ϕ_2 and the line current to a new value I_2 as illustrated (figure 11.7b). The power or active component remains the same, $VI_2 \cos \phi_2 = 40\ 000$ as before.

$$\therefore I_2 = \frac{40 \times 10^3}{500 \times 0.95} = 84.2\text{A}$$

Since $\cos \phi_2 = 0.95$ (lagging) $\sin \phi_2 = 0.312$ and $I_2 \sin \phi_2 = 84.2 \times 0.312 = 26.1\text{A}$.



▲ **Figure 11.7**

The line reactive current component is reduced from 80A to $26.1\text{A} = 53.9\text{A}$. This will be the new value of I_C or a capacitor is used which takes a current of 53.9A.

Thus $X_C = \frac{500}{53.9} = \frac{10^6}{2\pi fC}$ (where C is in microfarads)

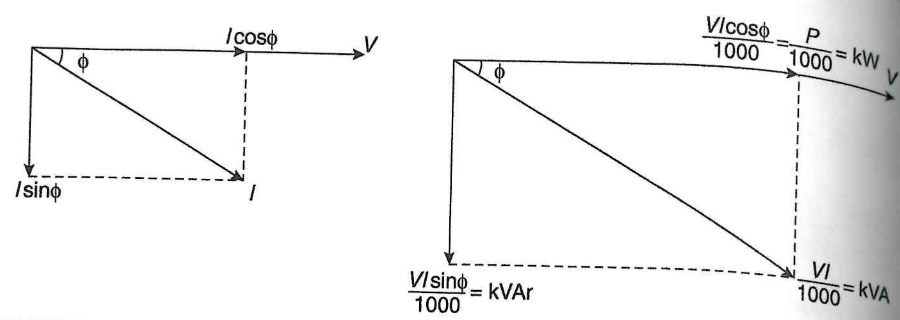
or $C = \frac{10^6 \times 53.9}{500 \times 2 \times 3.14 \times 50}$ $C = 343\mu\text{F}$.

Since 343μF brings the line power factor to 0.95 a further $(510 - 363) = 167\mu\text{F}$ would bring the value to unity. As the capacitor cost depends on its capacitance value and little advantage is gained by improving the power factor above 0.95, it is not necessary to achieve unity power factor working. It is vital to note that, although power factor is improved, an increased power output is *not* obtained from the load. Students often have the wrong idea, for example, that the power factor of a circuit supplying a 5kW motor is improved, a motor will give an output greater than 5kW. This is wrong. All that is achieved is that, by connecting an extra capacitor across the motor, the total line current is reduced, i.e. a condition is attained when the *minimum* supply current required for a specified power output is used. This minimum supply current reduces all the disadvantages already stated but the motor current itself is unaltered.

kW, kVA and kVAr

As mentioned kW, kVA and kVAr terminology is still used in electrical engineering and some revision is needed, so the diagram (figures 11.8a and 11.8b) is considered. For a circuit, where current and voltage are out-of-phase, the phasor diagram is as shown. Current I is resolved into an in-phase or active component $I \cos \phi$ and an out-of-phase or reactive component $I \sin \phi$. $I \cos \phi$ is responsible for all the power dissipated in the circuit, since $P = VI \cos \phi$, while $I \sin \phi$ is responsible for no power, being at right angles to the voltage.

From the expression $P = VI \cos \phi$ it is seen that P can be the 'active' component of VI (symbol S – see *Note*). The term volt amperes or kilovolt amperes is used for VI then the kVA (kilovolt amperes) is regarded as resolved into 2 components, one of which is the power component. The term W or kW (kilowatts) describes this component and the other component termed the 'volt amperes reactive' or 'reactive kilovolt amperes' and designated by VAr or kVAr. If the current phasors of figure 11.8a are multiplied by V, the new condition becomes more apparent and leads to a power diagram. The product VI (S) is shown as the volt amperes (VA) or $\frac{VI}{1000} = \text{kVA}$ and referred to as the 'apparent power'. Since $VI \cos \phi = P$ then $VA \cos \phi = \text{W}$ and $\text{kVA} \cos \phi = \text{kW}$. kW



▲ Figure 11.8

is a measure of 'active power', in line with the original definition for power factor, i.e. the ratio of active power to apparent power.

Thus: power factor or $\cos\phi = \dfrac{\text{kW}}{\text{kVA}}$

Similarly $VI \sin\phi$ (Q – see *Note*), or kVA $\sin\phi$ is the 'reactive power' or volt amperes reactive designated by kVAr and from the power diagram (figure 11.8b).

$$\text{Apparent power} = \sqrt{\text{Active Power}^2 + \text{Reactive Power}^2}$$

Summarising kW = kVA $\cos\phi$.   kVAr = kVA $\sin\phi$

$$\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAr}^2}$$

$$\cos\phi = \frac{\text{kW}}{\text{kVA}} \qquad \sin\phi = \frac{\text{kVAr}}{\text{kVA}}$$

Note. Symbols S, P and Q are recommended substitutes for VI, $VI\cos\phi$ and $VI\sin\phi$ but it is likely the units – kVA, kW and KVAr – will continue to be shown on phasor diagrams, as this is the older electrical power engineer practice. The appropriate alternative has been introduced and is shown where appropriate.

In summary we have:

$P = VI\cos\phi$ $Q = VI\sin\phi$ and $S = VI$

Thus $S = \sqrt{P^2 + Q^2}$ and $P = S\cos\phi$ $Q = S\sin\phi$

$\cos\phi = \dfrac{P}{S}$ and $\sin\phi = \dfrac{Q}{S}$.

It is noted that the kVA values of various loads are *not* in-phase and don't add arithmetically. kW values are all active components, are in-phase and add. kVAr values are reactive components, they can be in-phase or in anti-phase and are added, provided allowance is made for the sign as shown in the following examples.

Example 11.6. Two loads are connected in parallel. Load A is 800kVA at 0.6 (lagging). Load B is 700kVA at 0.8 power factor (lagging). Find the total kW, kVA and overall power factor of the joint loads.

See the diagram (figure 11.9).

For load A. $\cos\phi_A = 0.6$, $\sin\phi_A = 0.8$

Active power, $P_A = VI_A \cos\phi_A = 800 \times 0.6 = 480\text{kW}$

Reactive power, $Q_A = VI_A \sin\phi_A = 800 \times 0.8 = 640\text{kVAr}$

For load B. $\cos\phi_B = 0.8$, $\sin\phi_B = 0.6$

Active power, $P_B = VI_B \cos\phi_B = 700 \times 0.8 = 560\text{kW}$

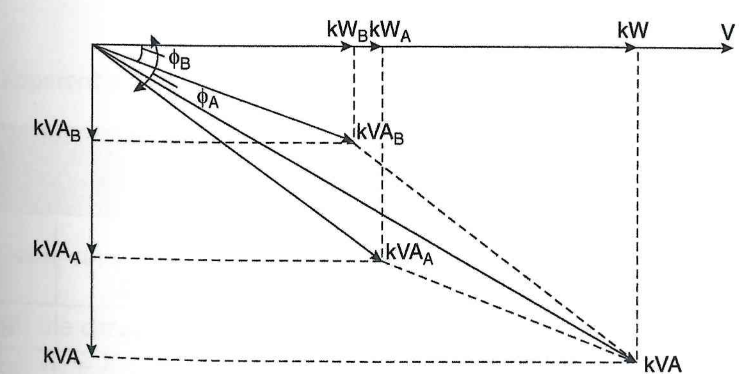
Reactive power, $Q_B = VI_B \sin\phi_B = 700 \times 0.6 = 420\text{kVAr}$

Total active power, $P = 480 + 560 = 1040\text{kW}$

Total reactive power, $Q = 640 + 420 = 1060\text{kVAr}$

Total apparent power, $S = \sqrt{(1040^2 + 1060^2)} = 1485\text{kVA}$

$$\text{Overall power factor} = \frac{1040}{1485} = 0.7\ (\text{lagging})$$



▲ Figure 11.9

Problems involving multiple loads are best treated by setting out the power-diagram components in tabular fashion. An arrow illustrates the phasor direction and reminds students which columns can be added arithmetically.

Example 11.7. A 220V, single-phase alternator supplies the following loads:

(A) 20kW at unity power factor for lighting and heating.

(B) A 75kW induction motor having an efficiency of 90.5% operating at a power factor of 0.8 (lagging).

(C) A synchronous motor taking 50kVA at a power factor of 0.5 (leading).

Find the total kVA, current and the power factor of the combined load.

Load A can be set into the columns directly as shown.

Load B Motor power output = rating as given = 75kW

$$\text{Motor input active power} = \frac{75}{0.905} = 82.9\text{kW}$$

$$\text{Apparent power} = \frac{82.9}{0.8} = 103.6\text{kVA}$$

Load C can also be set into the columns directly.

$$\text{Total apparent power (S)} = \sqrt{127.9^2 + 18.9^2} = 129\text{kV A}$$

$$\text{Total current} = \frac{129 \times 1000}{220} = 588\text{A}$$

Table 11.1

Load	kVA (s) ↘	kW or → kVA cos ϕ (P)	kVAr or ↕ kVA sin ϕ (Q)	cos ϕ	sin ϕ
a	20 →	20 →	0	1	0
b	103.6 ↘	82.9 →	−62.16 ↓	0.8	0.6
c	50 ↗	25 →	43.3 ↑	0.5	0.866
		127.9 →	−18.86 ↓		

$$\text{Resultant power factor} = \frac{127.9}{129} = 0.99 \text{ (lagging)}$$

Note. The reactive inductive load component dominates, hence a resultant lagging power-factor condition.

Power-factor improvement (kVA method)

Treatment of power-factor improvement problems is similar to the 'current method'. The diagram for the load condition is made by splitting the original load kVA into its kW (real expended power) and kVAr (imaginary) components. Since the kW remains the same, then for a new power-factor condition for the supply, the final kVAr value is obtained by reducing the original kVAr by an amount equal to the kVAr of the components added. Such components must use no power and a static capacitor is such a device component. The added kVAr leading, reduces the lagging supply kVAr. If a synchronous motor is used to obtain a better overall power factor, this contributes output power which must be taken into account.

Example 11.8. A 400V, 50Hz, 20kW, single-phase induction motor has a full-load efficiency of 91.15% and operates at a power factor of 0.87 (lagging). Find the kVAr value of the capacitor to be connected in parallel to improve the circuit power factor to 0.95 (lagging) (2 decimal places). Find also the capacitance value of this capacitor (3 significant figures). Figure 11.10 illustrates the problem and solution.

Motor output = motor rating as given = 20kW

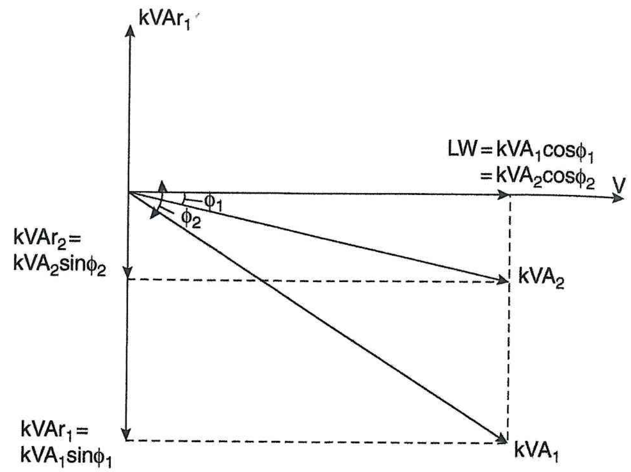
$$\text{Motor active input power } P_1 = \frac{20}{0.9115} = 21.94\text{kW}$$

$$\text{Motor apparent power } S_1 = \frac{21.94}{0.87} = 25.22\text{kVA}$$

As $\cos\phi_1 = 0.87$ then $\sin\phi_1 = 0.493$

Thus Q_1 or $S_1 \sin\phi_1, = 25.22 \times 0.493 = 12.44\text{kVAr}$

Although the circuit power factor is improved to $\cos\phi_2$ the *power of the circuit* is not altered $\therefore P_1 = P_2$ or $S_1 \cos\phi_1 = S_2 \cos\phi_2$ whence



▲ **Figure 11.10**

$$S_2 = S_1 \cos\phi_1 / \cos\phi_2$$

$$\text{or } S_2 = 25.22 \times \frac{0.87}{0.95} = 23.1\text{kVA}$$

Again $\cos\phi_2 = 0.95$ therefore, from tables, $\sin\phi_2 = 0.3123$ and

$$Q_2 = 23.1 \times 0.3123 = 7.21\text{kVAr}$$

Required Q value $= 12.44 - 7.21 = 5.23$kVAr. This should be the capacitor's rating.

$$\text{Capacitor current } I_c = \frac{5230}{400} = 13.75\text{A}$$

$$\text{Capacitor reactance } X_c = \frac{400}{13.75} = 30.59\Omega$$

$$\text{or } X_c = 30.59 = \frac{10^6}{2 \times \pi \times 50 \times C}$$

(where C is in microfarads.)

$$\text{Hence } C = \frac{10^4}{30.59 \times \pi} = 104\mu\text{F}.$$

Polyphase Working

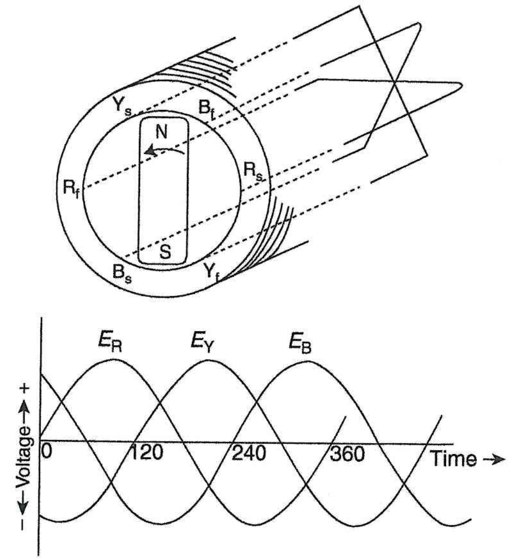
The student who intends to have a good practical knowledge of electrotechnology must understand the terms, relationships and theory of polyphase working. The importance of the work now covered cannot be too strongly stressed. Most students consider this part of theory to be 'that small straw which breaks the camel's back' and accordingly give it little attention. The result is that much hasty revision is needed later when A.C. machines are studied. Although Volume 7 is devoted to more advanced A.C. technology, the subject matter must now be considered as *basic and essential* to further studies.

Polyphase or split-phase power systems achieve their high conductor efficiency and improved safety by splitting up the total voltage into smaller parts and powering multiple loads at these reduced voltages, while drawing currents at levels typical of a full-voltage system, achieving a balance between system efficiency (low conductor current) and safety (low load voltage). Incidentally this approach will work just as well for D.C. power systems as well as A.C.

Three-phase systems

Universal practice has established 3-phase systems to be the most advantageous for polyphase working. A single-phase supply, as used for small installations, can always be obtained from a 3-phase system and in this way the relative advantage of either system is available. 2-phase systems are rarely used and don't warrant study here. More than 3-phase arrangements, such as 6-phase, have fewer and more specialised applications and we will confine our investigation to 3-phase working only.

Consider a 2-pole magnet, as shown (figure 11.11), rotated inside an external stationary armature or stator. Three coils are shown equally displaced, with 'starts' and 'finishes' marked symmetrically in a regular arrangement. Induced e.m.f.s result in each coil, identical in magnitude but displaced in-phase by 120 electrical degrees. A phase sequence Red–Yellow–Blue (R–Y–B) is assumed, i.e. the rotor turns so that the red-phase voltage reaches its maximum 120° before the yellow-phase voltage reaches its maximum and the latter 120° before the blue-phase voltage, as shown by the waveform diagram. A sinusoidal distribution of rotor flux is assumed and sine-wave e.m.f.s induced. The methods by which this is achieved will be discussed for the alternator. The 3 separate coils can be used to supply 3 independent single-phase loads, but



▲ Figure 11.11

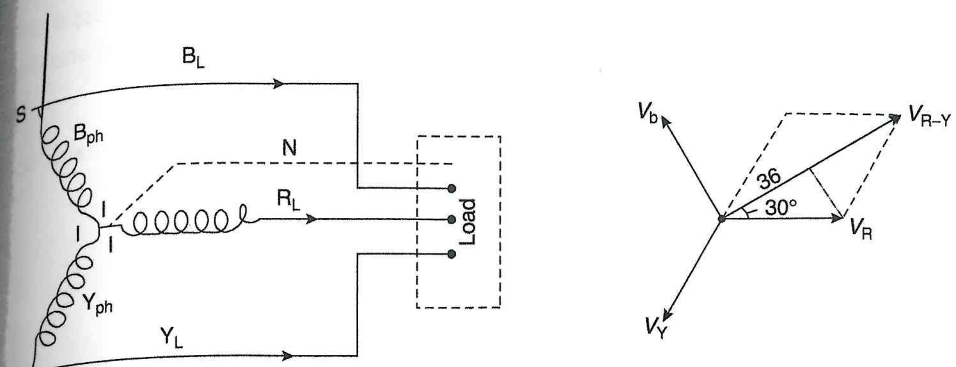
advantages are obtained by connecting the coils or phase windings. Two important methods are described: either the STAR or the DELTA connection.

Star or Y connection

Figure 11.12 shows the star arrangement with the 3 coils or phase windings connected so that either all the *starts* or *finishes* are joined together to form a star-point, i.e. 'corresponding ends' are connected together. Similarly supply lines are connected to the free ends, remote from the star-point.

The phasor diagram is drawn in terms of voltage with the red-phase voltage (V_R) as the reference. This notation is used from now on. The small letter suffix denotes the phase value, while the capital letter denotes the line value. Lines have been identified with the colours of the phases to whose 'starts' are connected. The double suffix such as $V_{R\text{-}Y}$ denotes the voltage *between* lines, the example being the red to yellow line voltage.

Assume the condition when the red-phase voltage is positive and the 'start' of the red-phase winding +ve with respect to the 'finish' or neutral point. Current flows through the lines and load as shown. For this example it is possible because, for the yellow phase at the same instant, its start is –ve with respect to its finish, as the yellow-phase waveform is in its –ve half cycle. Thus for the phasor diagram, the voltage between the red and yellow lines is obtained by the phasor *difference* of V_r and V_y.



▲ Figure 11.12

As a phasor difference is considered, the resultant is obtained by *reversing* one phasor with respect to the other and completing the parallelogram. From the deduction below, it is seen that the line voltage is √3 times the phase voltage. This relation holds for the other lines and their associated phases. A further point of importance for the star connection is that the line current equals the phase current or $I_L = I_{ph}$

Consider the phasordiagram. Let the line voltage $V_{R\text{-}Y} = 2x$

$$\text{But } \frac{x}{V_r} = \cos 30° \;\therefore\; x = \frac{\sqrt{3}}{2}V_r \quad \text{or} \quad 2x = \sqrt{3}V_r$$

Hence $V_{R\text{-}Y} = \sqrt{3}V_r$

or the voltage between lines $= \sqrt{3} \times$ the phase voltage.

Thus $V_L = \sqrt{3} \times V_{ph}$

For a star connection the following must be remembered.

$$\text{Line voltage} = \sqrt{3}\text{-phase voltage or } V = \sqrt{3}V_{ph} = 1.732V_{ph}$$

$$\text{Line current} = \text{Phase current or } I = I_{ph}$$

Note. The subscript L, as in V_L and I_L, is omitted when generalising – this is usual and both *V* and *I* are assumed to be line values. Again the relations are derived for an alternator or supply source

Example 11.9. Three 50Ω resistors are connected in a star configuration across 415V, 3-phase mains supply. Calculate the line and phase currents (1 decimal place) and the power taken from the supply (2 significant figures).

As the load is balanced, the voltage across each resistance is the phase voltage, so

$$V_{ph} = \frac{415}{\sqrt{3}} = 240V$$

Phase current = line current or $I_{ph} = I = \frac{240}{50} = 4.8A$

Power dissipated by 1 phase of load = $I_{ph}^2 R_{ph} = 4.8^2 \times 50W$

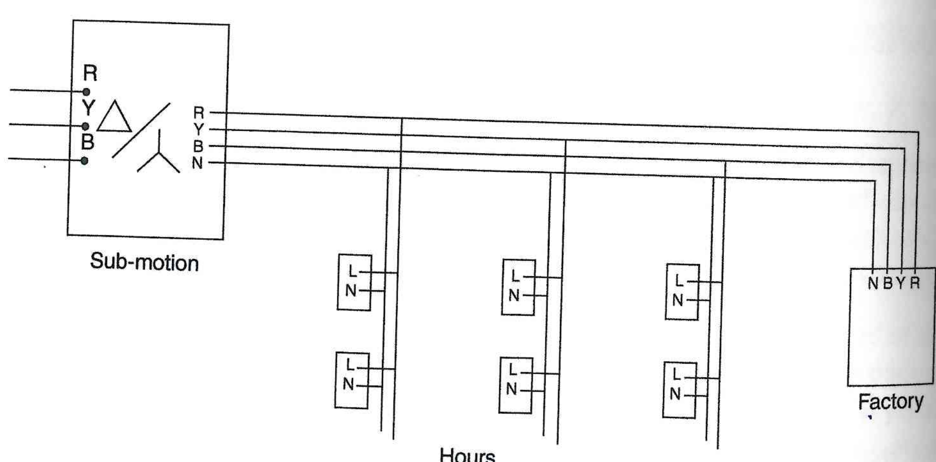
$= 1152W$ or $1.152kW$

and 3-phase power from the supply

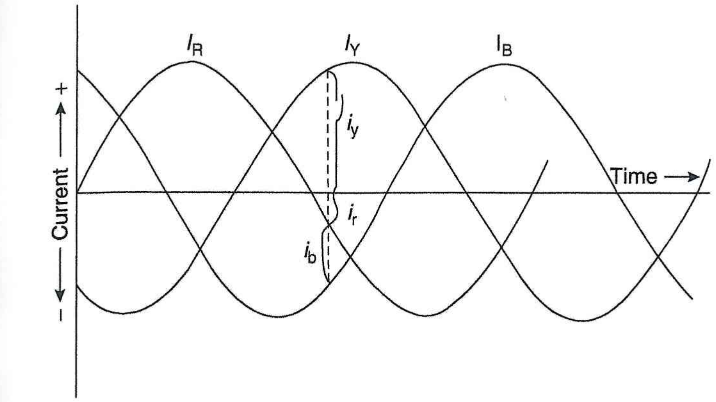
$= 3 \times 1.152 = 3.456kW = 3.5kW.$

Use of the neutral

An obvious use of the star connection is for distribution, as 2 voltages are available to the consumer, one for lighting and the other for power. Either 1-phase or 3-phase loading is possible and shown (figure 11.13).



▲ Figure 11.13



▲ Figure 11.14

Balanced load

A 3-phase load is said to be 'balanced' if the currents in all 3 phases are equal and their phase angles the same. If an instant in time is considered on the diagram (figure 11.14), the sum of *instantaneous* values of the currents $i_r + i_y + i_b = 0$. As these currents meet at the load neutral point and the resultant flows through the neutral line then the neutral carries *no current* and need not be used for balanced loading.

Unbalanced load

A neutral must be used if the load phase currents are unequal or the phase angles different. The neutral line carries the unbalanced current, i.e. the resultant of the 3 line currents. Since this neutral current is a phasor sum, it is obtained graphically or mathematically, as shown.

Example 11.10. The loads of a 4-wire, 3-phase system are: red line to neutral current = 50A, P.F. = 0.707 (lagging).

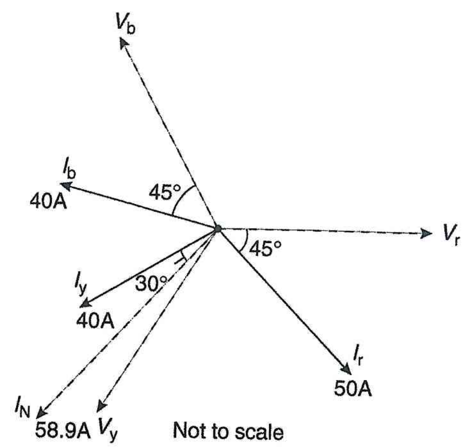
Yellow line to neutral current = 40A, power factor = 0.866 (lagging)

Blue line to neutral current = 40A, power factor = 0.707 (leading)

Determine the value of the current in the neutral wire. The solution is worked with reference to figure 11.15.

I_r = 50A lagging the voltage by 45° since cos 45 = 0.707

I_y = 40A lagging the voltage by 30° since cos 30 = 0.866



▲ **Figure 11.15**

I_b = 40A leading the voltage by 45° since cos 45 = 0.707

Resolving into horizontal and vertical components.

$I_H = (50 \times \cos 45) - (40 \times \cos 30) - (40 \times \cos 15)$

$= (50 \times 0.707) - (40 \times 0.866) - (40 \times 0.966)$

$= 35.35 - 34.64 - 38.64 = -37.93\text{A}$

$I_v = -(50 \times \sin 45) - (40 \times \sin 30) + (40 \times \sin 15)$

$= -(50 \times 0.707) - (40 \times 0.5) + (40 \times 0.259)$

$= -35.35 - 20 + 10.36 = -44.99$

Current in the neutral is the resultant.

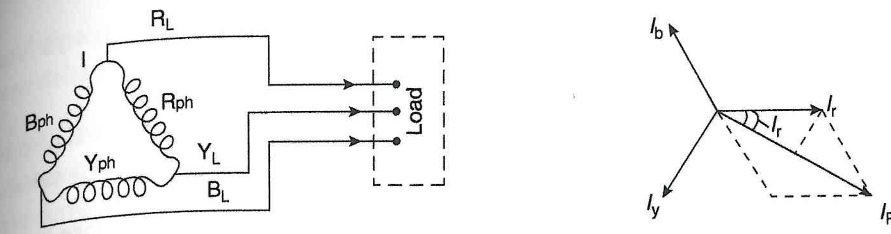
$$\text{or } I_N = \sqrt{37.93^2 + 44.92^2}$$

$$= 58.9\text{ A}$$

$$\cos\theta = \frac{-37.93}{58.9} = -0.653$$

$$\theta = 49.5°$$

Note. The –ve sign gives the quadrant in which I_N lies, as shown on the diagram.



▲ **Figure 11.16**

Delta Δ or (mesh) connection

This arrangement is shown (figure 11.16). For this connection, the 3-phase windings are arranged into a closed circuit by connecting 'uncorresponding ends', i.e. the *start* of one phase to the *finish* of another phase. Thus R start is connected to B finish, Y start to R finish, etc. The same reasoning as introduced for figure 11.17 is applied here, except that voltages are considered. Thus if the diagram represented the 3 equal phase voltages, it is seen that at any instant the sum of the instantaneous voltage values $v_r + v_y + v_b = 0$. For the mesh or closed winding, as the sum of the instantaneous voltages is zero, no circulating current flows round the mesh. Lines are taken from the junction points and for this connection, it is clear that the voltage developed across a phase is the voltage provided for the connected lines.

Thus V_L or $V = V_{ph}$

The lines have been identified with the colours of the phases to whose *starts* they are connected. Assume the condition when the red-phase voltage is positive, i.e. the start of the red-phase winding is +ve with respect to the finish. Current flows through from R phase into R line as shown. At this same instant the voltage in B phase is negative, i.e. its finish is +ve with respect to its start. Thus it is also correctly connected for feeding current into the R line and a line current is obtained by considering the phasor *difference* of 2 phase currents. The resultant line current is obtained by reversing a phase current (I_b) and combining it with I_r as shown (figure 11.16).

As before $x = I_r \cos 30° = I_{ph} \cos 30°$

Or $x = \frac{\sqrt{3}}{2} I_{ph}$ and as $I_L = 2x = \frac{2\sqrt{3}I_{ph}}{2}$

Hence I_L or $I = \sqrt{3}I_{ph}$

Thus for a delta connection, Line voltage = Phase voltage and Line current = √3 Phase current.

This relationship is deduced for *any* line and the connected phases and will give the same result. As before V and I are used for line values and V_{ph} and I_{ph} for phase values.

Example 11.11. Three 50Ω resistors are delta-connected across 415V, 3-phase lines as for Example 11.9. Calculate the line and phase currents (1 decimal place) and the power taken from the mains supply in 1 phase of the load (4 significant figures).

Voltage across 1-phase resistor = 415V

$$\text{Current in 1 phase of load} = \frac{415}{50} = 8.3\text{A}$$

Since the load is balanced, line current = √3 × 8.3 amperes = 14.4A

Power in 1 phase of load = $8.3^2 \times 50 = 3445$W

In addition the power in all 3 phases of load will be = 3 × 3.445 = 10.3kW.

In addition the power in all 3 phases of load will be = 3 × 3.445 = 10.3kW.

Three-phase power

For a star-connected load, $V = \sqrt{3}V_{ph}$ and $I = I_{ph}$

The power expended in 1 phase $= V_{ph} I_{Ph} \cos\phi$ and the power expended in 3 phases $= 3\, V_{ph} I_{ph} \cos\phi$

Converting to line values, the above becomes:

$$\text{Three-phase power} = 3\frac{V}{\sqrt{3}} I \cos\phi$$

$$\text{or } P = \sqrt{3}VI\cos\phi$$

For a Delta-connected load $V = V_{ph}$ and $I = \sqrt{3}I_{ph}$

The power expended in 1 phase $= V_{ph} I_{ph} \cos\phi$ and the power expended in 3 phases $= 3V_{ph} I_{ph} \cos\phi$

Converting to line values the above becomes:

$$\text{Three-phase power} = 3V\frac{I}{\sqrt{3}} \cos\phi$$

$$\text{or } P = \sqrt{3}VI\cos\phi$$

Thus the general expression holds, irrespective of the type of connection, namely either star or delta-Connected: 3-phase power is given by $\sqrt{3}\,VI\cos\phi$.

Example 11.12. A 75kW, 400V, 3-phase, delta-connected induction motor has a full-load efficiency of 91% and operates at a power factor of 0.9 (lagging). Calculate the line and phase currents at full load.

$$\text{Output power} = 75 \times 10^3 \text{ watts}$$

$$\text{Input power} = \frac{75 \times 10^3 \times 100}{91} \text{watts}$$

$$\text{also } P = \sqrt{3}\, VI \cos\phi$$

$$\text{So } \sqrt{3} \times 400 \times I \times 0.9 = \frac{75 \times 10^5}{91}$$

$$\text{And } I = \frac{75 \times 10^4}{1.732 \times 4 \times 9 \times 91} \text{amperes}$$

$$\text{or } I = 132.2\text{A}$$

$$\text{Motor phase current } \frac{132.A}{\sqrt{3}} = 76.3\text{A}$$

Three-phase kVA, kW *and* kVAr

As power factor can be defined as the ratio of true power to apparent power it can be applied to 3-phase working.

Thus:

$$\text{power factor} = \frac{\text{active power}}{\text{apparent power}}$$

Irrespective of star or delta connection $P = \sqrt{3}\, VI \cos\phi$.

So:

$$\cos\phi = \frac{P}{\sqrt{3}VI}$$

It follows that for 3-phase working, in order that the definition for power factor should apply,

apparent power (S) = $\sqrt{3}\,VI$

Note introduction of $\sqrt{3}$ – distinguishes this condition from single-phase working.

Again it is known that $\cos\phi = \frac{P}{S}$ or $\frac{KW}{KVA}$ so it follows that: *3-phase* kVA $= \frac{\sqrt{3}VI}{1000}$

Example 11.13. A 3-phase, 400V motor takes a current of 16.5A when the output is 9kW. Calculate (a) the kVA input (2 decimal places) and (b) the power factor, if the efficiency at this load is 89% (2 decimal places).

(a) $kVA \text{ input} = \frac{\sqrt{3}VI}{1000} = \frac{\sqrt{3} \times 400 \times 16.5}{1000}$

$= 11.43\text{kVA}$

(b) Output power = 9kW

$$\text{True active power} = \frac{9.0}{0.89} = 10.11\text{kW}$$

$$\text{So Power factor} = \frac{\text{active power}}{\text{apparent power}} = \frac{10.11}{11.43} = 0.88 \text{ (lagging)}.$$

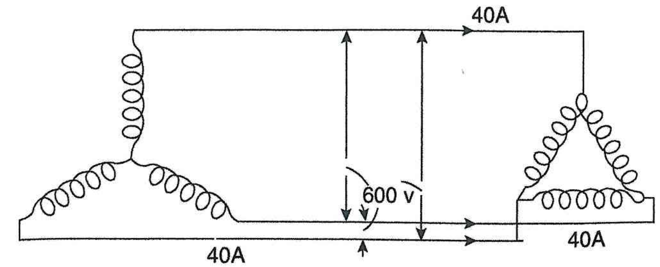
Combining star and delta networks provides an interesting example.

Example 11.14. A 3-phase, star-connected alternator supplies a delta-connected induction motor at 600V. The current taken is 40A. Find (a) the alternator's phase voltage (3 significant figures) and (b) the current in each phase of the motor (1 decimal place) refer to figure 11.17.

(a) For a star connection $V = \sqrt{3}\,V_{ph}$

$$\therefore V_{ph} = \frac{V}{\sqrt{3}} = \frac{600}{\sqrt{3}} = 346\text{V}. \text{ This is the alternator phase voltage.}$$

(b) For a delta connection $I_{ph} = \frac{I}{\sqrt{3}} = \frac{40}{\sqrt{3}} = 23.1\text{A}$



▲ Figure 11.17

(c) If the motor operates at a power factor of 0.8 (lagging) and an efficiency of 88%. Find the alternator's kVA rating (1 decimal place) and the motor's power output (2 decimal places).

$$\text{Apparent power rating of alternator} = \frac{\sqrt{3}VI}{1000}$$

$$= \frac{\sqrt{3} \times 60 \times 40}{1000} = 41.6\text{KVA}$$

$$\text{Motor apparent input power} = \frac{\sqrt{3}VI}{1000}$$

$$= \frac{1.732 \times 600 \times 40}{1000} = 41.6\text{KVA}$$

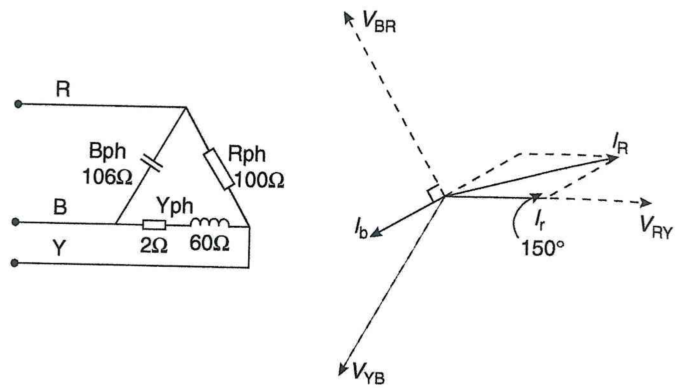
$$\text{True active input power} = 41.6 \times 0.8 = 33.28\text{kW}$$

$$\text{Output power} = 33.28 \times \frac{88}{1000} = 29.21\text{kW}$$

Example 11.15. A delta-connected load is as shown (figure 11.18). If the supply voltage is 400V, 50Hz, calculate the red-line current (I_R) (1 decimal place). Assume the currents as shown and maintain the correct phase sequence: R, Y and B. If the red-line current feeds current into the red phase of the load, the blue-phase current will be in the opposite sense so a phasor difference is involved.

Here $I_r = \frac{400}{100}$ 4A in-phase with V_{RY}

Also $I_b = \frac{400}{106} = 3.7\text{A}$ leading V_{BR} by 90°



▲ **Figure 11.18**

$$I_R = \sqrt{I_r^2 + I_b^2 + 2I_rI_b\cos 150^\circ}$$

$$= \sqrt{4^2 + 3.7^2 - 2\times 4\times 3.7\cos 150^\circ}$$

or I_R $= 7.4A.$

Practice Examples

11.1. A coil consumes 300W when connected into a 60V D.C. circuit. It consumes 1200W when connected into an A.C. circuit of 130V. What is the coil's reactance (1 significant figure)?

11.2. A circuit consists of 2 branches A and B in parallel. Branch A has a resistance of 12Ω and a reactance of 3Ω, while the values of branch B are 8Ω and 20Ω respectively. The circuit is supplied at 100V. Calculate the current in each branch (1 decimal place) and the supply current (1 decimal place).

11.3. An inductive circuit of resistance 50Ω and inductance 0.02H is connected in *parallel* with a 25µF capacitor across a 200V, 50Hz mains supply. Find the total current taken from the supply (1 decimal place) and its phase angle.

11.4. Two coils of resistances 8Ω and 10Ω and inductances 0.02H and 0.05H respectively are connected in parallel across 100V, 50Hz mains supply. A capacitor of capacitance 80µF in *series* with a 20Ω resistor is then connected in *parallel* with the coils. Find the total current taken from the mains supply (2 decimal places) and its phase angle with respect to the applied voltage.

11.5. A single-phase motor has an input of 50.6A at 240V, the power input being 10kW and the output 9kW. Calculate the value of the apparent power (1 decimal place), power factor (2 decimal place) and the efficiency.

11.6. A single-phase motor running from a 230V, 50Hz mains supply takes a current of 11.6A when giving an output of 1.5kW, the efficiency being 80%. Calculate the capacitance required to bring the power factor of the supply current to 0.95 (lagging) (2 significant figures). Calculate the capacitor's kVAr rating (2 decimal places).

11.7. The load taken from a single-phase supply consists of:

(a) Filament lamp load of 10kW at unity power factor.

(b) Motor load of 80kVA at 0.8 power factor (lagging).

(c) Motor load of 40kVA at 0.7 power factor (leading).

Calculate the total load taken from the supply in kW (3 significant figures) and in kVA (2 decimal places) and the power factor of the combined load (2 decimal places). Find the 'mains' current if the supply voltage is 250V (3 significant figures).

11.8. Three equal impedances of 10Ω, each with a phase angle of 30° (lagging), constitute a load on a 3-phase alternator, giving 100V per phase. Find the current per line and the total power when connected as follows: (a) Alternator in star, load in star, (b) Alternator in star, load in delta, (c) Alternator in delta, load in delta, (d) Alternator in delta, load in star (all 1 decimal place).

11.9. A 500V, 3-phase, star-connected alternator supplies a star-connected induction motor rated 45kW. The motor efficiency is 88% and the power factor 0.9 (lagging). The alternator efficiency at this load is 80%. Determine (a) the line current (1 decimal place), (b) the alternator power output (2 decimal places) and (c) the output power of the prime-mover (2 significant figures).

11.10. A 400V, 3-phase system takes 40A at a power factor of 0.8 (lagging). An over-excited synchronous motor is connected to raise the power factor of the combination to unity. If the motor's mechanical output is 12kW with efficiency 91%, find the kVA input to the motor and its power factor. Find also the total power taken from the supply mains (all 2 decimal places).

12

THE D.C. GENERATOR

The origin as well as the progress and improvement of civil society is founded in mechanical and chemical inventions.

Sir Humphry Davy

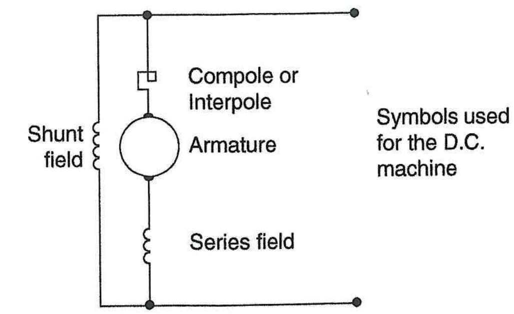
This chapter will not cover D.C. machine construction, operation and maintenance in detail, as this is better dealt with practically. The basic features of the D.C. generator will be covered and help to consolidate the theory introduced in Chapter 7. Once the D.C. generator construction is outlined, attention can then be given to further theory in Volume 7, and will be considered from a functional viewpoint, under the headings: (1) The Generator and (2) The Motor. Recently there has been a trend to remove detailed study of the D.C. generator from marine electrotechnology syllabi. Nevertheless, study is valued as it helps define the differences between the generator – a machine to convert mechanical into electrical energy, from the motor – a machine to convert electrical into mechanical energy.

D.C. Machine Construction

The principal features of D.C. machines are described under (1) the field system or stator and (2) the armature or rotor. Figure 12.1 is a representation for a D.C. compound-wound generator.

Field system

This includes the magnet arrangement of poles and yoke, the field coils and interpoles (if fitted). Interpoles are part of the armature electrical circuit and are mentioned under the heading of POLES AND YOKE.



▲ Figure 12.1

electromagnets and are often fitted with pole-shoes to concentrate the field across the narrow air gaps in which conductors move. The yoke is an extension of a magnet system forming the main frame of the machine carrying flux from and to the poles. Figure 12.2 shows typical field system construction.

Poles and yoke are constructed from cast steel or fabricated from mild steel sheet cut and rolled into shape. Poles may be part of the yoke, but are now usually built up in thin laminate form, riveted together and shaped to include the pole-shoes. Interpoles are similar to the main field poles but located on the yolk between them, having windings in series with the armature to reduce armature reaction effects.

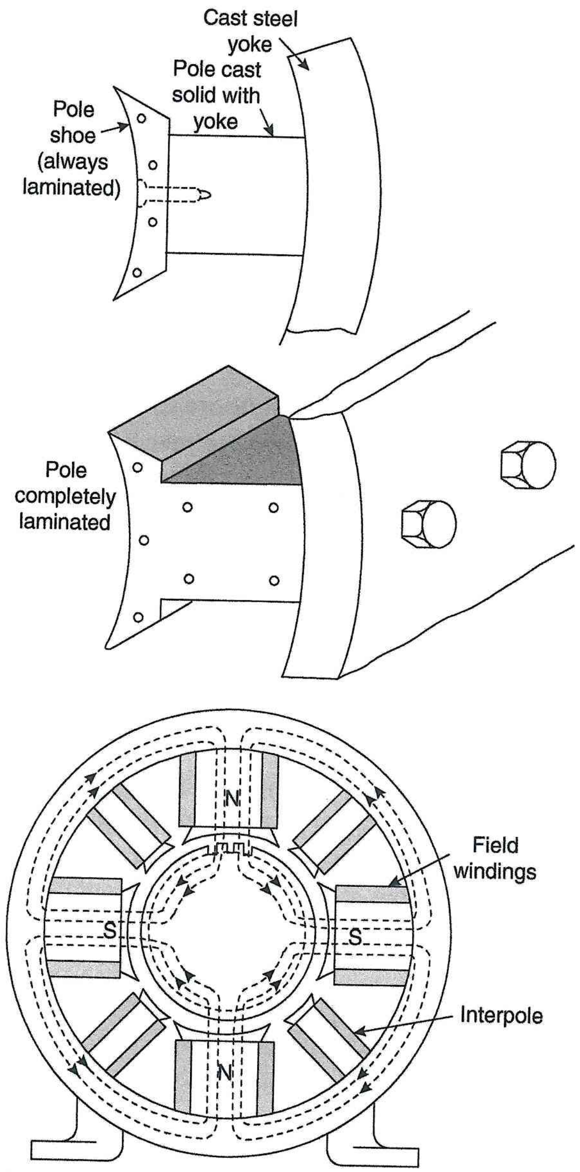
FIELD COILS. Field coils are of 2 types: (1) shunt coils – made of many turns of fine wire and (2) series coils – made from only a few turns of thick cable or conductor. Shunt coils are built on a 'bobbin' or 'former' while series coils may be self-supporting. Figure 12.3 shows a typical construction cross-section with insulation determined by the machine class and its duty.

The armature

This consists of armature core, windings, shaft and commutator. The brushes, although not part of the armature, are considered as they work with the commutator.

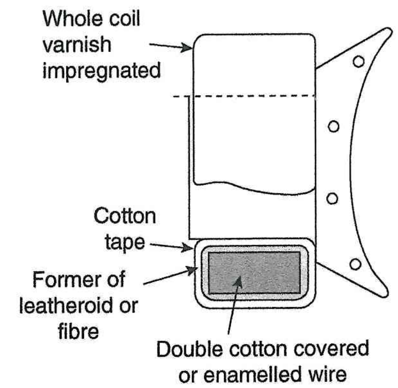
ARMATURE CORE. This is built up from iron laminations clamped between 2 end plates. Laminations are insulated from each other to minimise induced circulating or 'eddy currents'. If clamping bolts pass through the core as shown (figure 12.4), they must be insulated.

Modern methods use stamped laminations pressed onto and 'keyed' to a shaft, with end plates screwed onto small machine shafts. For larger designs a 'spider' shape helps ventilation and keeps the iron volume needed to a minimum.

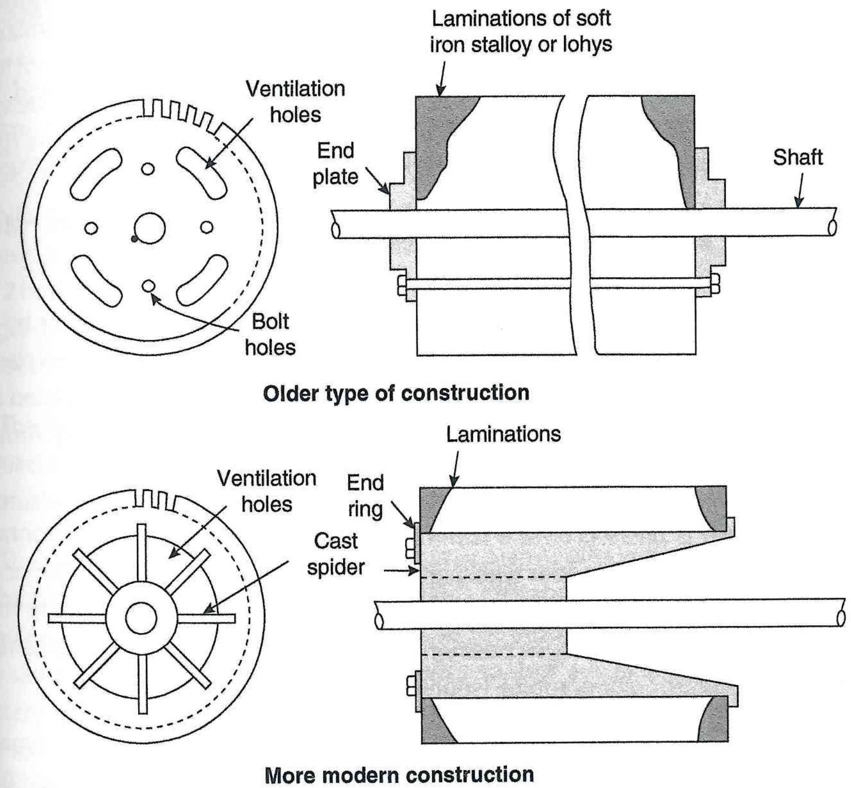


▲ Figure 12.2

WINDINGS. The number of conductors, their size, shape, etc. are decided by the machine's design requirements. Figure 12.5 shows a typical method of locating and holding coil sides in place. A wedge of bakelised paper or fabric is shown, but open slots with a piece of fibre and 'binders', made from high-tensile steel wire, can be used. For small machines, for example, motors for vacuum-cleaners, cabin-fans etc. armature



▲ Figure 12.3



▲ Figure 12.4