4.5.2 Nonideal or Actual Transformer

In the previous section, certain assumptions were made in characterizing an ideal transformer. These assumptions are no longer applicable when analyzing the performance of an actual transformer.

A *nonideal transformer* is illustrated in Fig. 4.10. It can be described as having resistances in its windings. Not all of the flux produced by one winding will link the other winding because of flux leakage. The core of the actual transformer is not perfectly permeable; it has a finite permeability. Thus, it requires a finite mmf for its magnetization. Because the flux in the core is alternating, there are hysteresis and eddy current losses, collectively called *core losses* or *iron losses*.

In deriving the equivalent circuit for the two-winding transformer of Fig. 4.10, the characteristics of an actual transformer described earlier need to be modeled.

Consider the primary circuit. A voltage equation around the loop may be written as

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} = R_1 i_1 + N_1 \frac{d\phi_1}{dt} \tag{4.39}$$

where

R_1 = resistance of primary winding

N_1 = number of turns for primary winding

The primary winding flux ϕ_1 may be expressed as the sum of the mutual flux ϕ_m and the primary leakage ϕ_{l1}:

$$\phi_1 = \phi_m + \phi_{l1} \tag{4.40}$$

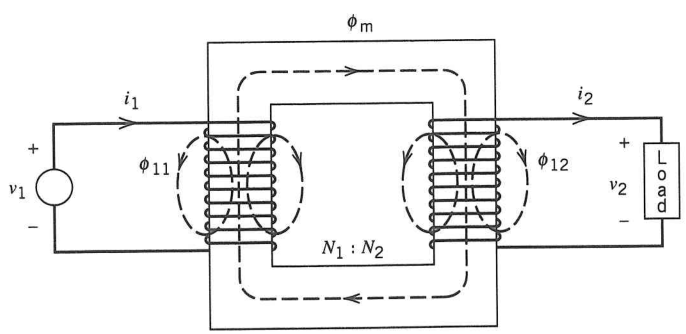


FIGURE 4.10 An actual transformer.

Thus, Eq. 4.39 reduces to

$$v_1 = R_1 i_1 + N_1 \frac{d\phi_{l1}}{dt} + N_1 \frac{d\phi_m}{dt} \tag{4.41}$$

Since the leakage flux ϕ_{l1} is a linear function of the primary current i_1, the second term on the right-hand side of Eq. 4.41 may be expressed in terms of the inductance of the primary winding. Thus,

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + N_1 \frac{d\phi_m}{dt} \tag{4.42}$$

The secondary circuit is considered next. From Fig. 4.10, the voltage equation in the secondary may be written as follows:

$$v_2 = -R_2 i_2 + \frac{d\lambda_2}{dt} = -R_2 i_2 + N_2 \frac{d\phi_2}{dt} \tag{4.43}$$

From the flux directions, the secondary flux may be represented by the difference between the mutual flux and the secondary leakage flux:

$$\phi_2 = \phi_m - \phi_{l2} \tag{4.44}$$

Substituting Eq. 4.44 into 4.43 yields

$$v_2 = -R_2 i_2 - N_2 \frac{d\phi_{l2}}{dt} + N_2 \frac{d\phi_m}{dt} \tag{4.45}$$

Similarly, the leakage flux ϕ_{l2} is a linear function of the secondary current i_2. Thus, Eq. 4.45 may be written using the inductance of the secondary winding as

$$v_2 = -R_2 i_2 - L_2 \frac{di_2}{dt} + N_2 \frac{d\phi_m}{dt} \tag{4.46}$$

In Eqs. 4.42 and 4.46, the last terms represent the induced voltages across the primary and secondary windings, respectively; that is,

$$e_1 = N_1 \frac{d\phi_m}{dt} \tag{4.47}$$

$$e_2 = N_2 \frac{d\phi_m}{dt} \tag{4.48}$$

Dividing Eq. 4.47 by Eq. 4.48 yields the voltage ratio:

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = a \tag{4.49}$$

Figure 4.11 shows the equivalent circuit of the two-winding transformer of Fig. 4.10. The circuit elements that are used to model the core magnetization and the core losses can be added to either the primary side or the secondary side. In Fig. 4.11, the inductor L_{m1} representing the core magnetization and the resistor R_{c1} representing the core losses (hysteresis and eddy current losses) have been connected in parallel and located in the primary side of the transformer equivalent circuit.

The core-related circuit elements R_{c1} and L_{m1} are usually determined at rated voltage and are referred to the primary side in Fig. 4.11. They are assumed to remain essentially constant when the transformer operates at, or near, rated conditions.

In phasor form, the transformer equivalent circuit takes the form shown in Fig. 4.12. The reactances are derived by multiplying the inductances by the radian frequency $\omega = 2\pi f$, where f is the frequency. The turns ratio $a = N_1/N_2$ is approximately equal to the voltage ratio V_1/V_2, the ratio of the rated primary voltage to the rated secondary voltage provided by the manufacturer.

A phasor diagram for a lagging power factor (inductive) load connected across the secondary of the transformer of Fig. 4.12 is shown in Fig. 4.13.

The notation used is as follows:

$\mathbf{E}_1$ = primary induced voltage
$\mathbf{E}_2$ = secondary induced voltage
$\mathbf{V}_1$ = primary terminal voltage

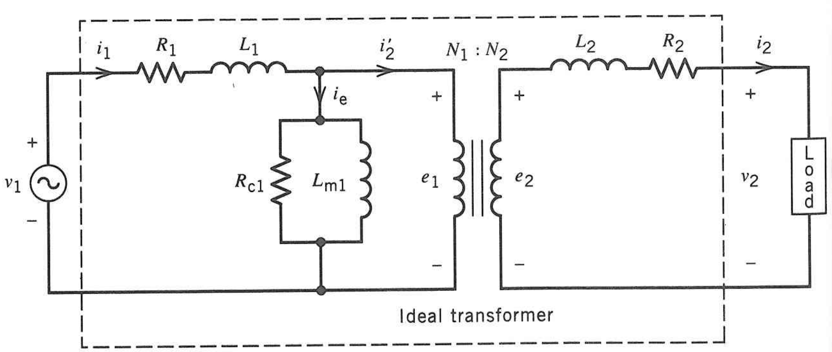


FIGURE 4.11 Equivalent circuit of a transformer.

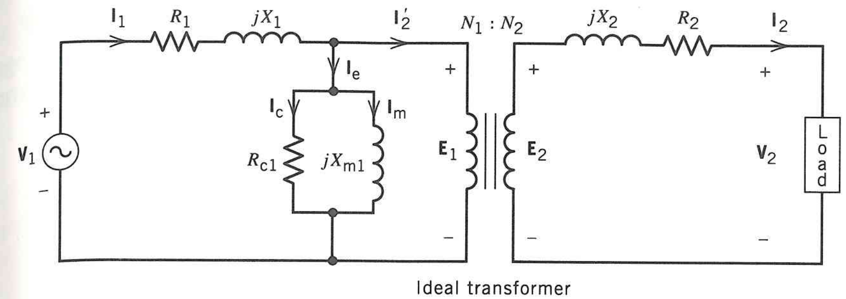


FIGURE 4.12 Transformer equivalent circuit in phasor form.

$\mathbf{V}_2$ = secondary terminal voltage
$\mathbf{I}_1$ = primary current
$\mathbf{I}_2$ = secondary current
$\mathbf{I}_e$ = excitation current
$\mathbf{I}_m, X_m$ = magnetizing current and reactance
$\mathbf{I}_c, R_c$ = current and resistance representing core loss
R_1 = resistance of the primary winding
R_2 = resistance of the secondary winding
X_1 = primary leakage reactance
X_2 = secondary leakage reactance

In the transformer equivalent circuit of Fig. 4.12, the ideal transformer can be moved out to the right or to the left of the equivalent circuit by referring all quantities to the primary or secondary, respectively, as shown in Fig. 4.14. This is almost always done because of the great simplicity it introduces in transformer performance analysis.

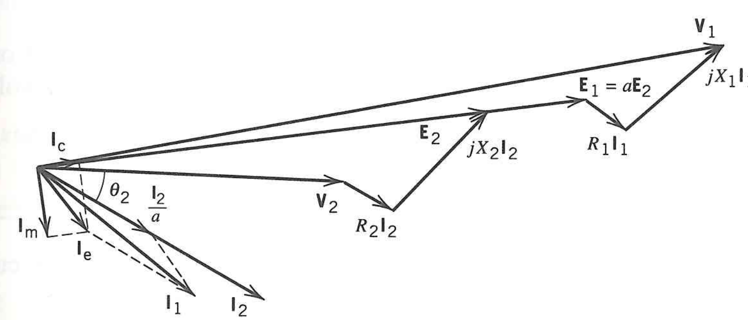


FIGURE 4.13 Phasor diagram for Fig. 4.12.

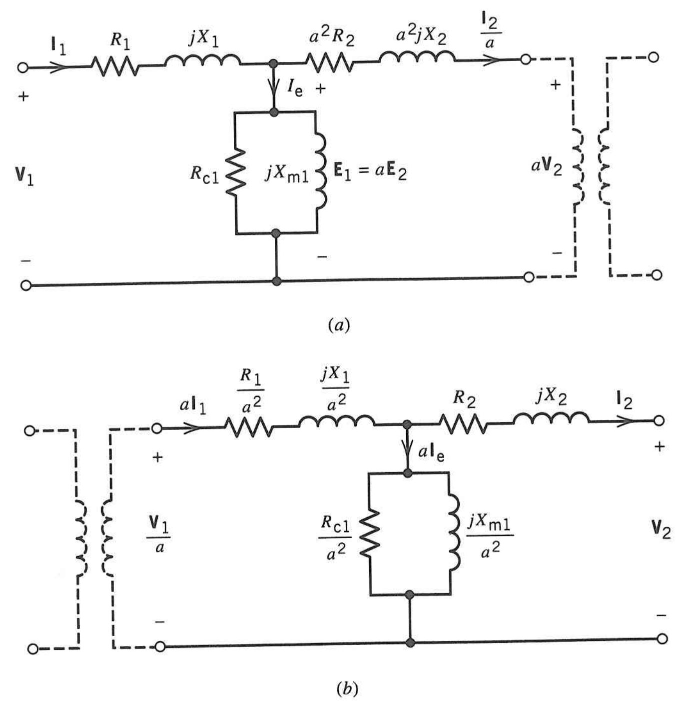


FIGURE 4.14 Referred transformer equivalent circuit: (*a*) referred to primary; (*b*) referred to secondary.

EXAMPLE 4.6

A 25-kVA, 440/220-V, 60-Hz transformer has the following parameters:

$$R_1 = 0.16\ \Omega \quad R_2 = 0.04\ \Omega \quad R_{c1} = 270\ \Omega$$
$$X_1 = 0.32\ \Omega \quad X_2 = 0.08\ \Omega \quad X_{m1} = 100\ \Omega$$

The transformer delivers 20 kW at 0.8 power factor lagging to a load on the low-voltage side with 220 V across the load. Find the primary terminal voltage.

Solution The voltage across the load is taken as reference phasor; thus,

$$\mathbf{V}_2 = 220\angle 0^\circ \text{ V}$$

For a load $P_2 = 20{,}000$ W at 0.8 power factor lagging, the secondary current is computed as follows:

$$\mathbf{I}_2 = \frac{20{,}000}{(220)(0.8)}\angle -\cos^{-1} 0.8 = 113.64\angle -36.9^\circ \text{ A}$$

The transformer turns ratio is $a = 440/220 = 2$. Thus, the secondary voltage and current and the winding resistance and reactance are referred to the primary side as follows:

$$a\mathbf{V}_2 = 2(220\angle 0^\circ) = 440\angle 0^\circ \text{ V}$$
$$\mathbf{I}_2/a = (113.64\angle -36.9^\circ)/2 = 56.82\angle -36.9^\circ \text{ A}$$
$$a^2R_2 = (2)^2(0.04) = 0.16\ \Omega$$
$$a^2X_2 = (2)^2(0.08) = 0.32\ \Omega$$

Referring to the phasor diagram of Fig. 4.13, the primary induced voltage is calculated as follows:

$$\begin{aligned}\mathbf{E}_1 &= a\mathbf{V}_2 + (\mathbf{I}_2/a)(a^2R_2 + ja^2X_2)\\ &= 440\angle 0^\circ + (56.82\angle -36.9^\circ)(0.16 + j0.32)\\ &= 458.2 + j9.07 = 458.3\angle 1^\circ \text{ V}\end{aligned}$$

The shunt branch currents are

$$\mathbf{I}_c = \mathbf{E}_1/R_{c1} = (458.2 + j9.07)/270 = 1.7 + j0.03 \text{ A}$$
$$\mathbf{I}_m = \mathbf{E}_1/jX_{m1} = (458.2 + j9.07)/j100 = 0.09 - j4.58 \text{ A}$$
$$\mathbf{I}_e = \mathbf{I}_c + \mathbf{I}_m = 1.79 - j4.55 \text{ A}$$

Thus, the primary current is

$$\begin{aligned}\mathbf{I}_1 &= \mathbf{I}_e + \mathbf{I}_2/a\\ &= (1.79 - j4.55) + (56.82\angle -36.9^\circ) = 61.04\angle -39.3^\circ \text{ A}\end{aligned}$$

Therefore, the primary voltage is found from

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{E}_1 + \mathbf{I}_1(R_1 + jX_1)\\ &= (458.2 + j9.07) + (61.04\angle -39.3^\circ)(0.16 + j0.32)\\ &= 478.1 + j18 = 478.4\angle 2.2^\circ \text{ V}\end{aligned}$$

4.5.3 Approximate Equivalent Circuits

The derivation of approximate equivalent circuits begins with the diagrams shown in Fig. 4.14. All quantities have been referred to the same side of the transformer, and the ideal transformer may be omitted from the equivalent circuit.

The first step in the simplification process is to move the shunt magnetization branch from the middle of the T circuit to either the primary or secondary terminal, as shown in Fig. 4.15*a* and *b*. This step neglects the voltage drop across the primary or secondary winding caused by the exciting current. The voltage drop caused by the load component of the current is still included, of course. The error introduced in this step is generally very small in most problems involving power transformers.

The primary and secondary winding resistances are combined to give either the equivalent resistance referred to the primary side $R_{e1} = R_1 + a^2R_2$ or the equivalent resistance referred to the secondary side $R_{e2} = R_1/a^2 + R_2$. Similarly, the primary and secondary winding reactances are combined to obtain either the equivalent reactance referred to the primary side $X_{e1} = X_1 + a^2X_2$ or the equivalent reactance referred to the secondary side $X_{e2} = X_1/a^2 + X_2$.

The next step in deriving the approximate equivalent circuit is the deletion of the shunt magnetizing branch completely. Thus, the transformer equivalent circuit reduces to a simple equivalent series impedance referred to either primary or secondary, as shown in Fig. 4.15*c* and *d*.

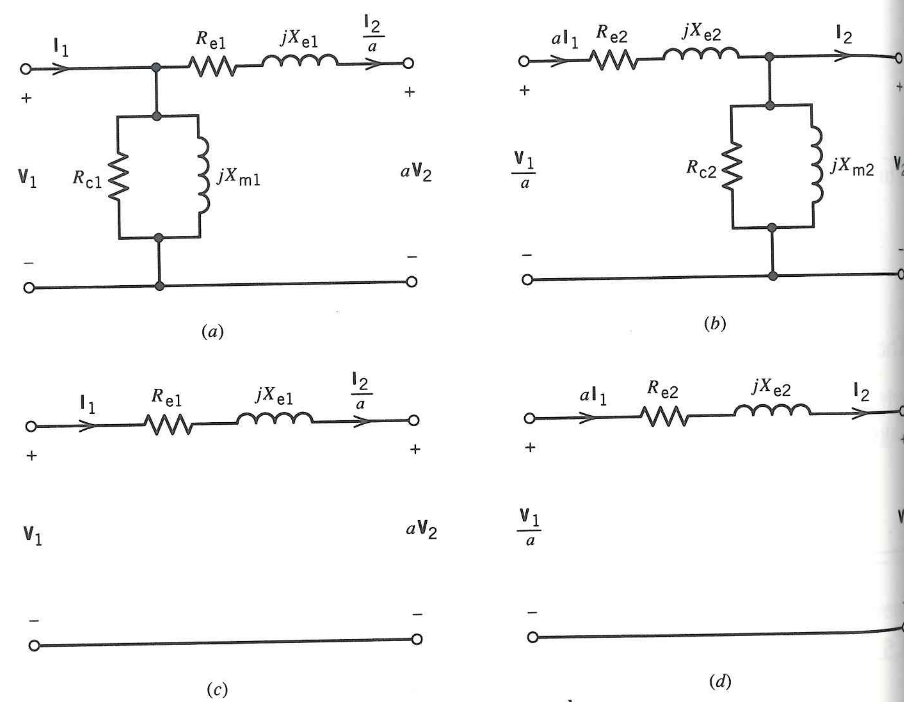


FIGURE 4.15 Approximate equivalent circuits.

4.5.4 Voltage Regulation

Distribution and power transformers are often used to supply loads that are designed to operate at essentially constant voltage. The amount of secondary (load) current drawn depends on the load connected to the transformer secondary terminals. As this current changes with changing load, with the same applied primary voltage, the load voltage likewise changes. This change is due to the voltage drop across the internal impedance of the transformer. A measure of how much the voltage will change as load is varied is called *voltage regulation*.

The voltage regulation of a transformer is defined as the change in the magnitude of the secondary voltage as the current changes from full load to no load with the primary voltage held fixed. This is expressed mathematically as

$$\text{Voltage regulation} = \frac{|\mathbf{V}_{2,\text{nl}}| - |\mathbf{V}_{2,\text{fl}}|}{|\mathbf{V}_{2,\text{fl}}|}100\% \tag{4.50a}$$

$$= \frac{|\mathbf{V}_1| - |a\mathbf{V}_2|}{|a\mathbf{V}_2|}100\% \tag{4.50b}$$

$$= \frac{|\mathbf{V}_1/a| - |\mathbf{V}_2|}{|\mathbf{V}_2|}100\% \tag{4.50c}$$

The use of Eq. 4.50 in calculating the voltage regulation implies that the primary voltage V_1 is adjusted to a value that provides rated secondary voltage V_2 across the load when rated secondary current is supplied to the load. Then as current is reduced to zero or no load, the increase in secondary voltage is determined.

4.5.5 Efficiency

The percent *efficiency* of the transformer is defined as the ratio of the power output P_{output} to the power input P_{input}, both expressed in watts, multiplied by 100%. Thus,

$$\eta = \frac{P_{\text{output}}}{P_{\text{input}}}\ 100\% \tag{4.51}$$

This can also be expressed as

$$\eta = \frac{P_{\text{output}}}{P_{\text{output}} + \Sigma(\text{losses})}\ 100\% \tag{4.52}$$

or

$$\eta = \frac{P_{\text{input}} - \Sigma(\text{losses})}{P_{\text{input}}} 100\% \tag{4.53}$$

where

$$\Sigma(\text{losses}) = \text{core loss} + \text{copper loss} \tag{4.54a}$$
$$\Sigma(\text{losses}) = P_{\text{core}} + (I_1^2 R_1 + I_2^2 R_2) \tag{4.54b}$$
$$\Sigma(\text{losses}) = P_{\text{core}} + I_1^2 R_{e1} \tag{4.54c}$$
$$\Sigma(\text{losses}) = P_{\text{core}} + I_2^2 R_{e2} \tag{4.54d}$$

EXAMPLE 4.7

A 150-kVA, 2400/240-V transformer has the following parameters referred to the primary side: $R_{e1} = 0.5\ \Omega$ and $X_{e1} = 1.5\ \Omega$. The shunt magnetizing impedance is very large and can be neglected. At full load, the transformer delivers rated kVA at 0.85 lagging power factor and the secondary voltage is 240 V. Calculate (a) the voltage regulation and (b) the efficiency assuming core losses amount to 600 W.

Solution

a. The transformer turns ratio is

$$a = 2400/240 = 10$$

Take the secondary voltage $\mathbf{V}_2$ as the reference phasor.

$$\mathbf{V}_2 = 240\angle 0^\circ \text{ V}$$
$$a\mathbf{V}_2 = 2400\angle 0^\circ \text{ V}$$

At rated load and 0.85 PF lagging:

$$\mathbf{I}_2 = (150{,}000/240)\angle -\cos^{-1} 0.85^\circ = 625\angle -31.8^\circ \text{ A}$$
$$\mathbf{I}_1 = \mathbf{I}_2/a = (625/10)\angle -31.8^\circ = 62.5\angle -31.8^\circ \text{ A}$$

The primary voltage is calculated as follows:

$$\mathbf{V}_1 = a\mathbf{V}_2 + (\mathbf{I}_2/a)(R_{e1} + jX_{e1})$$
$$= 2400\angle 0^\circ + (62.5\angle -31.8^\circ)(0.5 + j1.5) = 2476.8\angle 1.5^\circ \text{ V}$$

Thus, the percent voltage regulation is found by using Eq. 4.50b as follows:

$$\text{Voltage regulation} = \frac{V_1 - aV_2}{aV_2} 100\%$$
$$= \frac{2476.8 - 2400}{2400} 100\% = 3.2\%$$

b. At rated output,

$$P_{\text{output}} = (150{,}000)(0.85) = 127{,}500 \text{ W}$$
$$P_{\text{cu}} = I_1^2 R_{e1} = (62.5)^2(0.5) = 1953 \text{ W}$$
$$P_{\text{core}} = 600 \text{ W}$$
$$P_{\text{input}} = P_{\text{output}} + \Sigma(\text{losses}) = 130{,}053 \text{ W}$$

Therefore, the efficiency is found by using Eq. 4.51 as follows:

$$\eta = \frac{\text{power output}}{\text{power input}} 100\%$$
$$= \frac{127{,}500}{130{,}053} 100\% = 98\%$$

4.5.6 Determination of Equivalent Circuit Parameters

Two simple tests are used to determine the values for the parameters of the transformer equivalent circuit of Fig. 4.15*a* and *b*. If it is desired to find the parameters of the exact equivalent circuit of Fig. 4.12, it is customary to assume $R_1 = a^2 R_2$ and $X_1 = a^2 X_2$. This assumption allows the decomposition of the equivalent resistance and reactance into the primary and secondary components.

The two tests are the short-circuit and open-circuit tests. Let the primary (winding 1) be the high-voltage side and the secondary (winding 2) the low-voltage side for the transformer of Fig. 4.10.

Open-Circuit Test In the *open-circuit test*, the transformer rated voltage is applied to the low-voltage side of the transformer with the high-voltage side left open. Measurements of power, current, and voltage are made on the low-voltage side as shown in Fig. 4.16.

Since the high-voltage side is open, the input current I_{oc} is equal to the exciting current through the shunt excitation branch as shown in the equivalent circuit of Fig. 4.17. Because this current is very small, about 5% of rated value, the voltage drop across the low-voltage winding and the winding copper losses are neglected.

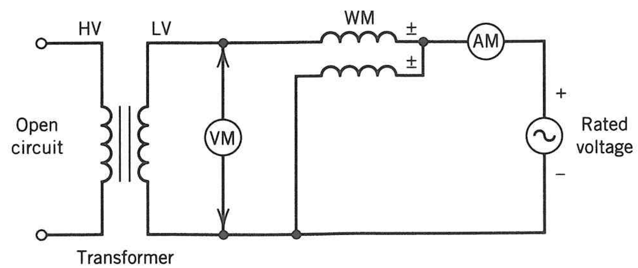


FIGURE 4.16 Connections for open-circuit test.

The magnitude of the admittance of the shunt excitation branch of the equivalent circuit referred to the low-voltage side is calculated as follows:

$$|Y_{o2}| = \frac{I_{oc}}{V_{oc}} \tag{4.55}$$

The phase angle of the admittance is found as

$$-\theta_{o2} = -\cos^{-1}\left(\frac{P_{oc}}{V_{oc}I_{oc}}\right) \tag{4.56}$$

Thus, the complex admittance may be expressed as

$$Y_{o2} = |Y_{o2}|\angle{-\theta_{o2}} = G_{c2} - jB_{m2} \tag{4.57}$$

The corresponding resistance and reactance parameters of Fig. 4.15*b* are derived from the conductance and susceptance, respectively:

$$R_{c2} = \frac{1}{G_{c2}} \tag{4.58}$$

$$jX_{m2} = \frac{1}{-jB_{m2}} \tag{4.59}$$

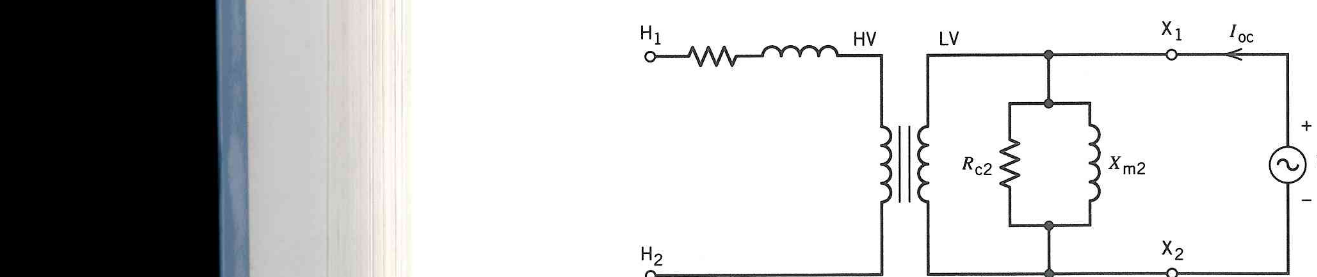


FIGURE 4.17 Equivalent circuit for open-circuit test.

These parameters may be referred to the high-voltage side to give the parameters of the equivalent circuit of Fig. 4.15*a*.

$$R_{c1} = a^2R_{c2} \tag{4.60}$$

$$X_{m1} = a^2X_{m2} \tag{4.61}$$

Note that when rated voltage at rated frequency is applied during the open-circuit test, the power input P_{oc} is practically equal to the rated core loss. In most applications, this value of core loss is typically assumed to remain constant for different load levels.

Short-Circuit Test In the *short-circuit test*, the low-voltage side is short-circuited and the high-voltage side is connected to a variable, low-voltage source. Measurements of power, current, and voltage are made on the high-voltage side as shown in Fig. 4.18.

The applied voltage is adjusted until rated short-circuit current flows in the windings. This voltage is generally much smaller than the rated voltage, in the range of 0.05 to 0.10 per unit. Therefore, the current through the magnetizing branch is negligible, and the applied voltage may be assumed to occur wholly as a voltage drop across the transformer series impedance as shown in the equivalent circuit of Fig. 4.19.

The magnitude of the series impedance referred to the high-voltage (primary) side may be calculated as follows:

$$|Z_{e1}| = \frac{V_{sc}}{I_{sc}} \tag{4.62}$$

The equivalent series resistance referred to the high-voltage side is

$$R_{e1} = \frac{P_{sc}}{I_{sc}^2} = R_1 + a^2R_2 \tag{4.63}$$

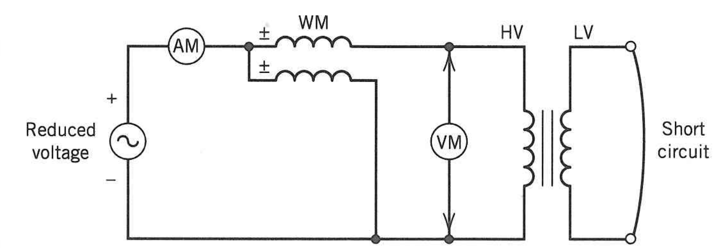


FIGURE 4.18 Connections for short-circuit test.

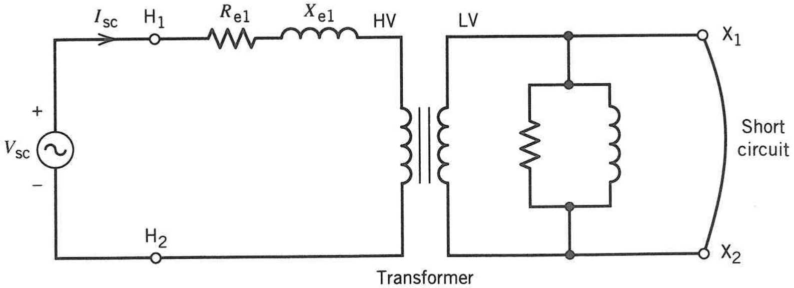


FIGURE 4.19 Equivalent circuit for short-circuit test.

Correspondingly, the equivalent series reactance referred to the high-voltage side is

$$X_{e1} = \sqrt{|Z_{e1}|^2 - R_{e1}^2} = X_1 + a^2X_2 \tag{4.64}$$

The values of these parameters derived from the short-circuit test are used in conjunction with Fig. 4.15*a*. These parameters may be referred to the low-voltage (secondary) side to derive the corresponding values for use with Fig. 4.15*b* as follows:

$$R_{e2} = \left(\frac{1}{a^2}\right)R_{e1} \tag{4.65}$$

$$X_{e2} = \left(\frac{1}{a^2}\right)X_{e1} \tag{4.66}$$

Note that when rated current flows through the windings during the short-circuit test, the power input P_{sc} is equal to the rated copper loss.

Tests on Three-Phase Transformers When a three-phase transformer undergoes open-circuit and short-circuit tests, it must be remembered that the power being measured is total three-phase power, the measured voltage is line-to-line voltage, and the measured current is line current. The impedance parameters of interest are to be calculated on a per-phase basis. Therefore, before using the formulas derived above to calculate the resistances and reactances, the measured values must also be converted to per-phase values. Three-phase transformers are discussed later in Section 4.7.

EXAMPLE 4.8

A 50-kVA, 2400/240-V, 60-Hz single-phase transformer has a short-circuit test performed on its high-voltage side. An open-circuit test is performed on the low-voltage side. The following test results were obtained:

	Voltage (V)	Current (A)	Power (W)
Short-circuit (LV shorted)	48	20.8	620
Open-circuit (HV open)	240	5.4	186

a. Draw the transformer's equivalent circuit.

b. Determine its voltage regulation, and efficiency at rated load, 0.8 power factor lagging, and rated voltage at the secondary terminals.

Solution The ratings of the transformer, with the high-voltage side as primary, are as follows:

Rated primary voltage $V_1 = 2400$ V
Rated secondary voltage $V_2 = 240$ V
Rated primary current $I_1 = 50{,}000/2400 = 20.83$ A
Rated secondary current $I_2 = 50{,}000/240 = 208.3$ A
Turns ratio $a = 2400/240 = 10$

a. The equivalent circuit for the short-circuit test is shown in Fig. 4.19. The series impedance parameters are calculated as follows:

$$|Z_{e1}| = V_{sc}/I_{sc} = 48/20.8 = 2.30\ \Omega$$
$$R_{e1} = P_{sc}/I_{sc}^2 = 620/(20.8)^2 = 1.43\ \Omega$$
$$X_{e1} = \sqrt{|Z_{e1}|^2 - R_{e1}{}^2} = \sqrt{(2.30)^2 - (1.43)^2} = 1.80\ \Omega$$

The equivalent circuit for the open-circuit test is shown in Fig. 4.17. The parameters of the shunt magnetizing branch are computed as follows:

$$|Y_{o2}| = \frac{I_{oc}}{V_{oc}} = \frac{5.4}{240} = 0.0225\ \text{S}$$
$$-\theta_2 = -\cos^{-1}\left(\frac{P_{oc}}{V_{oc}I_{oc}}\right) = -\cos^{-1}\left[\frac{186}{(240)(5.4)}\right] = -81.8°$$
$$Y_{o2} = 0.0225\ \angle{-81.8°} = (3.23 - j22.3) \times 10^{-3}\ \text{S} = G_{c2} - jB_{m2}$$
$$R_{c2} = \frac{1}{G_{c2}} = \frac{1}{3.23 \times 10^{-3}} = 309.6\ \Omega$$
$$X_{m2} = \frac{1}{B_{m2}} = \frac{1}{22.3 \times 10^{-3}} = 44.8\ \Omega$$

The shunt magnetizing branch may be referred to the high-voltage side as

$$R_{c1} = a^2R_{c2} = (10)^2(309.6) = 30.96\ \text{k}\Omega$$
$$X_{m1} = a^2X_{m2} = (10)^2(44.8) = 4.48\ \text{k}\Omega$$

Finally, the transformer equivalent circuit is drawn as shown in Fig. 4.20.

b. At rated secondary conditions, and 0.8 power factor lagging:

$$a\mathbf{V}_2 = (10)(240\angle 0^\circ) = 2400\angle 0^\circ \text{ V}$$

$$\frac{\mathbf{I}_2}{a} = \frac{208.3\angle -36.9^\circ}{10} = 20.83\angle -36.9^\circ \text{ A}$$

The primary voltage is computed as follows:

$$\begin{aligned}\mathbf{V}_1 &= a\mathbf{V}_2 + (\mathbf{I}_2/a)(R_{e1} + jX_{e1}) \\ &= 2400\angle 0^\circ + (20.83\angle -36.9^\circ)(1.43 + j1.80) \\ &= 2446.3 + j12.1 = 2446.4\angle 0.3^\circ \text{ V}\end{aligned}$$

The percent voltage regulation is found by using Eq. 4.50b as follows:

$$\begin{aligned}\text{Voltage regulation} &= \frac{V_1 - aV_2}{aV_2} 100\% \\ &= \frac{2446.4 - 2400}{2400} 100\% = 1.9\%\end{aligned}$$

At rated load and 0.8 power factor, the power input is computed as the sum of the power output and losses as follows:

$$\begin{aligned}P_{\text{output}} &= (50{,}000)(0.80) = 40{,}000 \text{ W} \\ \Sigma(\text{losses}) &= \text{core loss} + \text{copper loss} = P_{oc} + I_1^2 R_{e1} \\ &= 186 + (20.83)^2(1.43) = 806 \text{ W} \\ P_{\text{input}} &= P_{\text{output}} + \Sigma(\text{losses}) = 40{,}000 + 806 = 40{,}806 \text{ W}\end{aligned}$$

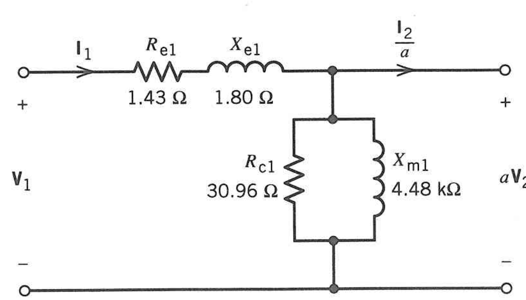


FIGURE 4.20 Transformer equivalent circuit for Example 4.7.

Therefore, the efficiency of the transformer is found by using Eq. 4.51 as follows:

$$\begin{aligned}\eta &= \frac{\text{power output}}{\text{power input}} 100\% \\ &= \frac{40{,}000}{40{,}806} 100\% = 98\%\end{aligned}$$

4.5.7 Polarity

Transformer windings are marked to identify terminals with the same polarity. Polarity marks may either be dots or + marks. Alternatively, polarity marks may be shown by assigning the same subscripts to corresponding primary and secondary labels, H and X, respectively. Thus, when the primary terminals are named H_1 and H_2, the corresponding secondary terminals are identified as X_1 and X_2, respectively.

Primary and secondary terminals have the same polarity when, at a given instant of time, current enters the primary terminal and leaves the secondary terminal as shown in Fig. 4.21.

The polarity marks also indicate that at the instant the primary dotted terminal H_1 is positive with respect to the undotted end H_2, so is the secondary dotted terminal X_1 positive with respect to its undotted end X_2.

The terms additive polarity and subtractive polarity merely have reference to the relative position of the locations of the H and X terminals. Figure 4.22 illustrates both conditions.

If the top terminals of the transformer of Fig. 4.22 are connected together and one winding is excited by a sinusoidal voltage source, the voltage measured across the bottom terminals will be either

a. The difference between the induced voltage across the H and X windings as in Fig. 4.22*a* or

b. The sum of the induced voltages as in Fig. 4.22*b*.

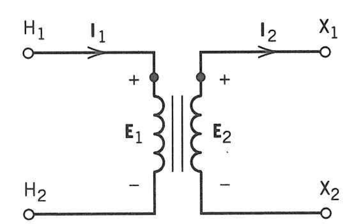


FIGURE 4.21 Transformer polarity markings.

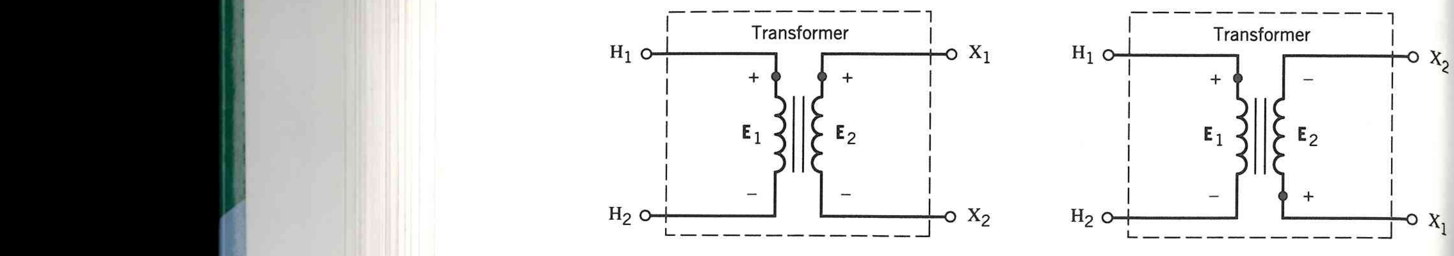


FIGURE 4.22 Transformer polarity: (*a*) subtractive polarity; (*b*) additive polarity.

DRILL PROBLEMS

D4.9 A 240/120-V 60-Hz ideal transformer is rated at 5 kVA.

a. Calculate the turns ratio.

b. Calculate the rated primary and secondary currents.

c. Calculate the primary and secondary currents when the transformer delivers 3.2 kW at rated secondary voltage and at 0.8 lagging power factor.

D4.10 A 50-kVA, 2400/240-V, 60-Hz distribution transformer has the following resistances and leakage reactances in ohms referred to its own side.

$$R_1 = 3.5 \qquad X_1 = 5.7$$
$$R_2 = 0.035 \qquad X_2 = 0.057$$

The subscript 1 denotes the primary or high-voltage winding and subscript 2 denotes the secondary or low-voltage winding.

a. Find the equivalent impedance Z_{e1} referred to the primary.

b. Find the equivalent impedance Z_{e2} referred to the secondary.

c. The transformer secondary is connected to an electrical load, and it delivers its rated current at rated voltage and 0.85 power factor lagging. Find the primary voltage.

D4.11 A single-phase, 10-kVA, 2200/220-V transformer has the following parameters:

$$R_1 = 4.0\ \Omega \qquad R_2 = 0.04\ \Omega$$
$$X_1 = 5.0\ \Omega \qquad X_2 = 0.05\ \Omega$$
$$R_{c1} = 35\ \text{k}\Omega \qquad X_{m1} = 4.0\ \text{k}\Omega$$

The transformer is supplying its rated current to a load at 220 V and 0.8 PF lagging.

a. Draw the equivalent circuit of this transformer, showing values of the elements referred to the primary side.

b. Determine the input voltage of the transformer.

c. Calculate the transformer voltage regulation.

d. Find the efficiency of the transformer.

D4.12 Solve Problem D4.11 by using the approximate equivalent circuit of a transformer given in Fig. 4.15*c* or Fig. 4.15*d*.

D4.13 A single-phase, 25-kVA, 2300/230-V, 60-Hz distribution transformer has the following characteristics:

Core loss at full voltage = 250 W
Copper loss at half load = 300 W

a. Determine the efficiency of the transformer when it delivers rated load at 0.866 power factor lagging.

b. The transformer has the following (24-h) load cycle:

1/4 full load for 4 h at 0.8 PF
1/2 full load for 10 h at 0.8 PF
3/4 full load for 6 h at 0.8 PF
Full load for 4 h at 0.9 PF

Find the all-day (or energy) efficiency, that is, the ratio of the energy output to the energy input over a 24-h period.

D4.14 A 5-kVA, 440/220-V, single-phase transformer was subjected to the short-circuit and open-circuit tests, and the following test results were obtained:

	Voltage (V)	Current (A)	Power (W)
Short-circuit test	28.5	11.4	65
Open-circuit test	220	1.25	50

a. Find the circuit parameters of the transformer.

b. Draw the equivalent circuit of the transformer.

4.6 AUTOTRANSFORMER

The *autotransformer* serves a function similar to that of the ordinary transformer: to raise or lower voltage. It consists of a single continuous winding with a tap brought out at some intermediate point as shown in Fig. 4.23. Because the primary and secondary windings of the autotransformer are

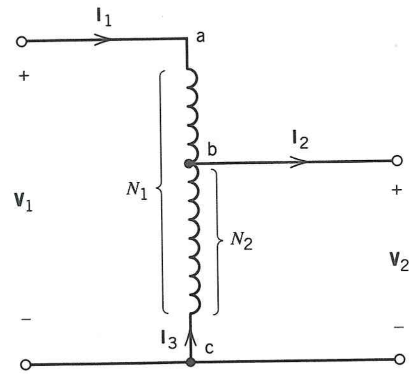


FIGURE 4.23 Autotransformer.

physically connected, the supply and output voltages are not insulated from each other.

When a voltage V_1 is applied to the primary of the autotransformer, the induced voltages are related by

$$\frac{E_1}{E_2} = \frac{E_{ac}}{E_{bc}} = \frac{N_1}{N_2} = a \tag{4.67}$$

Neglecting voltage drops in the windings,

$$\frac{V_1}{V_2} = a \tag{4.68}$$

When a load is connected to the secondary of the autotransformer, a current I_2 flows in the direction shown in Fig. 4.23. By Kirchhoff's current law,

$$I_2 = I_1 + I_3 \tag{4.69}$$

As in the ordinary transformer, the primary and secondary ampere-turns balance each other, except for the small current required for core magnetization:

$$N_1 I_1 = N_2 I_2 \tag{4.70}$$

Equation 4.70 may also be written as

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = a \tag{4.71}$$

Substituting Eq. 4.71 into Eq. 4.69, the ratio of the winding currents is found as

$$\frac{I_3}{I_1} = a - 1 \tag{4.72}$$

In an autotransformer, the total power transmitted from the primary to the secondary does not actually pass through the whole winding. This means that a greater amount of power can be transferred without exceeding the current ratings of the windings of the transformer.

The input apparent power is given by

$$S_1 = V_1 I_1 \tag{4.73}$$

Similarly, the output apparent power is given by

$$S_2 = V_2 I_2 \tag{4.74}$$

However, the apparent power in the transformer windings is

$$S_w = V_2 I_3 = (V_1 - V_2) I_1 = S_{ind} \tag{4.75}$$

This power is the component of the power transferred by transformer action or by electromagnetic induction.

The difference $(S_2 - S_w)$ between the output apparent power and the apparent power in the windings is the component of the output transferred by electrical conduction. This is equal to

$$S_{cond} = V_2 I_2 - V_2 I_3 = V_2 I_1 \tag{4.76}$$

EXAMPLE 4.9

A single-phase, 10-kVA, 440/110-V, two-winding transformer is connected as an autotransformer to supply a load at 550 V from a 440 V supply as shown below. Calculate the following:

a. kVA rating as an autotransformer
b. Apparent power transferred by conduction
c. Apparent power transferred by electromagnetic induction

Solution

The single-phase, two-winding transformer is reconnected as an autotransformer as shown in Fig. 4.24. The current ratings of the windings are given by

$$I_{ab} = 10{,}000/110 = 90.9 \text{ A}$$
$$I_{bc} = 10{,}000/440 = 22.7 \text{ A}$$

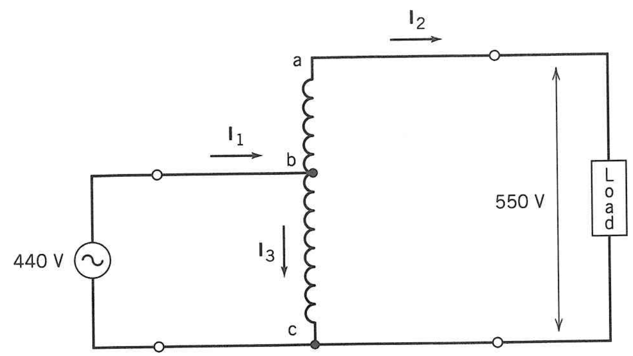


FIGURE 4.24 Autotransformer circuit of Example 4.9.

At full or rated load, the primary and secondary terminal currents are

$$I_2 = 90.9 \text{ A}$$
$$I_1 = I_2 + I_3 = 90.9 + 22.7 = 113.6 \text{ A}$$

Therefore, the kVA rating of the autotransformer is

$$\text{kVA}_1 = (440)(113.6)/1000 = 50 \text{ kVA}$$
$$\text{kVA}_2 = (550)(90.9)/1000 = 50 \text{ kVA}$$

Note that this transformer, whose rating as an ordinary two-winding transformer is only 10 kVA, is capable of handling 50 kVA as an autotransformer. However, not all of the 50 kVA is transformed by electromagnetic induction. A large part is merely transferred electrically by conduction.

The apparent power transformed by induction is

$$S_{\text{ind}} = V_1 I_3 = (440)(22.7) \text{ VA} = 10 \text{ kVA}$$

The apparent power transferred by conduction is

$$S_{\text{cond}} = V_1 I_2 = (440)(90.9) \text{ VA} = 40 \text{ kVA}$$

4.7 THREE-PHASE TRANSFORMERS

Three-phase transformers are used quite extensively in power systems to transform a balanced set of three-phase voltages at a particular voltage level into a balanced set of voltages at another level. Transformers used between generators and transmission systems, between transmission and subtransmission systems, and between subtransmission and distribution systems are all three-

phase transformers. Most commercial and industrial loads require three-phase transformers to transform the three-phase distribution voltage to the ultimate utilization level.

Three-phase transformers are formed in either of two ways. The first method is to connect three single-phase transformers to form a three-phase bank. The second method is to manufacture a three-phase transformer bank with all three phases located on a common multilegged core. As far as analysis is concerned, there is no difference between the two methods.

The primary windings and secondary windings of the three-phase transformer may be independently connected in either a wye (Y) or delta (Δ) connection. As a result, four types of three-phase transformers are in common use:

1. Wye-wye (Y-Y)
2. Wye-delta (Y-Δ)
3. Delta-wye (Δ-Y)
4. Delta-delta (Δ-Δ)

The four possible connections are shown in Fig. 4.25. It should be noted that to form a wye connection, the undotted ends of the three windings (three primaries or three secondaries) are joined together and form the neutral point and the dotted ends become the three line terminals. In forming a delta connection, the three windings belonging to the same side are connected in series in such a way that the sum of the phase voltages in the closed delta is equal to zero; then the line terminals are taken off the junctions of the windings.

In Fig. 4.25, the primary and secondary windings that are drawn parallel to each other belong to the same phase. Also shown in the figure are the various voltages and currents, where V and I are secondary line-to-line voltage and line current, respectively, and a is the turns ratio of the single-phase transformer.

The Y-Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage level, as in distribution transformers. Conversely, the Δ-Y connection is used for stepping up to a high voltage, as in generation station transformers.

The Y-Y connection is seldom used because of possible voltage unbalances and problems with third harmonic voltages. The Δ-Δ connection is used because of its advantage that one of the three single-phase transformers can be removed for repair or maintenance. The remaining two transformers continue to function as a three-phase bank, although the kVA rating of the bank is reduced to 58% of the original three-phase bank rating. This mode of operation is known as open-delta connection, or V-V connection.

The open-delta connection is also used when the load is presently small but is expected to grow in the future. Thus, instead of installing a three-phase bank of three single-phase transformers right away, only two single-phase transformers are used for three-phase voltage transformation. The third single-phase transformer serves as a spare and is connected at a later date when the load has grown.

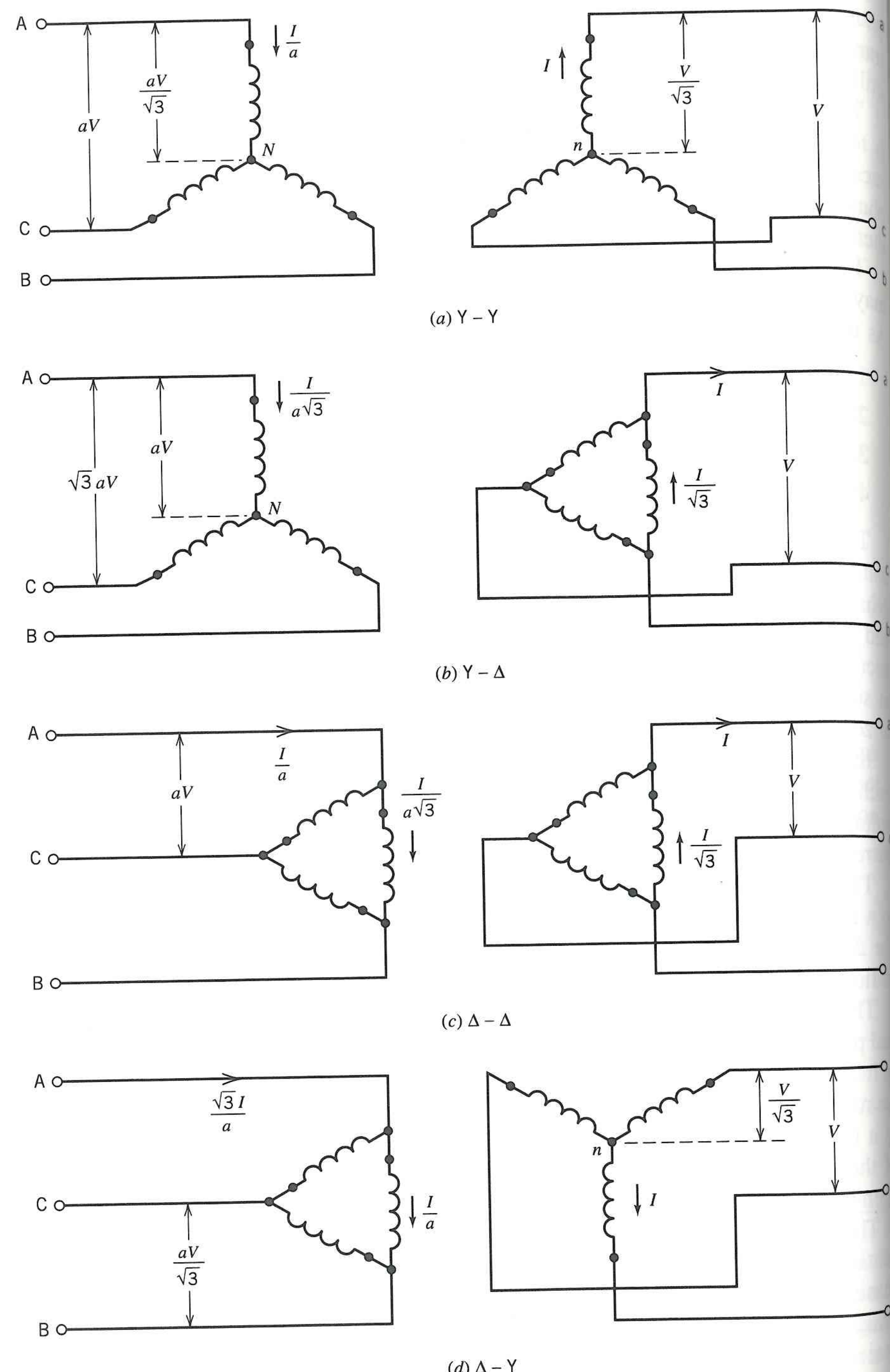


FIGURE 4.25 Three-phase transformer connections.

EXAMPLE 4.10

Three identical single-phase transformers are each rated 30 MVA, 200/40 kV, 60 Hz. They are connected to form a three-phase Y-Y transformer bank as shown in Fig. 4.26.

The bank is energized by a 345-kV three-phase source. A 60-MVA three-phase load, 0.9 PF lagging, is connected to the secondary of the transformer bank. Neglect exciting currents and voltage drops across the transformer. Choose $\mathbf{V}_{AB}$ at the primary as reference phasor.

a. Determine primary and secondary voltages and currents for this configuration.

b. Repeat part (a) when the secondary is Δ connected as shown in Fig. 4.27.

Solution

a. Assume an abc phase sequence. The primary line-to-line and phase voltages expressed in kV are as follows:

$$\mathbf{V}_{AB} = 345\underline{/0^\circ} \qquad \mathbf{V}_{AN} = 200\underline{/-30^\circ}$$
$$\mathbf{V}_{BC} = 345\underline{/-120^\circ} \qquad \mathbf{V}_{BN} = 200\underline{/-150^\circ}$$
$$\mathbf{V}_{CA} = 345\underline{/120^\circ} \qquad \mathbf{V}_{CN} = 200\underline{/90^\circ}$$

Since the turns ratio is $a = 200/40 = 5$ for each single-phase transformer, the secondary voltages in kV of the bank are given by

$$\mathbf{V}_{an} = 40\underline{/-30^\circ} \qquad \mathbf{V}_{ab} = 69\underline{/0^\circ}$$
$$\mathbf{V}_{bn} = 40\underline{/-150^\circ} \qquad \mathbf{V}_{bc} = 69\underline{/-120^\circ}$$
$$\mathbf{V}_{cn} = 40\underline{/90^\circ} \qquad \mathbf{V}_{ca} = 69\underline{/120^\circ}$$

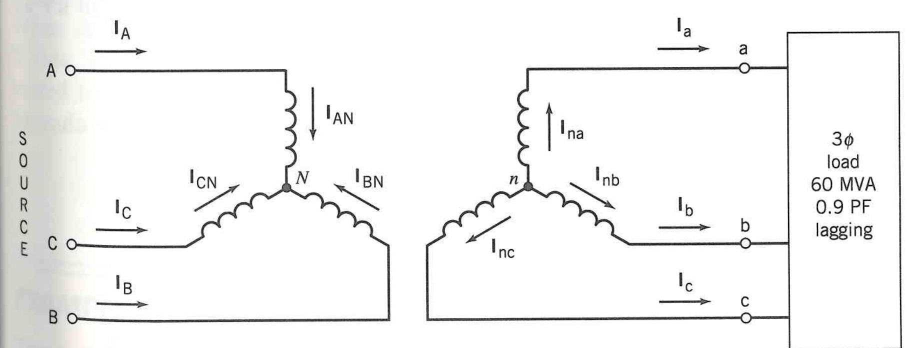


FIGURE 4.26 Y-Y transformer bank of Example 4.10.

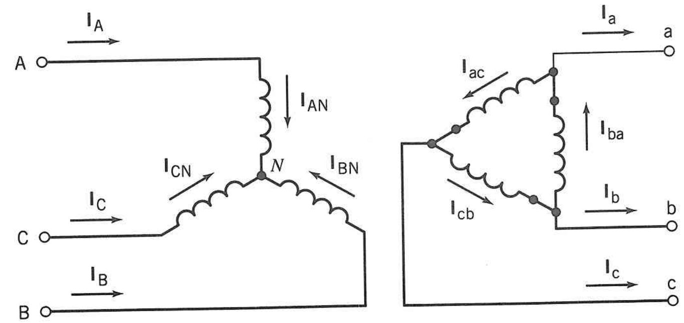


FIGURE 4.27 Y-Δ transformer bank.

The line currents are equal to the phase currents in the wye-connected transformer. At the secondary, the currents expressed in amperes are given by the following:

$$\mathbf{I}_a = \mathbf{I}_{na} = \frac{60{,}000}{\sqrt{3}\ 69}\angle{-30^\circ - \cos^{-1}0.9} = 500\angle{-55.8^\circ}$$

$$\mathbf{I}_b = \mathbf{I}_{nb} = 500\angle{-175.8^\circ}$$

$$\mathbf{I}_c = \mathbf{I}_{nc} = 500\angle{64.2^\circ}$$

The primary currents expressed in amperes are

$$\mathbf{I}_A = \mathbf{I}_{AN} = 100\angle{-55.8^\circ}$$

$$\mathbf{I}_B = \mathbf{I}_{BN} = 100\angle{-175.8^\circ}$$

$$\mathbf{I}_C = \mathbf{I}_{CN} = 100\angle{64.2^\circ}$$

b. For the Y-Δ connected transformer bank, the primary line-to-line and phase voltages are the same as in part (a). The secondary phase voltages are identical to the line-to-line voltages, and they are expressed in kV as follows.

$$\mathbf{V}_{ab} = 40\angle{-30^\circ}$$

$$\mathbf{V}_{bc} = 40\angle{-150^\circ}$$

$$\mathbf{V}_{ca} = 40\angle{90^\circ}$$

The secondary line currents in amperes are

$$\mathbf{I}_a = \frac{60{,}000}{\sqrt{3}\ 40}\angle{-30^\circ - 30^\circ - 25.8^\circ} = 866\angle{-85.8^\circ}\ \text{A}$$

$$\mathbf{I}_b = 866\angle{-205.8^\circ} = 866\angle{154.2^\circ}$$

$$\mathbf{I}_c = 866\angle{34.2^\circ}$$

and the secondary phase currents in amperes are

$$\mathbf{I}_{ba} = 500\angle{-55.8^\circ}$$

$$\mathbf{I}_{cb} = 500\angle{-175.8^\circ}$$

$$\mathbf{I}_{ac} = 500\angle{64.2^\circ}$$

The primary line currents in amperes are equal to the phase currents. Thus,

$$\mathbf{I}_A = \mathbf{I}_{AN} = 100\angle{-55.8^\circ}$$

$$\mathbf{I}_B = \mathbf{I}_{BN} = 100\angle{-175.8^\circ}$$

$$\mathbf{I}_C = \mathbf{I}_{CN} = 100\angle{64.2^\circ}$$

In either Y-Y or Δ-Δ connections, corresponding phase voltages are in phase. Similarly, corresponding line-to-line voltages in the primary and secondary are in phase. In other words, $\mathbf{V}_{AN}$ is in phase with $\mathbf{V}_{an}$, and $\mathbf{V}_{AB}$ is in phase with $\mathbf{V}_{ab}$. On the other hand, for both Y-Δ and Δ-Y connections, it is customary in the United States to have the primary phase or line-to-line voltage lead by 30°; thus, $\mathbf{V}_{AN}$ leads $\mathbf{V}_{an}$ by 30°, and $\mathbf{V}_{AB}$ leads $\mathbf{V}_{ab}$ by the same amount of phase shift.

Circuit analysis involving three-phase transformers under balanced conditions can be performed on a per-phase basis. This follows from the relationship that the per-phase real power and reactive power are one-third of the total real power and reactive power, respectively, of the three-phase transformer bank. It is convenient to carry out computations in a per-phase wye line-to-neutral basis. When Δ-Y, or Y-Δ connections are present, the parameters are referred to the Y side. In dealing with Δ-Δ connections, the Δ-connected impedances are converted to equivalent Y-connected impedances. The Δ-Y impedance conversion formula was given in Chapter 3 and is repeated here as Eq. 4.77.

$$Z_Y = \frac{1}{3}Z_\Delta \qquad (4.77)$$

EXAMPLE 4.11

Three single-phase 100-kVA, 2400/240-V, 60-Hz transformers are connected to form a three-phase, 4160/240-V transformer bank. The equivalent impedance of each transformer referred to its low-voltage side is $0.045 + j0.16\ \Omega$. The transformer bank is connected to a three-phase source through a three-phase feeder with an impedance of $0.5 + j1.5\ \Omega$ /phase. The transformer delivers 250 kW at 240 V and 0.866 lagging power factor.

a. Determine the transformer winding currents.

b. Determine the sending end voltage at the source.

Solution

a. The three-phase power system is connected as shown in Fig. 4.28.

The high-voltage windings are connected in wye so that the primary can be connected to the 4160-V source. The low-voltage windings are connected in delta to supply 240 V to the load.

The line current delivered to the load is

$$I_s = \frac{250{,}000}{\sqrt{3}(240)(0.866)} = 694.5 \text{ A}$$

The transformer secondary winding current is

$$I_2 = I_s/\sqrt{3} = 694.5/\sqrt{3} = 400 \text{ A}$$

The transformer winding ratio is

$$a = 2400/240 = 10$$

Therefore, the primary current is found as

$$I_p = I_1 = I_2/a = 400/10 = 40 \text{ A}$$

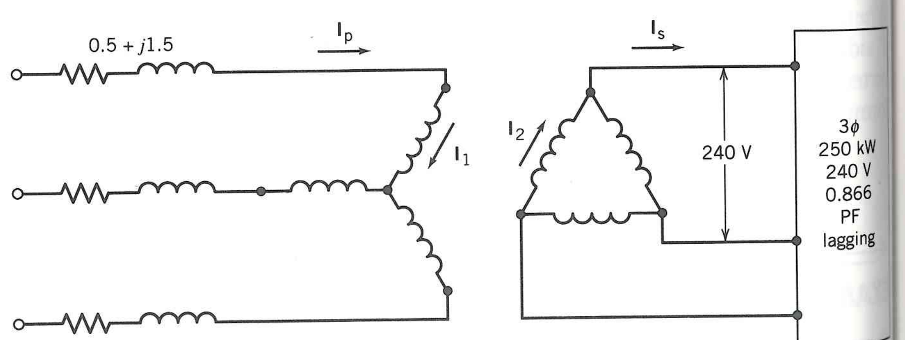


FIGURE 4.28 Three-phase connection diagram.

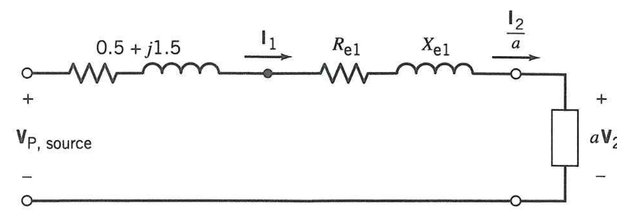


FIGURE 4.29 Single-phase equivalent circuit.

b. The equivalent impedance of the transformer referred to the high-voltage side is

$$Z_{e1} = a^2 Z_{e2} = (10)^2(0.045 + j0.16) = 4.5 + j16\ \Omega/\text{phase}$$

The single-phase equivalent circuit is shown in Fig. 4.29.

The sending end voltage is found as follows:

$$\begin{aligned} \mathbf{V}_{P,\text{source}} &= 2400\angle 0^\circ + (40\angle -30^\circ)(0.5 + j1.5 + 4.5 + j16) \\ &= 2923.2 + j506.2 = 2966.7\angle 9.8^\circ \text{ V (line-to-neutral)} \end{aligned}$$

The line-to-line voltage is given by

$$\begin{aligned} \mathbf{V}_{L,\text{source}} &= \sqrt{3}\,\mathbf{V}_{P,\text{source}} = \sqrt{3}\,2966.7\angle 39.8^\circ \\ &= 5138.5\angle 39.8^\circ \text{ V (line-to-line)} \end{aligned}$$

DRILL PROBLEMS

D4.15 Three single-phase transformers are connected to make a three-phase Δ-Y transformer bank. The rated line-to-line voltages of the bank are 2400 V and 208 V on the primary and secondary, respectively. The transformer supplies a wye-connected load that takes 18 kW at 208 V and 0.85 power factor lagging. Determine the primary and secondary line and phase currents.

D4.16 A three-phase transformer bank is formed by interconnecting three single-phase transformers. The high-voltage terminals are connected to a three-phase 69-kV feeder, and the low-voltage terminals are connected to a three-phase load rated at 1000 kVA and 4.16 kV. Specify the voltage, current, and kVA ratings of each transformer, both high-voltage and low-voltage windings, for the following connections:

a. Y-Y

b. Y-Δ

c. Δ-Y

d. Δ-Δ

REFERENCES

1. Bergseth, F. R., and S. S. Venkata. *Introduction to Electric Energy Devices*. Prentice Hall, Englewood Cliffs, N.J., 1987.
2. Brown, David, and E. P. Hamilton III. *Electromechanical Energy Conversion*. Macmillan, New York, 1984.
3. Chapman, Stephen J. *Electric Machinery Fundamentals*. 2nd ed. McGraw-Hill, New York, 1991.
4. Del Toro, Vincent. *Electric Machines and Power Systems*. Prentice Hall, Englewood Cliffs, N.J., 1985.
5. El-Hawary, Mohamed E. *Principles of Electric Machines with Power Electronics Applications*. Prentice Hall, Englewood Cliffs, N.J., 1986.
6. Fitzgerald, A. E., Charles Kingsley, and Stephen Umans. *Electric Machinery*. 5th ed. McGraw-Hill, New York, 1990.
7. Kosow, Irving L. *Electric Machinery and Transformers*. Prentice Hall, Englewood Cliffs, N.J., 1991.
8. Kraus, John D. *Electromagnetics*. 3rd ed. McGraw-Hill, New York, 1984.
9. Macpherson, George, and Robert D. Laramore. *An Introduction to Electrical Machines and Transformers*. 2nd ed. Wiley, New York, 1990.
10. Matsch, Leander W., and J. Derald Morgan. *Electromagnetic and Electromechanical Machines*. 3rd ed. Harper & Row, New York, 1986.
11. Nasar, Syed A. *Electric Energy Conversion and Transmission*. Macmillan, New York, 1985.
12. Ramshaw, Raymond, and R. G. van Heeswijk. *Energy Conversion Electric Motors and Generators*. Saunders College Publishing, Philadelphia, 1990.
13. Sen, P. C. *Principles of Electrical Machines and Power Electronics*. Wiley, New York, 1989.
14. Wildi, Theodore. *Electrical Machines, Drives, and Power Systems*. 2nd ed. Prentice Hall, Englewood Cliffs, N.J., 1991.

PROBLEMS

4.1 A long solenoid coil has its length much greater than its diameter as shown in Fig. 4.30. The magnetic field inside the coil may, therefore, be considered uniform.

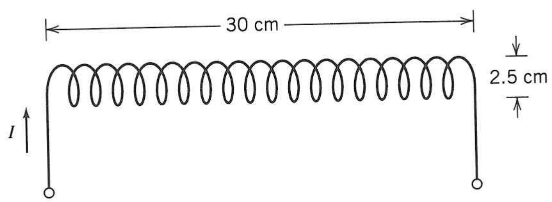


FIGURE 4.30 Solenoid coil of Problem 4.1.

The coil has 150 turns, its length is 30 cm, and its diameter is 2.5 cm. A current I = 25 A is supplied to the coil. Neglect the magnetic field outside the coil.

a. Determine the magnetic field intensity H and the magnetic flux density B inside the solenoid.

b. Determine the inductance of the coil.

4.2 A toroid has a circular cross section as shown in Fig. 4.31. It is made from cast steel with a relative permeability of 2500. Its outer diameter is 24 cm, and its inner diameter is 16 cm. The magnetic flux density in the core is 1.25 tesla measured at the mean diameter of the toroid.

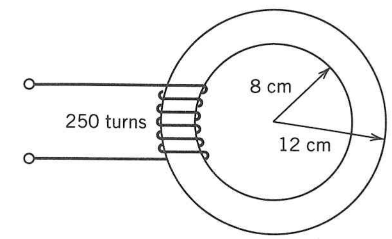


FIGURE 4.31 Magnetic circuit of Problem 4.2.

a. Find the current that must be supplied to the coil, which consists of 250 turns.

b. Determine the magnetic flux in the core.

c. A 10-mm air gap is cut across the toroid. Determine the current that must be supplied to the coil to produce the same value of magnetic flux density as in part (a).

4.3 A ferromagnetic circuit has a magnetic core with infinitely high relative permeability. It has three legs, and air gaps of 2 mm and 1 mm are cut from sections A and C, respectively, as shown in Fig. 4.32. A coil is wound on the center leg B, and it has 200 turns and a resistance of 2.5 Ω. The magnetic core has a 5 × 5 cm uniform cross-sectional area. A DC voltage is applied to the coil.

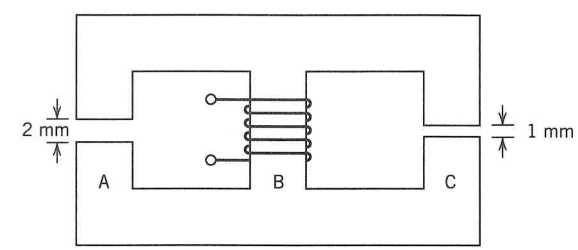


FIGURE 4.32 Magnetic circuit of Problem 4.3.

a. Determine the voltage that will produce a flux density of 0.75 T in the right leg C, which contains the 1-mm air gap.

b. Find the magnetic flux in the other two legs of the core.

4.4 Repeat Problem 4.3 if the coil is removed from the center leg B and placed on the right leg C, which contains the 1-mm air gap.

4.5 The electromagnetic system shown in Fig. 4.33 has a magnetic core of infinite relative permeability. Neglect fringing and leakage flux. Determine the expressions for

a. The magnetic flux in each of the three legs

b. The flux linkage and self-inductance of each coil

c. The mutual inductance

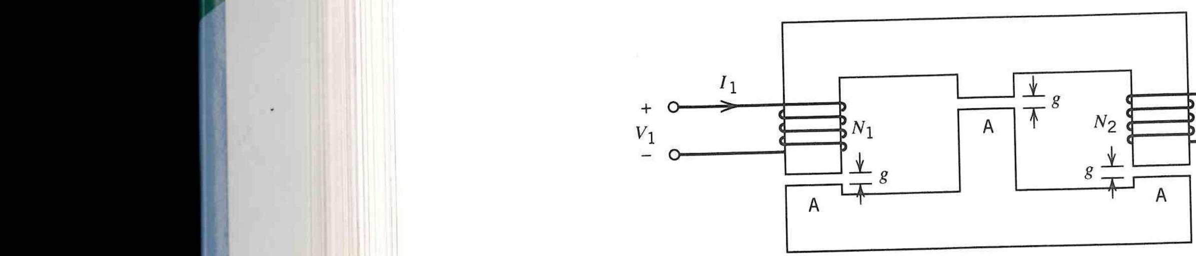


FIGURE 4.33 Magnetic circuit of Problem 4.5.

4.6 For the electromagnetic system of Fig. 4.33, the polarity of the voltage source V_1 is reversed so that the current I_1 flows in the opposite direction. The magnetic core has infinite relative permeability. Determine the expressions for

a. The magnetic flux in each of the three legs

b. The flux linkage of each coil

c. The self-inductance of each coil

d. The mutual inductance

4.7 The magnetic circuit shown in Fig. 4.34 has an infinitely permeable magnetic core. The following are given:

$g_1 = 5$ mm	$A_1 = 5$ cm^2	$N_1 = 80$ turns
$g_2 = 5$ mm	$A_2 = 5$ cm^2	$N_2 = 100$ turns
$g_3 = 10$ mm	$A_3 = 10$ cm^2	$N_3 = 125$ turns

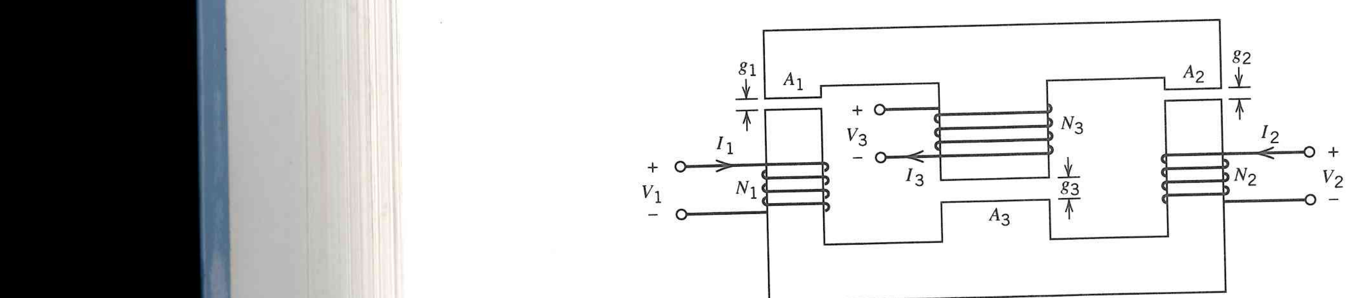


FIGURE 4.34 Magnetic circuit of Problem 4.7.

A current of 12 A flows in the first coil, N_1. The second and third coils, N_2 and N_3, respectively, are unexcited. Determine the flux densities in each of the air gaps g_1, g_2, and g_3.

4.8 In the electromagnetic circuit of Fig. 4.34, the three coils are excited simultaneously such that $I_1 = 12$ A, $I_2 = 10$ A, and $I_3 = 8$ A, with the directions of currents as shown. Determine the magnetic flux densities in the three air gaps.

4.9 In the electromagnetic circuit of Fig. 4.34, assume that there is no air gap in the center leg containing coil N_3, and only the first coil, N_1, carries a current of 12 A.

a. Determine the flux densities in the air gaps g_1 and g_2.

b. Suppose that the length of air gap g_2 is also reduced to zero; that is, there is also no air gap in the leg containing coil N_2. Determine the flux density in the first air gap g_1.

4.10 Refer to the electromagnetic system described in Problem 4.7 and shown in Fig. 4.34. Determine the self- and mutual inductances of the three coils.

4.11 The magnetic circuit shown in Fig. 4.35 has an iron core with infinite permeability. The core dimensions are:

$$A_c = 16 \text{ cm}^2 \qquad g = 2 \text{ mm} \qquad l_c = 80 \text{ cm}$$

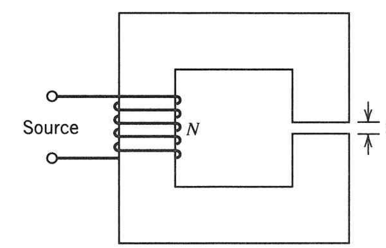


FIGURE 4.35 Magnetic circuit of Problem 4.11.

The coil has 500 turns and draws a current $I = 4$ A from the source. Neglect magnetic leakage and fringing. Calculate

a. The total magnetic flux

b. The flux linkages of the coil

c. The coil inductance

4.12 Repeat Problem 4.11 assuming that the core has a relative permeability of $\mu = 2000$.

4.13 A toroidal magnetic circuit consists of a coil of $N = 200$ turns each of circular cross section of radius $\rho = 0.25$ m, as shown in Fig. 4.36. The radius of the toroid is $R = 5$ m, as measured to the center of each circular turn. Assume that the magnetic field intensity is zero outside the toroid and the magnetic field intensity inside the toroid is given by

$$H = \frac{NI}{2\pi R}$$

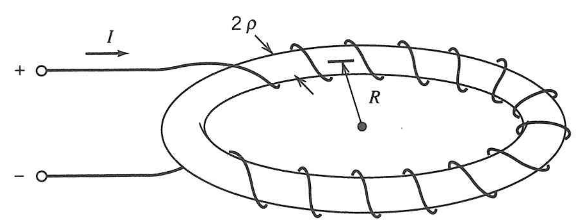


FIGURE 4.36 Magnetic circuit of Problem 4.13.

a. Calculate the coil inductance L.

b. A voltage source is connected to the coil, and the current is adjusted to produce a magnetic flux density of 1.0 T. Calculate the total stored magnetic energy.

4.14 A 4800/240-V, single-phase transformer is rated at 10 kVA, and it has an equivalent impedance referred to the primary side of $Z_{e1} = 120 + j300\ \Omega$.

a. Find the equivalent impedance referred to the secondary side Z_{e2}.

b. Calculate the voltage at the primary terminals if the secondary supplies rated secondary current at 230 V and unity power factor.

4.15 A 100-kVA, 2300/230-V, single-phase transformer has the following parameters:

$R_1 = 0.30$ ohm	$R_2 = 0.0030$ ohm	$R_c = 4.5\ \text{k}\Omega$
$X_1 = 0.65$ ohm	$X_2 = 0.0065$ ohm	$X_m = 1.0\ \text{k}\Omega$

The transformer delivers 75 kW at 230 V and 0.85 power factor lagging. Determine

a. The input current

b. The input voltage

c. The input power and power factor

4.16 A 15-kVA, 2400/240-V, single-phase transformer has an equivalent series impedance $Z_{e1} = 6 + j8.5\ \Omega$. The shunt magnetizing branches are given as $R_{c1} = 50\ \text{k}\Omega$ and $X_{m1} = 15\ \text{k}\Omega$. The transformer is delivering rated current to a load at 240 V and 0.8 lagging power factor. Find (a) the primary current and (b) the applied voltage.

4.17 A 25-kVA, 2300/230-V, single-phase transformer has a high-voltage winding with a resistance of 1.5 Ω and a leakage reactance of 2.4 Ω. The low-voltage winding has a resistance of 0.015 Ω and a leakage reactance of 0.024 Ω. The core loss is 175 W. Find the following:

a. Equivalent impedance referred to the high-voltage winding

b. Equivalent impedance referred to the low-voltage winding

c. Voltage regulation for full load at 230 V and 0.866 power factor lagging

d. The efficiency of the transformer under the conditions of part (c)

4.18 The transformer of Problem 4.15 has a secondary voltage of 230 V and the load on the transformer is 100 kVA. By using the approximate equivalent circuit of a transformer, determine the voltage that must be applied to the primary terminals if

a. The power factor of the load is 0.8 lagging

b. The power factor of the load is 0.8 leading

4.19 A 25-kVA, 2400/240-V, single-phase transformer has an equivalent resistance and reactance, both referred to the primary side, of $R_{e1} = 3.45\ \Omega$ and $X_{e1} = 5.75\ \Omega$, respectively. The core loss is 120 W. The transformer delivers rated kVA to a load at rated secondary voltage and 0.85 power factor lagging.

a. Determine the voltage applied to the primary side.

b. Determine the percent voltage regulation.

c. Find the efficiency of the transformer.

4.20 A 25-kVA, 2200/220-V, single-phase transformer has an equivalent series impedance of $Z_{e1} = 3.5 + j4.0\ \Omega$ referred to the primary side. The transformer is connected to a load whose power factor varies. The core loss is 160 W.

a. Calculate the voltage regulation at full load, 0.8 PF lagging.

b. Determine the highest value of voltage regulation for full-load output at rated secondary terminal voltage.

c. Determine the efficiency when the transformer delivers full-load output at rated secondary voltage and 0.8 power factor lagging.

4.21 A 10-kVA transformer has an iron loss of 150 W and a full-load copper loss of 250 W. Calculate the transformer efficiency for the following load conditions:

a. Full load at 0.8 power factor lagging

b. 75% of full load at unity power factor

c. 50% of full load at 0.6 power factor lagging

4.22 The transformer of Problem 4.21 operates on full load at 80% power factor for 4 h, on 75% of full load at unity power factor for 8 h, and on 50% of full load at 60% power factor for 12 h during 1 day. Determine the all-day efficiency. Refer to Drill Problem 4.13.

4.23 A 25-kVA, 2400/240-V, 60-Hz, distribution transformer was tested at 60 Hz, and the following data were obtained.

	Voltage (V)	Current (A)	Power (W)
Open-circuit test	240	3.2	165
Short-circuit test	55	10.4	375

a. Compute the efficiency for full-load output at rated voltage and 0.8 power factor lagging.

b. The power factor of the load is varied while the magnitudes of the current and the secondary voltage are held constant. Determine the largest value of voltage regulation and the power factor at which it occurs. Draw a phasor diagram depicting this condition.

4.24 A 50-kVA, 2400/240-V, single-phase transformer was tested, and the following test data were obtained.

	Voltage (V)	Current (A)	Power (W)
Short-circuit test	55	20.8	600
Open-circuit test	240	5.0	450

a. Calculate the voltage regulation and efficiency when the transformer is connected to a load that takes 156 A at 220 V and 0.8 power factor lagging.

b. Calculate the voltage regulation and efficiency at rated load conditions and 0.8 power factor lagging.

4.25 Two single-phase transformers are each rated 2400/120 V. Draw a circuit diagram showing the interconnections and polarity markings of these transformers.

a. When they are used for 4800/240-V operation

b. When they are used for 2400/120-V operation

4.26 A single-phase load is supplied through a 34.5-kV feeder and a 34.5/2.4-kV transformer. The feeder has an impedance of $50 + j180$ ohms, and the transformer has an equivalent impedance of $24 + j120$ ohms referred to its high-voltage side. The load takes 260 kW at 2.3 kV and 0.866 lagging power factor.

a. Find the voltage at the primary side of the transformer.

b. Determine the voltage at the sending end of the feeder.

c. Calculate the real and reactive power input at the sending end of the feeder.

4.27 A 10-kVA, 4160/240-V, single-phase transformer has a per-unit resistance of 0.01 and a per-unit reactance of 0.05. It has a core loss at rated voltage of 150 W. The transformer supplies 7.5 kVA at 240 V and 0.6 power factor lagging to a load connected to its secondary terminals.

a. Determine the equivalent impedance in ohms referred to the primary side.

b. Find the input voltage.

c. Determine the efficiency of the transformer.

4.28 A 15-kVA, 2200/220-V, single-phase transformer is connected to act as a booster from 2200 to 2420 V. Without exceeding the rated current of any winding and assuming an ideal transformer, determine (a) the kVA input and output, (b) the kVA transformed, and (c) the kVA conducted.

4.29 A 5-kVA, 480/120-V, single-phase transformer is to be used as an autotransformer to transform a 600-V source to a 480-V supply. As a single-phase transformer delivering rated load at 0.80 power factor lagging, its efficiency is 0.95%.

a. Show a connection diagram as an autotransformer.

b. Determine the kVA rating as an autotransformer.

c. Find the efficiency as an autotransformer delivering rated capacity at 480 V and 0.80 power factor lagging.

4.30 A 5-kVA, 220/220-V, 60-Hz, two-winding transformer has a full-load efficiency of 95% at unity power factor. The iron loss at 60 Hz is 100 W. This transformer is connected as an autotransformer and supplied by a 440-V source. The autotransformer delivers the maximum possible kVA without overloading the windings. Calculate (a) the primary current and (b) the efficiency of the autotransformer.

4.31 Three single-phase, 20/2.4-kV, ideal transformers are connected to form a three-phase, 10-MVA, 34.5/2.4-kV transformer bank. The transformer bank supplies a load of 6 MW at 2.4 kV and 0.85 power factor lagging.

a. Determine the line and phase currents at the primary and secondary sides of the transformer.

b. Determine the line-to-line and line-to-neutral voltages at the primary and secondary sides of the transformer.

4.32 Repeat Problem 4.31 if the three single-phase, 20/2.4-kV, ideal transformers are connected to form a three-phase, 10 MVA, 20/2.4-kV transformer bank and the bank supplies 6 MW at 2.4 kV and 0.85 power factor lagging.

4.33 Three single-phase, 10-kVA, 2400/120-V, 60-Hz transformers are connected to form a three-phase, 4160/208-V transformer bank. The equivalent impedance of each single-phase transformer referred to the primary side is $10 + j25\ \Omega$. The transformer bank delivers 27 kW at 208 V and 0.9 power factor leading.

a. Draw the three-phase schematic diagram showing the transformer connection. Draw a per-phase equivalent circuit.

b. Determine the primary current and power factor.

c. Determine the primary voltage.

d. Determine the voltage regulation.

4.34 Repeat Problem 4.33 if the three single-phase 2400/120-V transformers are connected to form a three-phase 2400/208-V transformer bank and the bank supplies 27 kW at 208 V and 0.9 power factor leading.

4.35 A three-phase, 300-kVA, 2300/230-V, wye-wye transformer bank has an iron loss of 2200 W and a full-load copper loss of 3800 W. Determine the efficiency of the transformer for 70% full load at 230 V and 0.85 power factor.

4.36 A three-phase transformer bank is formed by interconnecting three single-phase transformers. The three-phase bank is designed to be rated at 300 MVA and 230/34.5 kV. Find the voltage, current, and kVA ratings of each single-phase transformer, both high-voltage and low-voltage windings, if the transformer bank is connected:

a. Δ-Δ

b. Y-Δ

c. Y-Y

d. Δ-Y

Five

Fundamentals of Rotating Machines

5.1 INTRODUCTION

A transformer may be described as an energy transfer device; that is, energy is transferred from the primary to the secondary without changing its form. Both sides of the transformer, primary and secondary, carry energy in electrical form. On the other hand, rotating machines such as synchronous machines, induction machines, and DC machines are energy converters. They convert either mechanical energy to electrical energy, in the case of generators, or electrical energy to mechanical energy in the case of motors.

In converting energy from mechanical to electrical or electrical to mechanical, part of the energy is lost. If losses are neglected, however, the machine becomes an ideal energy converter and may be represented as shown in Fig. 5.1.

From the law of conservation of energy, the mechanical input is equal to the electrical output, since losses are neglected; that is,

$$\omega_m T = vi \tag{5.1}$$

where

$\omega_m T$ = input mechanical power

vi = output electrical power

T = mechanical input torque, expressed in N-m

ω_m = rotor speed, in rad/s

v = voltage (V)

i = current (A)

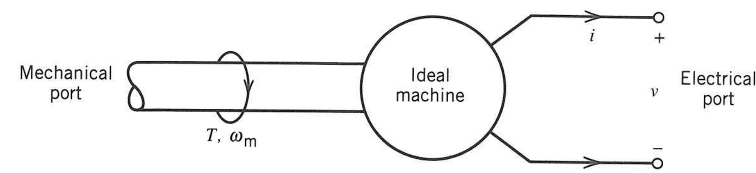


FIGURE 5.1 Ideal energy converter.

The energy losses consist of mechanical losses, which include windage and friction, and electrical losses, which include winding copper losses and magnetic core losses. These losses can be modeled externally. Therefore, in this chapter, the discussion will concentrate on the ideal energy converter. Another assumption made is infinite permeability for magnetic cores, both rotors and stators, used for the energy converters.

5.2 BASIC CONCEPTS OF ENERGY CONVERTERS

The source of a magnetic field may be a permanent magnet or an electric current. Hence, a magnetic field is produced near a conducting loop carrying a current i. Both magnitude and direction of the magnetic field will vary with position for a given current and loop dimensions. A current-carrying loop is shown in Fig. 5.2.

The magnetic field **H** at point P for the current loop of Fig. 5.2 is given by

$$\mathbf{H} = \oint \frac{i\,\mathbf{dl} \times \mathbf{u}_r}{4\pi r^2} \tag{5.2}$$

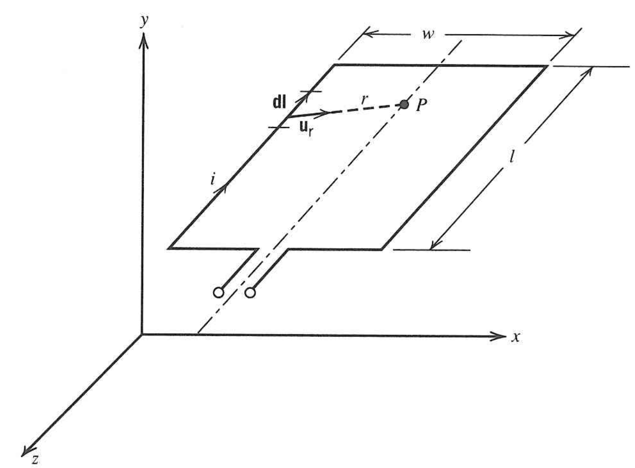


FIGURE 5.2 A current-carrying loop.

The integral in Eq. 5.2 is quite complicated. However, if length l is very long compared to the width w, then **H** along the centerline can be found using Ampère's law.

The magnetic field intensity **H** is directed downward, in the negative y direction, and has a magnitude given by

$$H = \frac{2i}{\pi w} \tag{5.3}$$

Hence, the magnetic flux density **B** may be expressed as

$$\mathbf{B} = \mu \mathbf{H} = -\frac{2\mu i}{\pi w}\mathbf{u}_y \tag{5.4}$$

where

μ = permeability

$\mathbf{u}_y$ = unit vector in the y direction

The foregoing discussion describes how a magnetic flux density is produced by an electric current in a conductor. Next, consider what will happen if the conducting loop shown in Fig. 5.3 is rotated about its axis (shown by the broken line) at a speed of ω_m rad/s under a magnetic field.

It can be seen from Fig. 5.3 that the magnetic flux ϕ linking the conducting loop varies from a minimum value to zero, to a maximum value, and back to the minimum, as the loop rotates. Therefore in accordance with Faraday's law a voltage is induced between terminals a and a′ and is given by

$$e_{aa'} = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = \frac{d\phi}{dt} \tag{5.5}$$

since the loop contains $N = 1$ turn.

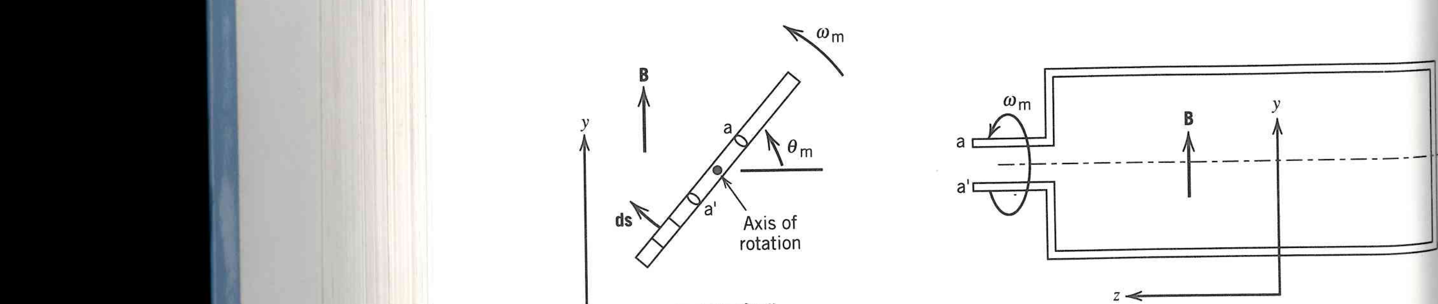


FIGURE 5.3 Rotating conducting loop.

From the given magnetic flux density **B**, the magnetic flux ϕ is found as

$$\phi = \int_S \mathbf{B} \cdot \mathbf{dS} \tag{5.6}$$

where **dS** is a differential area of the loop, and directed perpendicular to the loop and outward. Thus, **dS** makes an angle of θ_m with **B**, and Eq. 5.6 reduces to

$$\phi = \int B \cos \theta_m \, dS = Bwl \cos \theta_m \tag{5.7}$$

Since θ_m is equal to $\int_0^t \omega_m \, dt + \theta_m(0)$, and assuming θ_m is zero at $t = 0$, Eq. 5.7 may also be written as

$$\phi = Bwl \cos \omega_m t \tag{5.8}$$

Substituting Eq. 5.8 into Eq. 5.5 yields the expression for the induced emf.

$$e_{aa'} = -Bwl\omega_m \sin \omega_m t = Bwl\omega_m \cos(\omega_m t + 90°) \tag{5.9}$$

If a resistor R is connected across terminals a–a′, a current ($i = e_{aa'}/R$) will flow through both the resistor and the conducting loop. Hence, electrical power ($ie_{aa'} = i^2R$) is delivered to the resistor. This electrical power originates from the mechanical power required to keep the loop rotating at a speed of ω_m. The mechanical power is given by

$$P_m = \omega_m T_m = i e_{aa'} \tag{5.10}$$

The direction of the current i is such that the magnetic field produced by the loop current will oppose the change in the flux linking this loop due to the external magnetic flux density **B**. For example, when the loop is horizontal (z–x plane), current i will be clockwise to oppose the decrease in external flux linked by the loop.

Since the loop carries current and is situated in a magnetic flux density **B**, there will be an electromagnetic torque T_e acting on the loop. This torque tends to line up the magnetic axis of the loop with the external magnetic flux density **B**. When the loop is horizontal, T_e is clockwise. To maintain the speed ω_m, the applied mechanical torque T_m should be equal in magnitude and opposite in direction to T_e.

The preceding discussion describes the basic principle of operation of a generator. For motor action, on the other hand, a current i is supplied by an electric source. This current sets up a magnetic field. Interaction of this field and the external magnetic flux density **B** results in a torque T_e. This torque can be used to rotate a mechanical load that requires a torque T_m at some speed ω_m.

The basic concepts and principles of operation described in this section apply to both AC and DC machines. Although these concepts were discussed and derived with respect to the simple energy converter, the same concepts and principles remain valid for the more complicated and practical rotating machines.

EXAMPLE 5.1

A coil is formed by connecting 10 conducting loops, or turns, in series. Each turn has a length $l = 2$ m and width $w = 10$ cm. The 10-turn coil is rotated at a constant speed of 30 revolutions per second in a magnetic flux density $B = 2$ T directed upward.

a. Find an expression for the induced emf across the coil.

b. A resistor $R = 500\ \Omega$ is connected between the terminals of the coil. Determine the average power delivered to this resistor.

c. Calculate the average mechanical torque needed to turn the coil and generate power for the resistor.

Solution

a. The induced emf across a single loop, or turn, is found by using Eq. 5.9 as follows:

$$e_{\text{turn}} = (2)(0.10)(2)[(2\pi)(30)]\cos(60\pi t + \pi/2)$$
$$= 75.4\cos(188.5t + \pi/2)\text{ V}$$

Thus, the induced voltage across the coil of 10 turns is

$$e_{\text{coil}} = 10e_{\text{turn}} = 754\cos(188.5t + \pi/2)\text{ V}$$

b. The rms value of the induced voltage across the coil is

$$E_{\text{coil}} = 754/\sqrt{2} = 533\text{ V}$$

The rms current flowing through the resistor and coil is

$$I = E_{\text{coil}}/R = 533/500 = 1.066\text{ A}$$

Thus, the average power delivered to the resistor is given by

$$P = I^2R = (1.066)^2(500) = 568\text{ W}$$

c. The average mechanical torque required to rotate the coil of part (b) is obtained by using Eq. 5.10 as follows:

$$T_m = P/\omega_m = 568/188.5 = 3.0\text{ N-m}$$

DRILL PROBLEMS

D5.1 The machine of Example 5.1 can be used as a motor. The terminals of the coil are connected to a voltage source whose rms voltage is 600 V. The motor runs at 1800 revolutions per minute (rpm) and draws a current of 1.0 A. Find the torque supplied to the mechanical load.

D5.2 The coil of Fig. 5.3 has 100 turns and is rotated at a constant speed of 300 rpm. The axis of rotation is perpendicular to a uniform magnetic flux density of 0.1 T in the vertical direction. The coil has width $w = 10$ cm and length $l = 20$ cm. Calculate

a. The maximum flux passing through the coil

b. The flux linkage as a function of time

c. The maximum instantaneous voltage induced in the coil

d. The time-average value of the induced voltage

e. The induced voltage when the plane of the coil is 30° from the vertical

5.3 ROTATING MACHINES

In the last section, a description of an elementary electromechanical energy converter was presented. However, practical machines are quite different in construction from the simple energy converter. In these machines, voltages are generated in coils that each consist of several turns of conductors. These coils can be rotated mechanically through a magnetic field, or a magnetic field can be rotated mechanically past the coils. This relative motion between the coils and the magnetic field results in a time-varying voltage generated across the coils. A group of such coils interconnected so that their generated voltages add up to the desired value is called an *armature winding*. The armature of a DC machine is the rotating member, or *rotor*. The armature of an AC machine (e.g., a synchronous generator) is the stationary member, or *stator*.

Another group of coils is also present in the rotating machine. These are the excitation coils, or *field windings*. These field windings act as the primary source of magnetic flux for voltage generation in the armature. Also, the magnetic field produced by the field windings interacts with the field produced by the armature windings for torque production. In a DC machine the field windings are located in the stationary member, whereas in an AC machine the field winding is in the rotor.

These machines are described briefly in this section. Later chapters give more detailed discussions of each machine.

5.3.1 Induction Machines

Induction machines are used mostly as motors. They are seldom used as generators because their performance characteristics as generators are unsatisfactory for most applications. As motors, on the other hand, they are called the workhorse of the industry.

An *induction machine* consists of a stator and a rotor. The stator consists of a laminated core, as in transformers, with conductors embedded in slots. These conductors, when energized by an AC source of power, provide a magnetic flux density that is varying. The rotor is mounted on bearings and is separated from the stator by a small air gap. The rotor is cylindrical and carries either

a. Windings whose terminals are brought out to slip rings for external connections, as in wound-rotor machines, or

b. Conductor bars that are short-circuited at both ends for squirrel-cage motors.

In either case, voltage is induced in the rotor conductors. This voltage then causes current to flow. The current, in turn, creates a magnetic field. Interaction of the stator and rotor magnetic fields produces an electromagnetic torque. This torque, in turn, rotates a mechanical load.

Induction motors are sometimes called rotating transformers. This is because the rotor emfs are induced by transformer action. Squirrel-cage motors are cheaper than comparable wound-rotor motors and are highly reliable. These factors contribute to their immense popularity and widespread use. Induction motors are either three-phase or single-phase.

5.3.2 Synchronous Machines

The bulk of electric power is produced by three-phase synchronous generators. Synchronous generators with ratings of 1000 MVA are fairly common in the power industry. *Synchronous machines* may be used as generators or motors. From an analytical point of view, there is no difference between a synchronous generator and a motor.

A simple three-phase, two-pole synchronous generator is shown in Fig. 5.4. The field winding is excited by direct current conducted to it by means of stationary carbon brushes. The DC power required for excitation is 1% to 2% of the machine's rating.

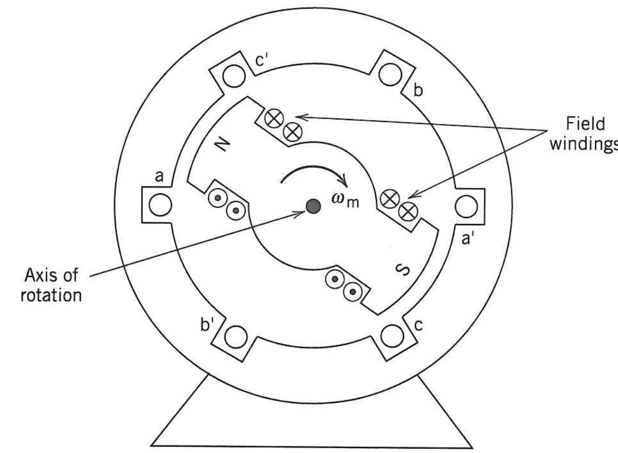


FIGURE 5.4 Elementary synchronous generator.

The armature winding consists of three coils of N turns each, shown by terminals aa′, bb′, and cc′. The conducting loops that form coils of each phase are parallel to the shaft and are connected in series by end connections (end connections are not shown in the figure). The rotor is rotated at a constant speed by the source of mechanical torque, the prime mover, connected to its shaft.

Consider phase a. As the rotor rotates, the flux linking coil aa′ changes, and therefore a voltage is induced between terminals a and a′. For every complete cycle of rotor motion, the induced voltage goes through one complete cycle. Therefore, the angular frequency ω of the induced voltage $e_{aa'}$ is the same as the mechanical speed ω_m (in rad/s). Thus, the frequency in Hz is the same as the speed of the rotor in revolutions per second; that is, the frequency is synchronized with the mechanical speed. This is why the adjective "synchronous" is used with synchronous machines. In order to produce a 60-Hz voltage, the rotor has to rotate at 3600 rpm.

Most synchronous machines have more than two poles. Figure 5.5 shows a four-pole machine. For every complete rotation of the prime mover, the flux linking coil aa′ goes through two complete cycles. Therefore, the angular frequency ω (in rad/s) of $e_{aa'}$ will be twice the speed of rotation ω_m of the prime mover. Thus, in general,

$$\omega = \left(\frac{p}{2}\right)\omega_m \tag{5.11}$$

where

ω = electrical angular frequency

ω_m = mechanical angular speed

p = number of magnetic poles set up by the field winding

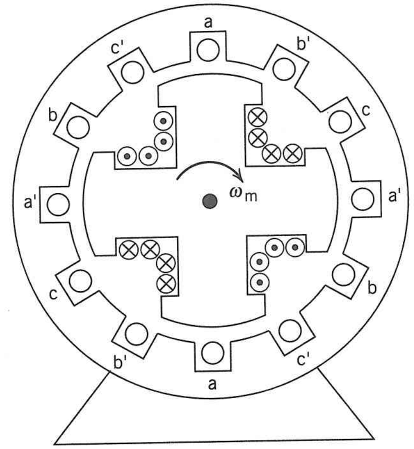


FIGURE 5.5 A four-pole synchronous machine.

In the United States, electric utilities supply their customers at 60 Hz (377 rad/s). From Eq. 5.11, it may be seen that to generate a 60-Hz voltage, the required speed of the prime mover is dependent on the number of poles. Since poles come in pairs, by law of nature, doubling the number of poles reduces the required speed by half. Therefore, to generate a 60-Hz voltage, a four-pole machine runs at 1800 rpm and an eight-pole machine runs at 900 rpm. Similarly, a six-pole machine runs at 1200 rpm and a 12-pole machine runs at 600 rpm.

The rotors shown in Figs. 5.4 and 5.5 have *salient*, or *projecting*, poles with concentrated windings. Another type of rotor configuration is called *nonsalient*, *round*, or *cylindrical*. The field winding is distributed over the surface of the rotor. A round-rotor synchronous machine is shown in Fig. 5.6.

Salient-pole construction is a characteristic of synchronous machines that have a large number of poles and operate at low speeds to produce the desired frequency of 60 Hz. Hydroelectric generators are salient-pole synchronous machines. On the other hand, the generators of steam turbines, such as those in coal and nuclear generating stations, and the generators of gas turbines are nonsalient synchronous machines. They operate best at high speeds, and they have few poles—generally two or four.

Almost all synchronous machines are three-phase machines. To generate a set of three voltages that are phase displaced by 120 electrical degrees, three coils phase displaced in space by 120 electrical degrees must be used. In Fig. 5.4, coils aa′, bb′, and cc′ are 120 mechanical degrees apart for the two-pole machine. The flux linking coil bb′ reaches its maximum 120 mechanical degrees after the flux linking coil aa′ reaches its maximum, and the flux linking cc′ reaches its maximum 120 mechanical degrees after the flux linking bb′ reaches its maximum. The induced coil voltages $e_{aa'}$, $e_{bb'}$, and $e_{cc'}$ are the derivatives of their respective flux linkages. Therefore, these voltages will also be 120 mechanical degrees apart. In this case, electrical degrees and mechanical degrees are the same.

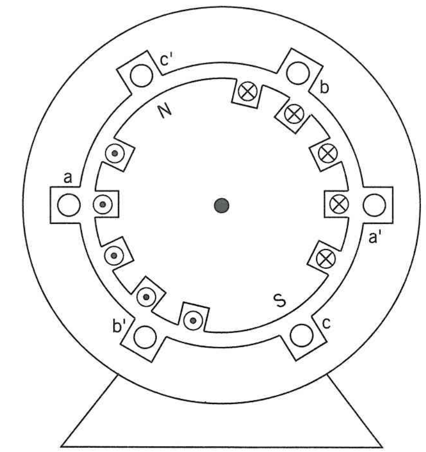


FIGURE 5.6 A two-pole, round-rotor synchronous machine.

In the four-pole machine shown in Fig. 5.5, the 360 mechanical degrees are equivalent to 720 electrical degrees. Hence, the coils are placed in the stator separated by 60 mechanical degrees, which are equivalent to 120 electrical degrees. Therefore, the induced coil voltages $e_{aa'}$, $e_{bb'}$, and $e_{cc'}$ are 120 electrical degrees apart just the same.

When a synchronous generator supplies electric power to an electrical load, the armature (stator) current creates a magnetic flux. This flux reacts with the magnetic flux produced by the field (rotor) current, and electromagnetic torque is produced. This torque tends to align the resultant fluxes of the rotor and stator currents. If an external mechanical torque is applied to counteract the electromagnetic torque, the rotor can keep rotating at constant speed. As a result, mechanical work is done on the rotor and electrical energy is delivered to the electrical load, for example, a resistor. Therefore, electromechanical energy conversion has occurred.

Synchronous motors are the counterparts of synchronous generators. A three-phase AC voltage source supplies three-phase currents to the armature windings, which are located on the stator. A DC voltage source provides DC current to the field winding on the rotor. Both stator and rotor windings produce magnetic fields and fluxes. The interaction of the stator and rotor magnetic fields (fluxes) results in an electromagnetic torque, which is used to rotate a mechanical load requiring a torque equal to the developed torque. The speed of the motor is determined by Eq. 5.11, where the electrical frequency ω is the frequency of the voltages applied to the stator windings, mostly 377 rad/s (60 Hz) in the United States.

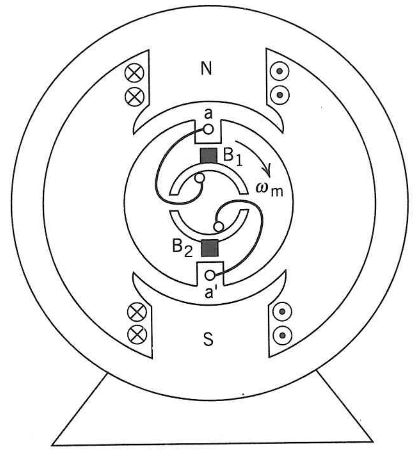


FIGURE 5.7 Elementary DC machine.

5.3.3 Direct-Current Machines

A DC machine can be either a generator or a motor. The armature of this machine is on the rotor, and the field winding is on the stator. This arrangement is opposite to that of an AC synchronous machine, or induction machine, where the armature is on the stator and the field is on the rotor. A simple two-pole DC machine is shown in Fig. 5.7.

For *DC machines*, the field and armature windings both have DC voltages at their terminals. Although the voltage induced in the armature winding is AC, the commutator segments are used to rectify this AC voltage. Hence, the voltage between brushes B_1 and B_2 is a DC voltage. This need for rectification is why the armature winding is located on the rotor.

The energy conversion process in DC machines is similar to that in AC machines. That is, the interaction of stator and rotor fields produces electromagnetic torque. In a DC generator, the rotor is rotated at constant speed by a mechanical energy source sufficient to overcome the electromagnetic torque to produce electricity, and thus mechanical energy is converted to electrical energy. On the other hand, in a DC motor, a DC voltage is applied to both armature and field windings and the electromagnetic torque produced is used to turn a mechanical load; thus, electrical energy is converted to mechanical energy.

DRILL PROBLEMS

D5.3 A synchronous generator has a rotor with six poles and operates at 50 Hz.

a. Determine the speed of the prime mover of the generator.

b. Repeat part (a) if the generator rotor has 12 poles.

c. Repeat part (a) if the generator rotor has two poles.

D5.4 A three-phase AC motor is connected to a 60-Hz voltage supply. It is used to drive a draft fan. At no load, the speed is 1188 rpm; at full load, the speed drops to 1128 rpm.

a. Determine the number of poles of this AC motor.

b. State whether this motor is an induction motor or a synchronous motor.

5.4 ARMATURE MMF AND MAGNETIC FIELD

Most armatures have distributed windings, that is, windings that are spread over a number of slots around the periphery of the machine. This is illustrated in Fig. 5.8.

To derive the magnetic field of a distributed winding, the single N-turn coil is considered first. Such a coil spanning 180 mechanical degrees is shown in Fig. 5.9. The dot and cross indicate current toward and away from the reader, respectively. The rotor is of cylindrical type.

By applying Ampère's law to the semicircular path shown by broken lines in Fig. 5.9, the magnetic field is related to the mmf F as follows:

$$\oint \mathbf{H} \cdot \mathbf{dl} = F \tag{5.12}$$

Assuming that the relative permeability of the iron core is infinitely high, Eq. 5.12 reduces to

$$H(2g) = Ni \tag{5.13}$$

where

g = length of air gap

N = number of turns in the coil

i = current through coil aa′

It is apparent that the mmf associated with the closed path takes the form of an mmf drop across the total air-gap length. This is because the mmf drop inside the iron, on both the stator and the rotor, is negligible, since the core is assumed to have infinitely high permeability. Half of the mmf appears as an mmf drop across the top half of the air gap, and the other half appears across the lower half of the air gap. As a result, the air-gap mmf looks like that of Fig. 5.9*b*.

The rectangular waveform of the mmf shown in Fig. 5.9*b* can be resolved into a Fourier series composed of a fundamental component and a series of

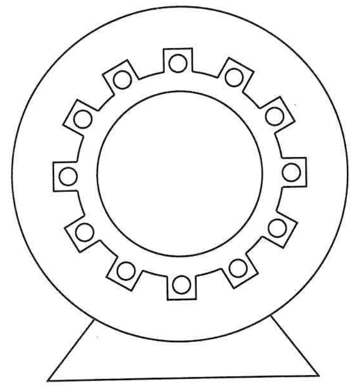


FIGURE 5.8 Distributed armature windings.

odd harmonics. For ease of calculations, the mmf is approximated as

$$F = F_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta_m = F_m\cos\theta_m \tag{5.14}$$

where F_{a1} is the fundamental component of F and $F_m = (4/\pi)(Ni/2)$.

This sinusoidal approximation of the armature winding mmf is an excellent choice for AC machines. If the winding is distributed among a number of slots, the resultant mmf will have a peak value F_p that is smaller than F_m.

The mmf waveform of the armature of a DC machine approximates a sawtooth waveform. This sawtooth waveform can also be approximated by a sinusoidal waveform.

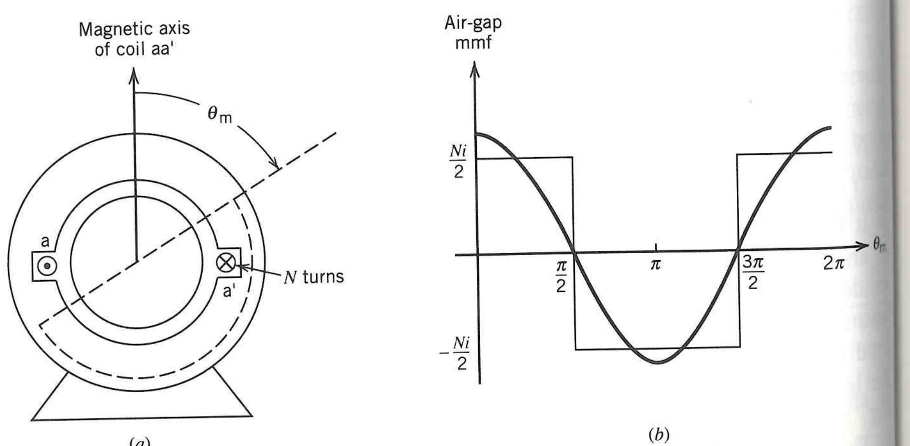


FIGURE 5.9 Single coil machine: (a) concentrated full-pitch coil; (b) air-gap magnetomotive force.

Therefore, in the analysis of AC and DC machines, the mmf of the armature windings will be assumed to have a sinusoidal space distribution. Also, for modeling of AC and DC machines, a two-pole machine will be considered.

From Eq. 5.13, it may be seen that the magnetic field intensity $\mathbf{H}$ is equal to the mmf drop $(Ni/2)$ across the air gap divided by the air-gap length g. Therefore, the fundamental component of $\mathbf{H}$ may be expressed as

$$H = H_{a1} = \frac{F_{a1}}{g} = \frac{F_m}{g}\cos\theta_m = H_m\cos\theta_m \tag{5.15}$$

where $H_m = F_m/g = (4/\pi)(Ni/2g)$.

For a distributed winding such as that shown in Fig. 5.10, the air-gap magnetic field intensity will have a peak value H_p, which is less than H_m.

5.5 ROTATING MMF IN AC MACHINES

To understand the theory of polyphase AC machines, it is necessary to study the nature of the mmf produced by a polyphase winding. Before analyzing the three-phase situation, the single-phase situation is considered.

5.5.1 Single-Phase Winding mmf

In the last section, the mmf of a single-phase winding was derived. It is rewritten here as

$$F_{a1} = F_m\cos\theta_m = \frac{4}{\pi}\frac{Ni}{2}\cos\theta_m \tag{5.16}$$

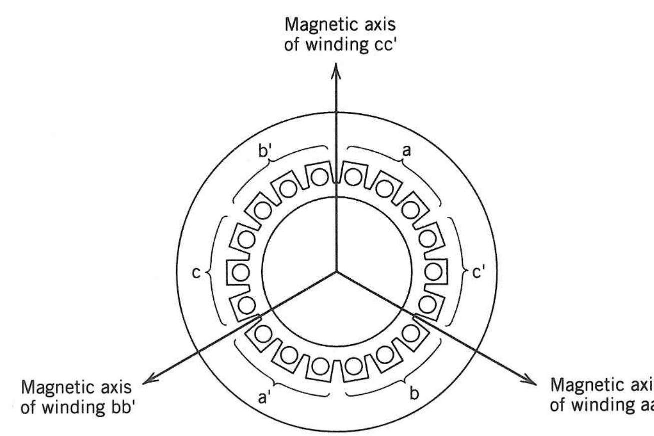


FIGURE 5.10 Three-phase distributed winding.

When the winding is excited by a sinusoidal current, $i = I_a \cos \omega t$, the expression for the mmf becomes

$$F_{a1} = \frac{2NI_a}{\pi} \cos \theta_m \cos \omega t = F_a \cos \theta_m \cos \omega t \qquad (5.17)$$

where $F_a = 2NI_a/\pi$.

Applying trigonometric identities for the product of cosines to Eq. 5.17 yields

$$F_{a1} = \tfrac{1}{2}F_a \cos(\theta_m - \omega t) + \tfrac{1}{2}F_a \cos(\theta_m + \omega t) = F^+ + F^- \qquad (5.18)$$

Equation 5.18 contains two variables: θ_m (space variable) and t (time variable). The first term, F^+, is a traveling wave with amplitude $\frac{1}{2}F_a$, traveling in the direction of increasing θ_m. This phenomenon can be seen by plotting the first term as a function of θ_m for two particular values of t as shown in Fig. 5.11.

Similarly, the second term of Eq. 5.18 represents a traveling wave in the negative θ_m direction. Hence, the single-phase winding mmf can be represented by two vectors traveling in opposite directions as shown in Fig. 5.12.

As seen from Figs. 5.11 and 5.12, the sum of the forward- and backward-traveling waves, F^+ and F^-, respectively, yields a pulsating wave F_{a1}. At any set of values of t and θ_m, the net mmf F_{a1} is found by adding the components of F^+ and F^- along the line that makes an angle θ_m with respect to the magnetic axis of winding aa′. For example at $t = t_1$ and $\theta_m = \pi/3$, the value of F_{a1} is given by

$$F_{a1} = \frac{F_a}{2} \cos(\frac{\pi}{3} - \omega t_1) + \frac{F_a}{2} \cos(\frac{\pi}{3} + \omega t_1)$$
$$= F_a \cos \frac{\pi}{3} \cos \omega t_1 \qquad (5.19)$$

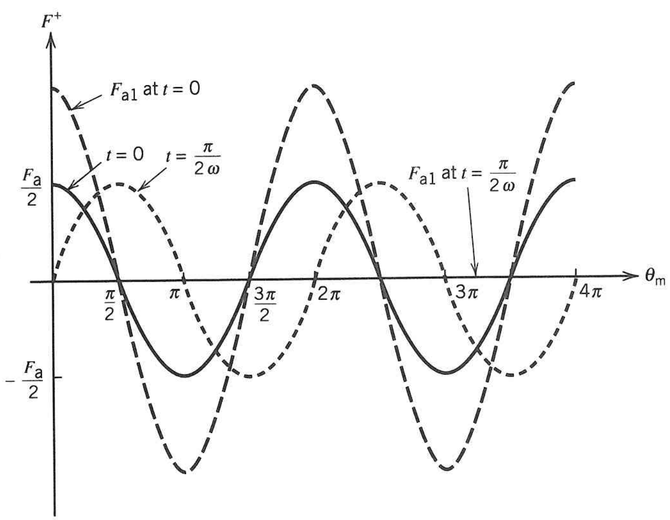


FIGURE 5.11 Traveling wave in the positive θ_m direction.

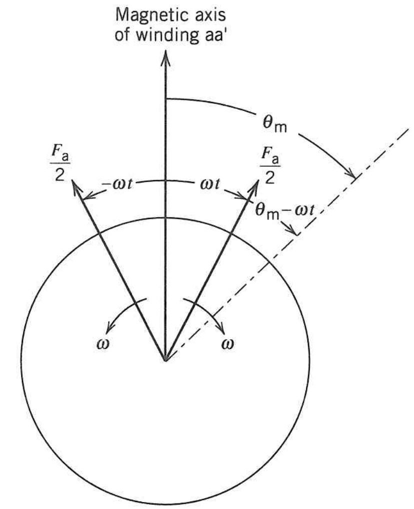


FIGURE 5.12 Double rotating waveforms representation of single-phase mmf.

5.5.2 Three-Phase Winding mmf

Armature windings of synchronous machines and induction machines are typically three-phase windings. Such an armature is illustrated in Fig. 5.13.

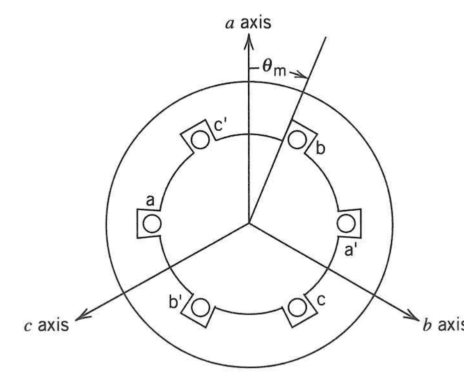


FIGURE 5.13 A three-phase armature winding.

The three phases have equal numbers of coils. At θ_m, the resultant mmf F due to all three phases is given by the sum of the individual phase mmfs. Thus,

$$\begin{aligned} F &= F_{a1} + F_{b1} + F_{c1} \\ &= \frac{2}{\pi} N i_a \cos\theta_m + \frac{2}{\pi} N i_b \cos(\theta_m - 120°) \\ &\quad + \frac{2}{\pi} N i_c \cos(\theta_m - 240°) \end{aligned} \tag{5.20}$$

Under balanced three-phase conditions, the instantaneous currents are expressed as follows:

$$i_a = I_p \cos\omega t \tag{5.21}$$

$$i_b = I_p \cos(\omega t - 120°) \tag{5.22}$$

$$i_c = I_p \cos(\omega t - 240°) \tag{5.23}$$

where I_p is the maximum current.

Substituting these current expressions into Eq. 5.20 yields

$$\begin{aligned} F(\theta_m, t) = \frac{2}{\pi} N I_p [&\cos\theta_m \cos\omega t \\ &+ \cos(\theta_m - 120°)\cos(\omega t - 120°) \\ &+ \cos(\theta_m - 240°)\cos(\omega t - 240°)] \end{aligned} \tag{5.24}$$

By using trigonometric identities to simplify the expression inside the brackets in Eq. 5.24, the resultant mmf may be expressed as

$$F(\theta_m, t) = \frac{3}{2} F_p \cos(\theta_m - \omega t) \tag{5.25}$$

where $F_p = 2NI_p/\pi$.

Equation 5.25 represents a traveling wave moving in the direction of positive θ_m. It is shown graphically in Fig. 5.14.

At any time t and position θ_m, the resultant mmf due to all three windings is found by taking the projection of the vector $\frac{3}{2}F_p$ on the line θ_m. For example, at time $t = \pi/(4\omega)$ seconds, the mmf along the a axis is the component of the vector $\frac{3}{2}F_p$ along the line $\theta_m = 0$. Thus,

$$F(\theta_m, t) = F\left(0, \frac{\pi}{4\omega}\right) = \frac{3}{2} F_p \cos\left(0 - \frac{\pi}{4}\right) = \frac{3}{2} F_p \cos\frac{\pi}{4} \tag{5.26}$$

In this derivation, it has been assumed that at $t = 0$, the mmf due to coil aa′ is maximum along the line $\theta_m = 0$.

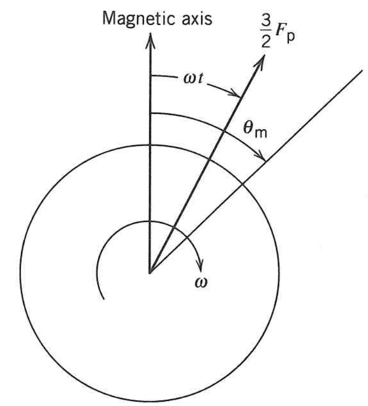


FIGURE 5.14 Graphical representation of three-phase resultant mmf.

DRILL PROBLEMS

D5.5 The windings of a three-phase machine are supplied with currents i_a, i_b, and i_c. The mmfs produced by these currents are given as follows:

$$F_a = N i_a \cos\theta_m$$

$$F_b = N i_b \cos(\theta_m - 120°)$$

$$F_c = N i_c \cos(\theta_m - 240°)$$

a. Assume that the three-phase windings are connected in series and supplied by one voltage source, that is, $i_a = i_b = i_c$. Find the resultant mmf due to all three windings as a function of θ_m.

b. The three-phase windings are connected to a balanced three-phase voltage supply, and they take the following currents:

$$i_a = I_p \cos\omega t$$

$$i_b = I_p \cos(\omega t - 120°)$$

$$i_c = I_p \cos(\omega t - 240°)$$

Find the resultant mmf.

c. Let $\theta_m = \omega_m t + \alpha$. Determine the relationship between ω_m and ω that results in maximum mmf.

D5.6 Show that

$$\begin{aligned} F\cos\alpha\cos\beta &+ F\cos(\alpha - 120°)\cos(\beta - 120°) \\ &+ F\cos(\alpha - 240°)\cos(\beta - 240°) = \tfrac{3}{2}F\cos(\alpha - \beta) \end{aligned}$$

D5.7 A synchronous generator has a three-phase winding with 10 turns per phase. The three phase currents are given by

$$i_a = 100 \cos 377t \text{ A}$$
$$i_b = 100 \cos(377t - 120°)$$
$$i_c = 100 \cos(377t - 240°)$$

Determine (a) the fundamental component of the mmf of each winding and (b) the resultant mmf.

5.6 GENERATED VOLTAGE IN ROTATING MACHINES

In Section 5.2, the calculation of the induced voltage (emf) in a coil was described. By Faraday's law, an emf is produced when the flux linking a coil changes. In this section, the generated voltage for a three-phase rotating machine will be derived. An elementary three-phase synchronous machine is shown in Fig. 5.15.

The rotor coil receives DC current through the slip rings. This current I_f produces an mmf that is maximum along the rotor field's magnetic axis. Its fundamental component is distributed sinusoidally around the rotor's periphery.

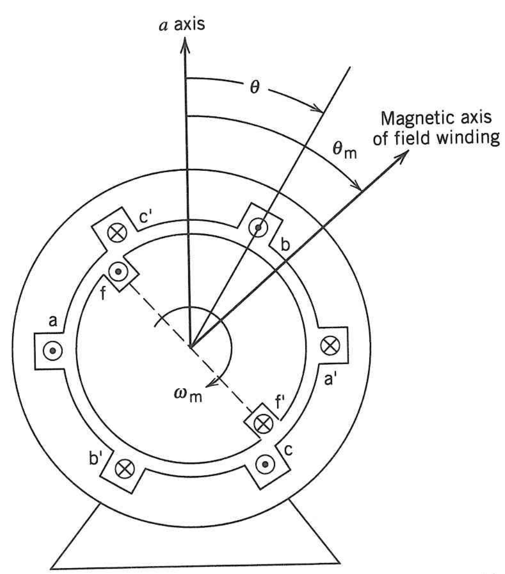


FIGURE 5.15 A three-phase AC synchronous machine.

Therefore, the mmf in the air gap along the line θ may be expressed as

$$F_{r1} = \frac{4}{\pi}\frac{N_f I_f}{2}\cos(\theta_m - \theta) \tag{5.27}$$

where N_f is the number of turns of the field winding.

The magnetic field and flux density are then obtained as follows:

$$H = \frac{F_{r1}}{g} = \frac{2N_f I_f}{\pi g}\cos(\theta_m - \theta) \tag{5.28}$$

$$B = \mu_0 H = \frac{2\mu_0 N_f I_f}{\pi g}\cos(\theta_m - \theta) \tag{5.29}$$

where g is the length of the air gap. Thus, the flux linking the stator windings aa′, bb′, and cc′ is computed as

$$\phi_{aa'} = \int_S \mathbf{B}\cdot d\mathbf{S} = \int_{-\pi/2}^{\pi/2} \frac{2\mu_0 N_f I_f}{\pi g}[\cos(\theta_m - \theta)]l\,r\,d\theta$$

$$= \frac{4\mu_0 N_f I_f}{\pi g} l r \cos\theta_m = \Phi_p \cos\theta_m \tag{5.30}$$

where

$\Phi_p = 4\mu_0 N_f I_f l r/(\pi g)$

= maximum flux linking one loop of winding aa′ due to I_f

l = axial length of the machine

r = rotor radius

Denoting by N_a the number of turns in each armature phase winding, the total flux linkages are obtained as follows:

$$\lambda_{aa'} = N_a \Phi_p \cos\theta_m \tag{5.31}$$
$$\lambda_{bb'} = N_a \Phi_p \cos(\theta_m - 120°) \tag{5.32}$$
$$\lambda_{cc'} = N_a \Phi_p \cos(\theta_m - 240°) \tag{5.33}$$

The angle θ_m is the relative position of the rotor magnetic axis with respect to the magnetic axis of the winding aa′. Since the rotor turns at a constant speed of ω rad/s, the angle θ_m may be expressed as

$$\theta_m = \int_0^t \omega\, dt + \theta_m(0) = \omega t + \theta_m(0) \tag{5.34}$$

Thus, the total flux linkages may be written as

$$\lambda_{aa'} = N_a\Phi_p \cos[\omega t + \theta_m(0)] \quad (5.35)$$
$$\lambda_{bb'} = N_a\Phi_p \cos[\omega t + \theta_m(0) - 120°] \quad (5.36)$$
$$\lambda_{cc'} = N_a\Phi_p \cos[\omega t + \theta_m(0) - 240°] \quad (5.37)$$

From these equations, and since the flux linkages are time varying, the induced voltages are derived as follows:

$$e_{aa'} = \frac{d\lambda_{aa'}}{dt} = -\omega N_a\Phi_p \sin[\omega t + \theta_m(0)] = \omega N_a\Phi_p \cos[\omega t + \theta_m(0) + 90°] \quad (5.38)$$
$$e_{bb'} = \frac{d\lambda_{bb'}}{dt} = \omega N_a\Phi_p \cos[\omega t + \theta_m(0) - 30°] \quad (5.39)$$
$$e_{cc'} = \frac{d\lambda_{cc'}}{dt} = \omega N_a\Phi_p \cos[\omega t + \theta_m(0) - 150°] \quad (5.40)$$

Equations 5.38–5.40 represent a set of balanced induced voltages. The maximum value of the voltage is given by

$$E_{max} = \omega N_a\Phi_p = 2\pi f N_a\Phi_p \quad (5.41)$$

and its rms value is

$$E_{rms} = \frac{2\pi}{\sqrt{2}} f N_a\Phi_p = 4.44 f N_a\Phi_p \quad (5.42)$$

where f is the frequency in hertz.

It has been assumed that the three-phase windings were full-pitch, concentrated windings. In full-pitch windings, the two sides of each coil are placed in slots that are 180 electrical degrees apart. Concentrated windings have the conductors of each phase winding placed in just one pair of slots.

In actual machines, the coils of each armature phase winding are distributed among a number of slots with corresponding coil sides in slots possibly less than 180 electrical degrees apart. This results in a reduction in the generated voltages for each phase because the induced voltages in the coils are no longer in time phase. Hence, the phasor sum of the coil voltages is less than it would be if all the coils in one phase were concentrated in one pair of full-pitch slots. Therefore, for distributed and fractional-pitch windings, a reduction factor K_w is introduced. K_w is called the machine *winding factor* and has a value from 0.85 to 0.95. Thus, Eq. 5.42 is modified as follows:

$$E_{rms} = 4.44 K_w f N_a\Phi_p \quad (5.43)$$

When the terminals of the machine are connected to a balanced electrical load, a set of balanced currents will flow. These balanced currents produce a magnetic field that can be represented as a rotating mmf at an angular velocity ω, the frequency of the generated voltage, which is also the angular velocity of the rotor in rad/s. If there are more than two poles, the rotor speed and the radian frequency of the voltage are related by Eq. 5.11.

The voltage induced in DC machines can also be analyzed by the same principles as in AC machines. In DC machines, however, the field winding is on the stator and the armature winding is on the rotor. The rotor coils are distributed over the periphery of the rotor. The voltage induced in each turn of the rotor winding is a sinusoidal function of time. Mechanical rectification is provided by a commutator and brush combination. The average, or DC, value of the voltage between brushes is given by

$$E_a = \frac{1}{\pi}\int_0^{\pi} \omega N\Phi_p \sin \omega t \, d(\omega t) = \frac{2}{\pi}\omega N\Phi_p \quad (5.44)$$

Equation 5.44 is more conveniently expressed in terms of the mechanical speed ω_m in rad/s, or n in rev/min (rpm). Thus, E_a may be written as

$$E_a = \frac{2}{\pi}\left(\frac{p}{2}\omega_m\right)N\Phi_p = \frac{p\omega_m N\Phi_p}{\pi} \quad (5.45)$$

Since $\omega_m = 2\pi n/60$, Eq. 5.45 may also be written as

$$E_a = \frac{2}{\pi}\left(\frac{p}{2}\frac{2\pi n}{60}\right)N\Phi_p = \frac{pnN\Phi_p}{30} \quad (5.46)$$

where

p = number of poles
N = total number of turns in series between armature terminals

In terms of the total number of armature conductors Z and the number of parallel paths a between armature terminals, the number of series turns is given by

$$N = \frac{Z}{2a} \quad (5.47)$$

Substituting Eq. 5.47 into Eq. 5.45 gives

$$E_a = \frac{pZ}{2\pi a}\Phi_p\omega_m \quad (5.48)$$

The commutator and brush combination converts the AC to a DC voltage in the case of a DC generator. In the DC motor, each conducting loop on the rotor is in contact with the applied DC voltage through the commutator and brush. Hence, the current in each loop will be a pulse that can be approximated to a sinusoidal function, that is, the fundamental component in a Fourier series representation.

DRILL PROBLEMS

D5.8 A two-pole, three-phase, wye-connected, round-rotor synchronous generator has the following data:

$$N_a = 24 \text{ turns per phase} \qquad N_f = 500 \text{ turns}$$
$$l = 4 \text{ m} \qquad r = 50 \text{ cm} \qquad g = 20 \text{ mm}$$

The field current, or rotor current, is adjusted to $I_f = 8$ A. Find (a) the peak fundamental mmf due to the field current and (b) the induced voltage in each phase and their rms values.

D5.9 The two-pole machine shown in Fig. 5.16 has a field winding located on the stator and an armature winding on the rotor, with $N_f = 800$ turns and $N_a = 50$ turns, respectively. It has a uniform air gap of 0.5 mm. The armature has a diameter of 0.5 m, and it is 1.5 m long. The field winding carries a current of 2 A. The rotor is driven at 3600 rpm. Determine (a) the frequency of the rotor induced voltage and (b) the instantaneous voltage and rms voltage induced in the rotor coil.

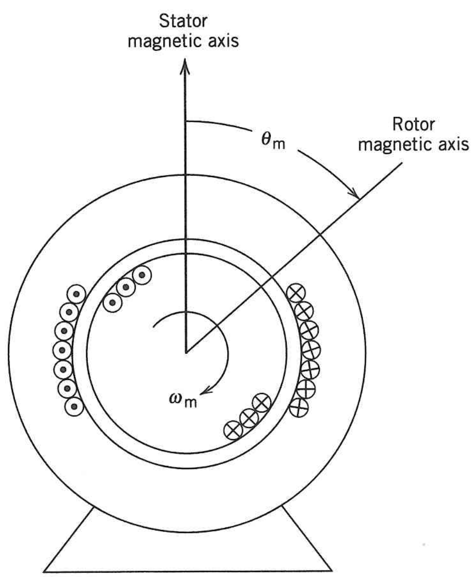


FIGURE 5.16 Rotating machine with uniform air gap.

5.7 TORQUE IN ROUND-ROTOR MACHINES

The behavior of an electromechanical energy conversion device can be described in terms of its equivalent circuit, which is governed by Kirchhoff's voltage law, and its torque equation. The previous section derived expressions for generated voltage that are sufficient to model equivalent circuits. In this section, the torque equations are derived.

Two points of view are presented. The first considers the machine as a set of coupled coils. The second point of view considers the machine as two groups of windings producing magnetic fields in the air gap—one group on the rotor and the other group on the stator. In this manner, the torque is expressed as the tendency for two magnetic fields to line up in the same manner as permanent magnets tend to align themselves. The generated voltage is expressed as the result of relative motion between a winding and a magnetic field, or mmf.

Before the two approaches are described in greater detail, the expression for electromagnetic torque for an electromechanical energy conversion device will be derived. This expression can be obtained from the energy conservation law.

The model of a lossless electromechanical energy conversion device is shown in Fig. 5.17. This schematic represents a motor.

For the energy conversion system shown, the energy conservation law may be expressed as

$$W_e = W_f + W_m \tag{5.49}$$

where

W_e = electrical energy input from the electrical source
W_f = energy stored in the magnetic field of the two coils associated with the electrical inputs
W_m = mechanical energy output

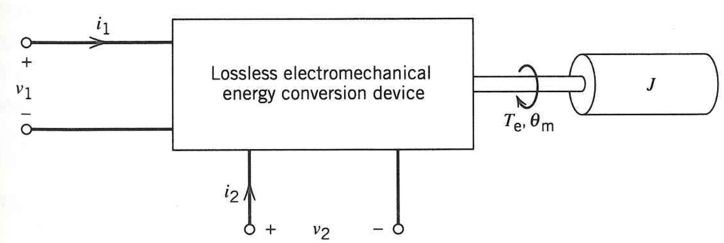


FIGURE 5.17 Model of electromechanical energy converter.

The electrical energy and mechanical energy are given by the following expressions:

$$W_e = \int_0^t v_1 i_1 \, d\tau + \int_0^t v_2 i_2 \, d\tau = \int_0^t p_1 \, d\tau + \int_0^t p_2 \, d\tau \tag{5.50}$$

$$W_m = \int_{\theta_m(0)}^{\theta_m} T_m \, d\phi_m = \int_{\theta_m(0)}^{\theta_m} T_e \, d\phi_m \tag{5.51}$$

Equations 5.50 and 5.51 can be written in their differential forms as Eqs. 5.52 and 5.53, respectively.

$$dW_e = v_1 i_1 \, dt + v_2 i_2 \, dt \tag{5.52}$$

$$dW_m = T_m \, d\theta_m = T_e \, d\theta_m \tag{5.53}$$

Since the windings are assumed to have negligible resistances, the terminal voltages are equal to the induced voltages in the coils:

$$v_1 = e_1 = \frac{d\lambda_1}{dt} \tag{5.54}$$

$$v_2 = e_2 = \frac{d\lambda_2}{dt} \tag{5.55}$$

Substituting Eqs. 5.54 and 5.55 into Eq. 5.52 yields

$$dW_e = i_1 \, d\lambda_1 + i_2 \, d\lambda_2 \tag{5.56}$$

Substituting Eqs. 5.53 and 5.56 into the differential form of Eq. 5.49, and solving for dW_f gives

$$dW_f = i_1 \, d\lambda_1 + i_2 \, d\lambda_2 - T_e \, d\theta_m \tag{5.57}$$

In Fig. 5.17, one independent variable can be specified for each of the terminal pairs; therefore, two electrical variables and one mechanical (or spatial) variable are specified. In Eq. 5.57, the variables λ_1, λ_2, and θ_m are selected as independent variables; thus, the magnetic *field energy* is expressed as a function of these variables: $W_f(\lambda_1, \lambda_2, \theta_m)$. However, the derivation of an expression for the energy function is not an easy task inasmuch as the flux linkages are not physically measurable.

At this point, a new function called *coenergy* is defined. It is designated as W_f', and it is associated with the field energy function W_f as follows:

$$W_f' + W_f = \lambda_1 i_1 + \lambda_2 i_2 \tag{5.58}$$

Equation 5.58 can be written in differential form, and the expression for the differential of the coenergy function dW_f' is obtained as follows:

$$\begin{aligned} dW_f' &= d(\lambda_1 i_1) + d(\lambda_2 i_2) - dW_f \\ &= \lambda_1 \, di_1 + i_1 \, d\lambda_1 + \lambda_2 \, di_2 + i_2 \, d\lambda_2 - dW_f \end{aligned} \tag{5.59}$$

Substituting Eq. 5.57 into Eq. 5.59 and simplifying yield

$$dW_f' = \lambda_1 \, di_1 + \lambda_2 \, di_2 + T_e \, d\theta_m \tag{5.60}$$

The coenergy function is seen to be dependent on i_1, i_2, and θ_m, and it is expressed as a function of these variables. Thus,

$$W_f' = W_f'(i_1, i_2, \theta_m) \tag{5.61}$$

The total differential of the coenergy function may be written as

$$dW_f' = \frac{\partial W_f'}{\partial i_1} \, di_1 + \frac{\partial W_f'}{\partial i_2} \, di_2 + \frac{\partial W_f'}{\partial \theta_m} \, d\theta_m \tag{5.62}$$

Upon term-by-term comparison of Eqs. 5.60 and 5.62, the expression for electromagnetic torque is obtained.

$$T_e = \frac{\partial W_f'(i_1, i_2, \theta_m)}{\partial \theta_m} \tag{5.63}$$

The torque expression of Eq. 5.63 can be generalized for the case of n windings, or n currents, as follows:

$$T_e = \frac{\partial W_f'(i_1, i_2, \ldots, i_n, \theta_m)}{\partial \theta_m} \tag{5.64}$$

When the magnetic cores on the rotor and stator are assumed to have linear characteristics, the energy function W_f may be expressed as

$$W_f = \tfrac{1}{2}\lambda_1 i_1 + \tfrac{1}{2}\lambda_2 i_2 \tag{5.65}$$

Substituting this expression for W_f into Eq. 5.58 yields

$$W_f' = \tfrac{1}{2}\lambda_1 i_1 + \tfrac{1}{2}\lambda_2 i_2 \tag{5.66}$$

Thus, it is seen that the field energy function and the coenergy function are the same in a linear magnetic circuit.

In *linear electromagnetic systems*, the relationships between flux linkages and currents are given by

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \tag{5.67}$$
$$\lambda_1 = L_{21}i_1 + L_{22}i_2 \tag{5.68}$$

where

L_{11} = self-inductance of winding 1

L_{22} = self-inductance of winding 2

$L_{12} = L_{21}$ = mutual inductance between windings 1 and 2

These inductances are functions of only the mechanical variable, or spatial variable, θ_m. Substituting Eqs. 5.67 and 5.68 into Eq. 5.66 yields the expression for the coenergy function.

$$W_f' = \tfrac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \tfrac{1}{2}L_{22}i_2^2 \tag{5.69}$$

Thus, the expression for the electromagnetic torque may be written as follows:

$$T_e = \left(\frac{\partial W_f'(i_1, i_2, \theta_m)}{\partial \theta_m}\right|_{i_1, i_2 = \text{ fixed}}$$
$$= \left(\frac{1}{2}\frac{\partial\left(L_{11}i_1^2\right)}{\partial \theta_m} + \frac{\partial(L_{12}i_1i_2)}{\partial \theta_m} + \frac{1}{2}\frac{\partial\left(L_{22}i_2^2\right)}{\partial \theta_m}\right|_{i_1, i_2 = \text{ fixed}} \tag{5.70}$$

Since the inductances are functions of only the spatial variable θ_m, Eq. 5.70 reduces to

$$T_e = \frac{1}{2}i_1^2\frac{dL_{11}(\theta_m)}{d\theta_m} + i_1i_2\frac{dL_{12}(\theta_m)}{d\theta_m} + \frac{1}{2}i_2^2\frac{dL_{22}(\theta_m)}{d\theta_m} \tag{5.71}$$

Note that no electromagnetic torque is developed if none of the inductances is a function of the spatial variable θ_m.

EXAMPLE 5.2

The two-pole machine shown in Fig. 5.16 has two mutually coupled coils. The first coil, located on the stator, is held fixed, and the second coil, on the rotor, is free to rotate. The inductances of the coils are given by

$$L_{11} = 10 \qquad L_{22} = 8 \qquad L_{12} = L_{21} = 6\cos\theta_m$$

Let the current in the first coil be denoted by i_1 and the current in the second coil by i_2. The magnetic axes of the two coils are initially displaced by an angle $\theta_m(0) = 30°$. Find the electrical torque for the following conditions:

a. $i_1 = 2$ A and $i_2 = 0$

b. $i_1 = i_2 = 2$ A

c. $i_1 = 2$ A and $i_2 = \sqrt{2}\sin\omega t$ A

d. $i_1 = i_2 = \sqrt{2}\sin\omega t$ A

e. $i_1 = 2$ A and coil 2 short-circuited

Solution Using Eq. 5.69, the expression for the coenergy function is given by

$$W_f' = \tfrac{1}{2}(10)i_1^2 + 6\cos\theta_m i_1i_2 + \tfrac{1}{2}(8)i_2^2$$

The general expression for the electromagnetic torque can be found by using Eq. 5.71 as follows:

$$T_e = \frac{1}{2}i_1^2\frac{d}{d\theta_m}(10) + i_1i_2\frac{d}{d\theta_m}(6\cos\theta_m) + \frac{1}{2}i_2^2\frac{d}{d\theta_m}(8) = -6i_1i_2\sin\theta_m$$

a. $T_e = -6i_1i_2\sin\theta_m = -(6)(2)(0)\sin 30° = 0$

b. $T_e = -6i_1i_2\sin\theta_m = -(6)(2)(2)\sin 30° = -12$ N-m

c. $T_e = -6i_1i_2\sin\theta_m = -(6)(2)(\sqrt{2}\sin\omega t)\sin 30°$
$= -6\sqrt{2}\sin\omega t$ N-m

d. $T_e = -6i_1i_2\sin\theta_m = -(6)(\sqrt{2}\sin\omega t)(\sqrt{2}\sin\omega t)\sin 30°$
$= -6\sin^2\omega t$

e. Since the rotor is short-circuited, the flux linkage of the rotor may be set equal to zero; thus,

$$0 = L_{21}i_1 + L_{22}i_2$$

Therefore, the rotor current can be expressed as

$$i_2 = -(L_{21}/L_{22})i_1 = -[(6\cos\theta_m)/8](2) = -1.5\cos\theta_m \text{ A}$$

Substituting in the general expression for torque yields

$$T_e = -6i_1i_2\sin\theta_m = -(6)(2)(-1.5\cos 30°)\sin 30° = 7.8 \text{ N-m}$$

EXAMPLE 5.3

A linear electromechanical energy conversion system has two electrical inputs and one mechanical output. The self- and mutual inductances of the coils are

$$L_{11} = 5 + \cos 2\theta_m \text{ mH}$$
$$L_{22} = 50 + 10 \cos 2\theta_m \text{ H}$$
$$L_{12} = L_{21} = 100 \cos \theta_m \text{ mH}$$

The first coil is supplied with a current $i_1 = 1$ A, and the second draws a current $i_2 = 10$ mA. Determine (a) the general expression for electromagnetic torque T_e and (b) the maximum torque.

Solution

a. For this linear system, the coenergy function is found by using Eq. 5.69. Substituting the expressions for the inductances yields

$$\begin{aligned} W_f' = W_f &= \tfrac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \tfrac{1}{2}L_{22}i_2^2 \\ &= \tfrac{1}{2}(5 + \cos 2\theta_m) \times 10^{-3} i_1^2 + 0.10 \cos \theta_m i_1 i_2 \\ &\quad + \tfrac{1}{2}(50 + 10 \cos 2\theta_m) i_2^2 \end{aligned}$$

The expression for the electromagnetic torque is obtained as follows:

$$\begin{aligned} T_e &= \left(\frac{\partial W_f'(i_1, i_2, \theta_m)}{\partial \theta_m}\right|_{i_1=1.0, i_2=0.01} \\ &= \left(\frac{1}{2}\frac{\partial(i_1^2 L_{11})}{\partial \theta_m} + \frac{\partial(i_1 i_2 L_{12})}{\partial \theta_m} + \frac{1}{2}\frac{\partial(i_2^2 L_{22})}{\partial \theta_m}\right|_{i_1=1.0, i_2=0.01} \\ &= \tfrac{1}{2}(1.0)^2(-2 \sin 2\theta_m) \times 10^{-3} + (1.0)(0.01)(-0.1 \sin \theta_m) \\ &\quad + \tfrac{1}{2}(0.01)^2(-20 \sin 2\theta_m) \\ &= -(2 \sin 2\theta_m + \sin \theta_m) \times 10^{-3} \end{aligned}$$

b. At maximum torque

$$\frac{dT_e}{d\theta_m} = 0$$

Differentiating T_e from part (a),

$$4 \cos 2\theta_m + \cos \theta_m = 0$$

Solving for θ_m by the quadratic formula,

$$\theta_m = 49.7°, 140.6° \quad \text{(extraneous)}$$

Substituting the value of $\theta_m = 49.7°$ into the expression for the electromagnetic torque yields

$$T_{e,max} = -[2 \sin 2(49.7°) + \sin 49.7°] \times 10^{-3} = -2.74 \times 10^{-3} \text{ N-m}$$

DRILL PROBLEMS

D5.10 An electromagnetic system has two windings and a nonuniform air gap. The self- and mutual inductances are given by

$$L_{11} = 6 + 1.5 \cos 2\theta \qquad L_{22} = 4 + \cos 2\theta \qquad M = 5 \cos \theta$$

The windings are connected to a DC voltage source. The first winding takes a current $I_1 = 10$ A, and the second draws a current $I_2 = 2$ A.

a. Determine the developed torque as a function of θ.

b. Find the stored energy as a function of θ.

c. For $\theta = 30°$, find the magnitude and direction of the torque on the rotor.

D5.11 An electromagnetic system consists of two mutually coupled coils as shown in Fig. 5.16. The inductances of the coils are

$$L_{ss} = 5 \text{ H}$$
$$L_{rr} = 3 \text{ H}$$
$$L_{sr} = 2 \cos \theta \text{ H}$$

Find the electromagnetic torque for the following cases.

a. $I_s = 10$ A and $I_r = 0$

b. $I_s = I_r = 10$ A

c. $I_s = 10$ A and $i_r = 10 \sin \omega t$

d. $i_s = i_r = 10 \sin \omega t$

5.7.1 The Coupled-Coils Approach

An elementary machine is shown in Fig. 5.18. It has one winding on the stator and one winding on the rotor. The slot openings for the stator and rotor coils

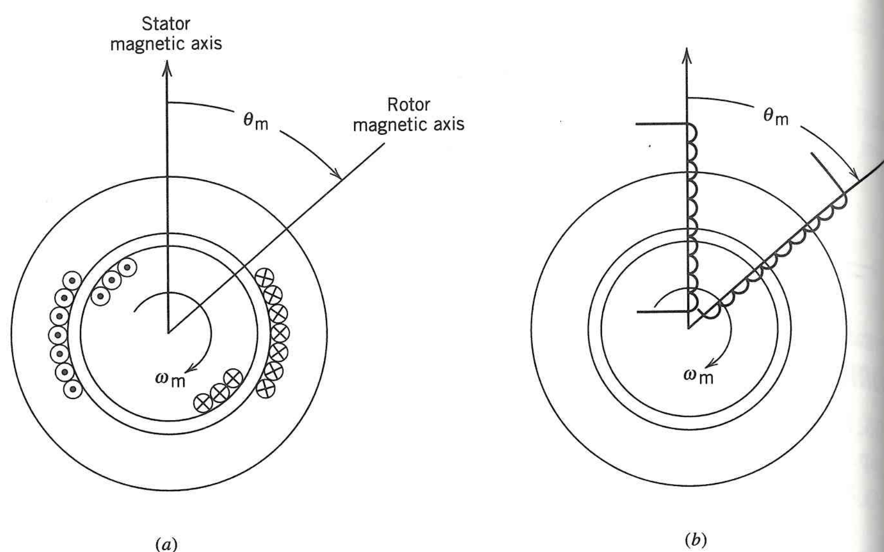


FIGURE 5.18 A round-rotor machine: (*a*) winding distribution; (*b*) schematic representation.

are not shown in the figure. Machines that have more than one coil on the stator may also be analyzed by representing the stator coils with one equivalent winding.

For this machine containing two coils, the electromagnetic torque T_e is given by

$$T_e = \frac{\partial W'(i_s, i_r, \theta_m)}{\partial \theta_m} \quad (5.72)$$

Since linearity has been assumed for both stator and rotor magnetic cores, the coenergy function W_f' may be expressed as

$$W_f' = \tfrac{1}{2}L_{ss}i_s^2 + \tfrac{1}{2}L_{rr}i_r^2 + L_{sr}i_s i_r \quad (5.73)$$

where

L_{ss} = self-inductance of the stator coil
L_{rr} = self-inductance of the rotor coil
L_{sr} = mutual inductance between stator and rotor coils
i_s = stator current
i_r = rotor current

Assuming infinite permeability for the magnetic cores, the inductance parameters are given as

$$L_{ss} = \frac{4\mu_0 N_s^2 l r}{\pi g} \quad (5.74)$$

$$L_{rr} = \frac{4\mu_0 N_r^2 l r}{\pi g} \quad (5.75)$$

$$L_{sr} = \frac{4\mu_0 N_s N_r l r}{\pi g}\cos\theta_m = M\cos\theta_m \quad (5.76)$$

where

μ_0 = permeability of free space
M = $(4\mu_0 N_s N_r l r)/(\pi g)$ = maximum mutual inductance that occurs when stator and rotor magnetic axes are aligned
N_s = number of turns on the stator
N_r = number of turns on the rotor
l = axial length
r = radius of the rotor
g = length of the air gap

Substituting Eqs. 5.74–5.76 into Eq. 5.73 and differentiating the result with respect to θ_m give the expression for the electromagnetic torque.

$$\begin{aligned} T_e &= -M i_s i_r \sin\theta_m \\ &= M i_s i_r \cos(\theta_m + 90°) \end{aligned} \quad (5.77)$$

It may be noted from Eq. 5.77 that the electromagnetic torque is directly proportional to the product of the stator current and the rotor current. In steady state operations, this torque is balanced by its counterpart mechanical torque T_m. For generator action, T_m is an input quantity; while for motor action, T_m is the output quantity.

EXAMPLE 5.4

For the two-pole machine of Fig. 5.18, the currents flowing in the stator and rotor windings are given by $i_s = I_s \cos\omega_s t$ and $i_r = I_r$ = constant, respectively, where ω_s is the angular frequency of the stator current.

a. Find the expression for the electromagnetic torque.

b. Find the speed at which average torque is nonzero.

c. Find the average torque $T_{e,ave}$ if $I_r = 4$ A, $I_s = 50$ A, $\theta_m(0) = 10°$, and $M = 2$ H in part (b).

Solution

a. The angle θ_m is found by using Eq. 5.34. Thus,

$$\theta_m = \int_0^t \omega_m\,dt + \theta_m(0) = \omega_m t + \theta_m(0)$$

Assume ω_m remains constant. Then the torque may be expressed as

$$T_e = -MI_s \cos \omega_s t \, I_r \sin \theta_m = -MI_s I_r \cos \omega_s t \sin[\omega_m t + \theta_m(0)]$$

b. The expression for T_e may also be written as

$$T_e = -MI_s I_r\{\tfrac{1}{2} \sin[(\omega_m + \omega_s)t + \theta_m(0)] + \tfrac{1}{2} \sin[(\omega_m - \omega_s)t + \theta_m(0)]\}$$

From this expression for torque, it is seen that a nonzero average will be present if and only if $\omega_m = \omega_s$. For a 60-Hz stator current frequency, the speed is

$$n = 120f/p = (120)(60)/2 = 3600 \text{ rpm}$$

c. For a nonzero average torque, $\omega_m = \omega_s$; thus,

$$T_e = -\tfrac{1}{2}MI_s I_r\{\sin[2\omega_s t + \theta_m(0)] + \sin \theta_m(0)\}$$

Since the average of a sinusoidal function of time is zero, the first term on the right-hand side of the preceeding expression reduces to zero; therefore,

$$T_{e,ave} = -\tfrac{1}{2}MI_s I_r \sin \theta_m(0) = -\tfrac{1}{2}(2)(50)(4) \sin 10° = -34.7 \text{ N-m}$$

DRILL PROBLEMS

D5.12 Derive Eqs. 5.74, 5.75, and 5.76. Assume that the permeabilities of rotor and stator ferromagnetic cores are infinitely high.

D5.13 Two mutually coupled coils, one mounted on a stator and the other on a rotor, are shown in Fig. 5.16. The self- and mutual inductances are given as follows:

$$L_{ss} = 0.50 \text{ H}$$
$$L_{rr} = 0.25 \text{ H}$$
$$L_{sr} = 1.0 \cos \theta \text{ H}$$

The coils are connected in series, and the combination is excited by a sinusoidal voltage source.

a. Determine the instantaneous torque if the coils draw a current $i(t) = 10\sqrt{2} \sin 377t$ A.

b. Determine the time-averaged torque T_{ave} as a function of the angular displacement θ.

c. Find the value of T_{ave} for $\theta = 60°$.

d. The series-connected coils are supplied from a DC source, and they take a current $I = 10$ A. Compute the torque at $\theta = 60°$, and compare with the result of part (c).

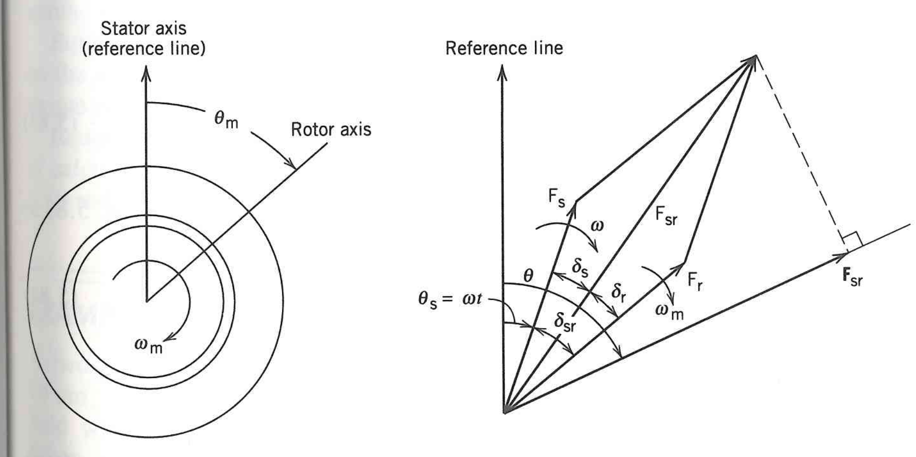
5.7.2 The Magnetic Field Approach

In this section, the machine model is based on the interaction of magnetic fields of the stator and rotor windings in the air gap. Since the core is assumed to have infinite permeability, the core magnetic field, or mmf drop, is approximately zero.

A two-pole synchronous machine is shown in Fig. 5.19. The currents in the stator and rotor windings create magnetic flux in the air gap. The stator and rotor mmfs can each be represented by a rotating mmf as shown in Fig. 5.19*b*. For the synchronous machine, the expressions for the mmfs of the stator and rotor are given by $F_s = \frac{3}{2}(2N_s I_s/\pi)$ and $F_r = 2N_r I_r/\pi$, respectively.

The interaction of the stator and rotor mmfs, or magnetic fields, creates an electromagnetic torque T_e. This torque tends to align the stator and rotor mmfs. As long as the angle between the two mmfs, δ_{sr}, is nonzero, an electromagnetic torque will exist.

At steady state, the angle δ_{sr} is a constant. In other words, ω and ω_m are equal. Note that ω and ω_m represent the angular speed of F_s and F_r, respectively, from the same reference line. In the derivation to follow, all losses and leakages are neglected and the permeability of the stator and rotor cores is assumed to be infinite.



(*a*) (*b*)

FIGURE 5.19 A two-pole machine: (*a*) elementary model; (*b*) vector diagram of mmf waves.

From Ampère's law, the magnetic field intensity **H** in the air gap is given by

$$\mathbf{H} = \frac{\mathbf{F}_{sr}}{g} \tag{5.78}$$

where

$\mathbf{F}_{sr}$ = resultant mmf at the air gap along the line θ

g = length of the air gap

It is seen from Fig. 5.19*b* that $\mathbf{F}_{sr}$ is the component of F_{sr} along the line θ. The expression for F_{sr} is found from Eq. 5.79.

$$F_{sr}^2 = F_s^2 + F_r^2 + 2F_sF_r \cos \delta_{sr} \tag{5.79}$$

Hence, the resultant mmf $\mathbf{F}_{sr}$ is given by

$$\mathbf{F}_{sr} = F_{sr} \cos(\theta - \omega t - \delta_s) \tag{5.80}$$

Substituting Eq. 5.80 into Eq. 5.78 yields

$$\mathbf{H} = \frac{F_{sr}}{g} \cos(\theta - \omega t - \delta_s) \tag{5.81}$$

Because of the assumption of magnetic core linearity, the coenergy function is equal to the energy function. The energy density W in the air gap is given by

$$W = \tfrac{1}{2}\mathbf{B} \cdot \mathbf{H} = \tfrac{1}{2}\mu_0 \mathbf{H} \cdot \mathbf{H} = \tfrac{1}{2}\mu_0 H^2 \tag{5.82}$$

Substituting the expression for **H** from Eq. 5.81 into 5.82 yields

$$W = \frac{\mu_0 F_{sr}^2}{2g^2} \cos^2(\theta - \omega t - \delta_s) \tag{5.83}$$

The average energy density (or coenergy density) is found from Eq. 5.83 as

$$W_{ave} = W'_{ave} = \frac{\mu_0 F_{sr}^2}{4g^2} \tag{5.84}$$

Finally, the total coenergy W' may be expressed as

$$W' = \frac{\mu_0 F_{sr}^2}{4g^2}(2\pi r l g) \tag{5.85}$$

where

r = radius of the rotor

l = axial length of the machine

g = air-gap clearance

Substituting the expression for F_{sr} from Eq. 5.79 into Eq. 5.85 yields

$$W' = \frac{\pi \mu_0 r l}{2g}(F_s^2 + F_r^2 + 2F_sF_r \cos \delta_{sr}) \tag{5.86}$$

The electromagnetic torque T_e is the rate of change of the coenergy W' with respect to rotor position θ_m, that is,

$$T_e = \frac{\partial W'}{\partial \theta_m} \tag{5.87}$$

Since θ_m can be expressed as the sum of θ_s and δ_{sr},

$$\theta_m = \theta_s + \delta_{sr} \tag{5.88}$$

Therefore, the partial derivative of W' with respect to δ_{sr} also gives the electromagnetic torque. Thus,

$$T_e = -\frac{\pi \mu_0 r l}{g} F_sF_r \sin \delta_{sr} \tag{5.89}$$

Equation 5.89 states that T_e is proportional to the peak value of the stator and rotor mmfs and the sine of δ_{sr}, which is the angle between the two rotating mmfs. The negative sign means that the fields tend to align themselves.

Equal and opposite torques are exerted on the stator and rotor. The torque on the stator is transmitted through the machine frame to the foundation. The torque on the rotor is balanced by the mechanical torque.

Equations 5.89 and 5.77 are similar in form. They represent alternative ways of calculating the torque. If F_s and F_r are expressed in terms of i_s and i_r, Eq. 5.89 reduces to Eq. 5.77.

EXAMPLE 5.5

A two-pole, three-phase, 60-Hz synchronous generator has a rotor radius of 10 cm, an air-gap length of 0.25 mm, and a rotor length of 40 cm. The rotor field winding has 300 turns, and the stator windings have 80 turns/phase. When connected to a three-phase electrical load, a balanced set of stator currents flow. At t = 0, the current in phase a is 15 A, its maximum value. At this same instant, the rotor axis makes an angle of 20 degrees with the reference line (refer to Fig. 5.19). The rotor current is constant at 2 A.

a. Calculate electromagnetic torque using Eq. 5.77.

b. Calculate electromagnetic torque using Eq. 5.89.

Solution

a. The mutual inductance is found by using Eq. 5.76 as follows:

$$M = \frac{4\mu_0 N_s N_r l r}{\pi g} = \frac{4(4\pi \times 10^{-7})(80)(300)(0.40)(0.10)}{\pi(0.00025)} = 6.14 \text{ H}$$

In deriving Eq. 5.77, it was assumed that there is only one winding on the stator. In this example, there are three windings on the stator. Hence, the one winding equivalent for the stator should carry a constant current of $\frac{3}{2}I_a$, where I_a is the current in phase a. Therefore, the electromagnetic torque T_e is given by

$$T_e = -M\left[\tfrac{3}{2}I_a\right]I_r \sin\theta_m(0) = -(6.14)\left[\tfrac{3}{2}(15)\right](2)\sin 20^\circ = -94.5 \text{ N-m}$$

b. From Eqs. 5.24 and 5.25, the stator mmf is given as

$$F_s = \frac{3}{2}F_p = \frac{3}{2}\left(\frac{2N_s I_a}{\pi}\right) = \frac{3}{2}\left[\frac{(2)(80)(15)}{\pi}\right] = 1146 \text{ A-t}$$

The rotor mmf is

$$F_r = \frac{2N_f I_f}{\pi} = \frac{(2)(300)(2)}{\pi} = 382 \text{ A-t}$$

Since $\omega_m = \omega_s$, and referring to Fig. 5.19,

$$\delta_{sr} = \theta_m - \theta_s = \omega_m t + \theta_m(0) - \omega_s t = \theta_m(0) = 20^\circ$$

Therefore, according to Eq. 5.89,

$$T_e = -\frac{\pi(4\pi \times 10^{-7})(0.10)(0.40)}{0.00025}(1146)(382)\sin 20^\circ = -94.5 \text{ N-m}$$

Thus, it is seen that Eqs. 5.77 and 5.89 yield the same value for electromagnetic torque.

DRILL PROBLEMS

D5.14 In Drill Problem D5.7, the mmf due to the generator rotor field winding may be expressed in terms of the rotor position θ_m and the peak value of rotor mmf F_r as follows:

$$F_{r1} = F_r \cos(\theta - \theta_m)$$

Let $\theta_m = (377t + \pi/2)$ radians and $F_r = 3000$ A-t. Find an expression for mmf due to both armature and field windings.

D5.15 Show that Eq. 5.89 is equivalent to Eq. 5.77.

REFERENCES

1. Bergseth, F. R., and S. S. Venkata. *Introduction to Electric Energy Devices*. Prentice Hall, Englewood Cliffs, N.J., 1987.
2. Del Toro, Vincent. *Electric Machines and Power Systems*. Prentice-Hall, Englewood Cliffs, N.J., 1985.
3. Fitzgerald, A. E., Charles Kingsley, and Stephen Umans. *Electric Machinery*. 4th ed. McGraw-Hill, New York, 1983.
4. Krause, Paul C., and Oleg Wasynczuk. *Electromechanical Motion Devices*. McGraw-Hill, New York, 1989.
5. Nasar, Syed A. *Electric Energy Conversion and Transmission*. Macmillan, New York, 1985.
6. Ramshaw, Raymond, and R. G. van Heeswijk. *Energy Conversion Electric Motors and Generators*. Saunders College Publishing, Philadelphia, 1990.
7. Sen, P. C. *Principles of Electrical Machines and Power Electronics*. Wiley, New York, 1989.
8. Shultz, Richard D., and Richard A. Smith. *Introduction to Electric Power Engineering*. Wiley, New York, 1988.

PROBLEMS

5.1 An electromagnet with two air gaps is shown in Fig. 5.20. The relative permeability of the iron is assumed to be infinite, and flux fringing in the air gaps can be neglected. The coil has 500 turns, and it is supplied with a current of 2.5 A. Find

a. The magnetic flux in each air gap

b. The total flux linkage of the coil

c. The magnetic energy stored in this system

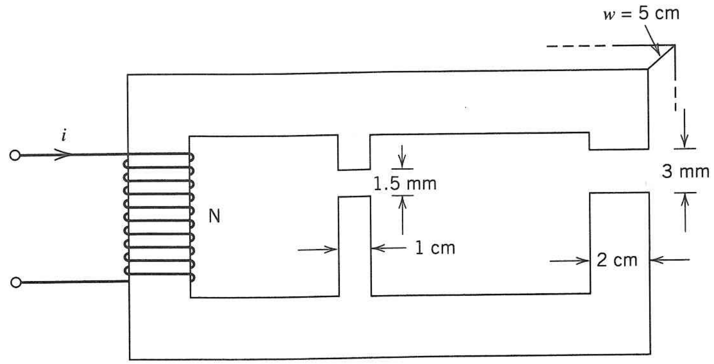


FIGURE 5.20 Electromagnet of Problem 5.1.

5.2 A cylindrical electromagnet is shown in Fig. 5.21. The plunger is free to move in the vertical direction. The air gap between the core shell and the plunger is uniform at 0.5 mm. It may be assumed that magnetic flux leakage and flux fringing in the air gaps are negligible and the relative permeability of the core is infinitely high. The coil contains 600 turns and is supplied with a DC current of 8 A.

a. Calculate the magnetic flux density in the air gap between the surfaces of the center core and plunger when the air-gap length $g = 1.0$ mm.

b. Find the inductance of the coil.

c. Determine the stored magnetic energy.

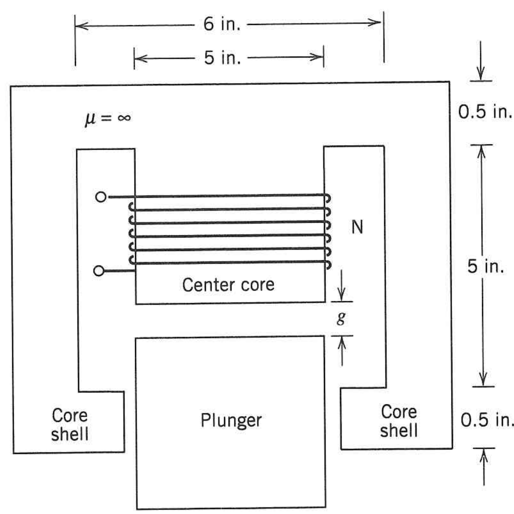


FIGURE 5.21 Cylindrical electromagnet of Problem 5.2.

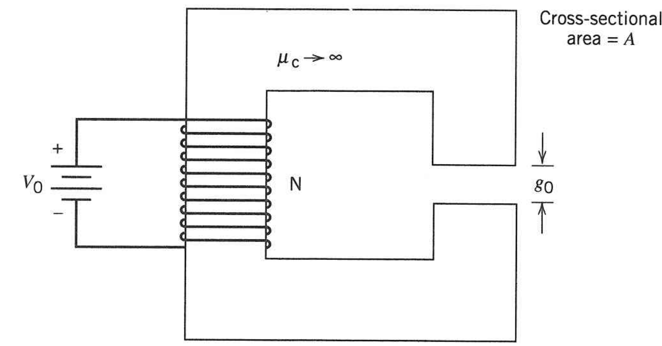


FIGURE 5.22 Inductor circuit of Problem 5.3.

5.3 An inductor is made from a magnetic core with an air gap as shown in Fig. 5.22. The winding has resistance R and is connected to a source with voltage V_0.

a. Show that the coil inductance is given by the expression

$$L = \frac{\mu_0 N^2 A}{g_0}$$

b. Calculate the magnetic stored energy in the inductor.

c. With the voltage held constant at V_0, the air-gap length is varied from g_0 to g_1. Calculate the change in the stored magnetic energy.

5.4 The magnetic circuit of Fig. 5.23 is connected to a source whose voltage varies sinusoidally with time, that is,

$$v(t) = V_0 \cos \omega t$$

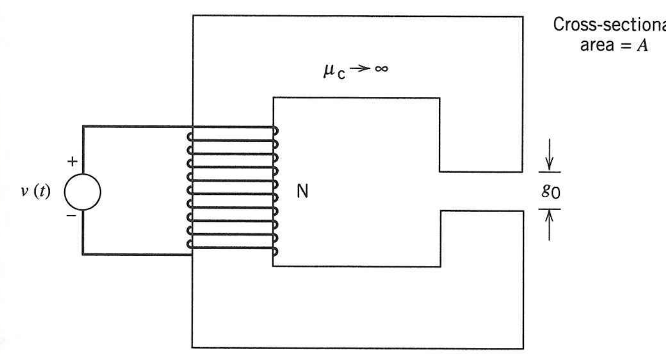


FIGURE 5.23 Magnetic circuit of Problem 5.4.

Assume that the air gap is fixed at g_0 and that the resistance of the coil is R. Calculate

a. The time-averaged magnetic energy stored in the inductor

b. The instantaneous power output of the voltage source as a function of R

c. The time-averaged power output of the voltage source as a function of R

d. The time-averaged power dissipated in the resistor

5.5 Twenty-five conducting loops, or turns, are connected in series to form a coil. Each turn has a length l = 2.5 m and width w = 20 cm. The coil is rotated at a constant speed of 1200 rpm in a magnetic flux density **B** directed upward. The induced voltage across the coil has an rms value of 1000 V. Determine the required value of flux density.

5.6 A coil of 50 turns is in the shape of a square with 15 cm on each side. It is driven at a constant speed of 600 rpm. The coil is placed in such a way that its axis of revolution is perpendicular to a uniform magnetic flux density **B** of 0.15 T in the vertical direction. The coil is in a position of maximum flux linkage at time $t = 0$. Determine (a) the time variation of the flux linkage and (b) the instantaneous voltage induced in the coil.

5.7 Two mutually coupled coils are shown in Fig. 5.24. The first coil is held fixed, and the second coil is free to rotate. The inductances of the coils are given by

$$L_{11} = \text{A} \qquad L_{22} = \text{B} \qquad L_{12} = L_{21} = M \cos\theta$$

Let the current in the first coil be denoted by i_1 and the current in the second coil by i_2. Find the electrical torque for the following conditions:

a. $i_1 = I_0$ and $i_2 = 0$

b. $i_1 = i_2 = I_0$

c. $i_1 = I_0$ and $i_2 = I_m \sin\omega t$

d. $i_1 = i_2 = I_m \sin\omega t$

e. $i_1 = I_0$ and coil 2 short-circuited

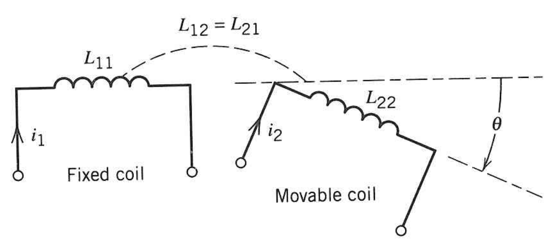


FIGURE 5.24 Mutually coupled coils of Problem 5.7.

5.8 Two mutually coupled coils are shown in Fig. 5.25. The first coil is held stationary while the second is free to rotate. The self-inductances of the two coils are L_{11} and L_{22} and the mutual inductance is L_{12}. The values of these inductances in henrys are given in the following table for two angular positions θ of the rotor, where θ is measured from the axis of the stationary coil used as reference. The inductances may be assumed to vary linearly with θ over the range 45° < θ < 75°.

θ	L_{11}	L_{22}	L_{12}
45°	0.6	1.1	0.3
75°	1.0	2.0	1.0

For each of the following cases, compute the electromagnetic torque when the rotor is at angular position θ = 60°. State whether the torque tends to turn the rotor clockwise or counterclockwise.

a. I_1 = 10 A and I_2 = 0

b. I_1 = 0 and I_2 = 10 A

c. I_1 = 10 A and I_2 = 10 A along the arrow directions

d. I_1 = 10 A in the arrow direction and I_2 = 10 A in the reverse direction

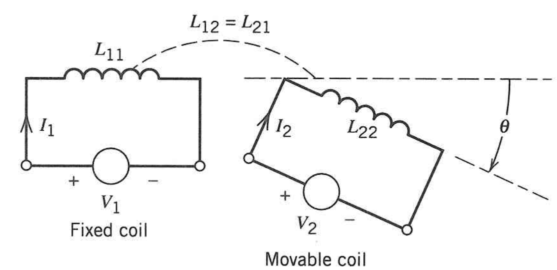


FIGURE 5.25 Magnetic system of Problem 5.8.

5.9 An electromagnetic system consists of two windings, one mounted on a stator and the other on a rotor. The self- and mutual inductances are given as

$$L_{11} = 1.0\ \text{H} \qquad L_{22} = 0.5\ \text{H} \qquad L_{12} = 1.414 \cos\theta\ \text{H}$$

where θ is the angle between the axes of the windings. The resistances of the windings are assumed to be negligible. Winding 2 is short-circuited, and the current supplied to winding 1 as a function of time is $i_1 = 14.14 \sin\omega t$.

a. Derive an expression for the instantaneous torque on the rotor in terms of the angle θ.

b. Compute the time-averaged torque when θ = 45°.

c. If the rotor is allowed to move, will it rotate continuously or will it tend to come to rest? If it comes to rest, at what angular position θ?

5.10 The two mutually coupled coils described in Problem 5.9 are connected in parallel across a 208-V, 60-Hz source. The coil resistances are negligible.

- **a.** Derive an expression for the instantaneous torque on the rotor in terms of the angle θ.
- **b.** Compute the average torque when $\theta = 30°$.

5.11 The doubly excited rotating system shown in Fig. 5.16 has a coil on the stator and a coil on the rotor. The self- and mutual inductances are given by

$$L_{ss} = L_{rr} = 2 \text{ H}$$
$$L_{sr} = \cos\theta \text{ H}$$

The resistances of the windings may be neglected. The rotor winding is short-circuited and the stator winding is connected to a current source $i_s(t) = 14.14 \sin \omega t$ A. The rotor is held stationary at an angular position of $\theta = 15°$. Determine

- **a.** The instantaneous electromagnetic torque
- **b.** The time-averaged electromagnetic torque
- **c.** The equilibrium positions of the rotor coil

5.12 Two coils, one mounted on a stator and the other on a rotor, have self- and mutual inductances of

$$L_{11} = 0.40 \text{ mH} \qquad L_{22} = 0.20 \text{ mH} \qquad L_{12} = 0.5 \cos\theta \text{ mH}$$

where θ is the angle between the axes of the coils. The coils are connected in series and carry a current

$$i(t) = \sqrt{2}\, I \sin \omega t$$

- **a.** Determine the instantaneous torque T on the rotor as a function of the angular position θ.
- **b.** Find the time-averaged torque T_{ave} as a function of θ.
- **c.** Compute the value of T_{ave} for $I = 5$ A, and $\theta = 90°$.

5.13 Two windings are mounted in a machine with a uniform air gap such as that shown in Fig. 5.16. The winding resistances are negligible. The self- and mutual inductances are given as follows:

$$L_{ss} = 2.5 \text{ H}$$
$$L_{rr} = 1.0 \text{ H}$$
$$M_{sr} = \sqrt{2} \cos\theta \text{ H}$$

The first winding is connected to a voltage source, and the second winding is short-circuited. The current in winding 1 is known to be $i_s(t) = 10\sqrt{2} \sin \omega t$, where ω is the frequency of the source. The rotor is held stationary.

- **a.** Derive an expression for the instantaneous torque in terms of the angular position θ.
- **b.** Compute the time-averaged torque at $\theta = 45°$.
- **c.** If the rotor is released, will it rotate continuously or will it tend to come to rest? If it comes to rest, at what angle θ?

5.14 A machine has self- and mutual inductances that can be described by

$$M_{12} = 1 - \sin\theta \text{ H}$$
$$L_{11} = 1 + \sin\theta \text{ H}$$
$$L_{22} = 2(1 + \sin\theta) \text{ H}$$

The two coils are connected to separate sources. Coil 1 is supplied with a constant current of $I_1 = 15$ A, and coil 2 is supplied with a constant current of $I_2 = -4$ A. Assume that the value of $\theta = 45°$.

- **a.** Find the value and direction of the developed torque.
- **b.** Compute the amount of energy supplied by each source.
- **c.** The current of coil 1 of part (a) is changed to a sinusoidal current of 10 A rms, and coil 2 is short-circuited. Find the rms value of the current in coil 2. The source has a voltage of 120 V at 60 Hz.
- **d.** Determine the instantaneous torque produced in part (c).
- **e.** Find the average value of the torque in part (c).

5.15 If the number of turns on the stator of Example 5.5 is doubled, calculate the electromagnetic torque using Eqs. 5.77 and 5.89.

Six

DC Machines

6.1 INTRODUCTION

The DC machine is a versatile electromechanical energy conversion device characterized by superior torque characteristics and a wide range of speed. Its efficiency is very good over its speed range. DC currents are required for both its field winding, located on the stator, and its armature winding on the rotor.

The DC machine is more costly than comparable AC machines, and its maintenance costs are also higher. Because of their costs, DC machines are not widely used in industry. Their use is limited to tough jobs, such as in steel mills and paper mills. They are also used as motors for control purposes.

Basic torque production and induced voltage were discussed in the previous chapter. In this chapter, the operational characteristics of various types of DC machines are discussed.

6.2 BASIC PRINCIPLES OF OPERATION

DC machines, like other electromechanical energy conversion devices, have two sets of electrical windings: field and armature windings. The field winding is on the stator, and the armature winding is on the rotor. A two-pole DC machine is shown in Fig. 6.1.

The magnetic field of the field winding is approximately sinusoidal. Thus, AC voltage is induced in the armature winding as the rotor turns under the magnetic field of the stator. This induced or generated voltage is also approximately sinusoidal. Since the armature windings are distributed over the armature

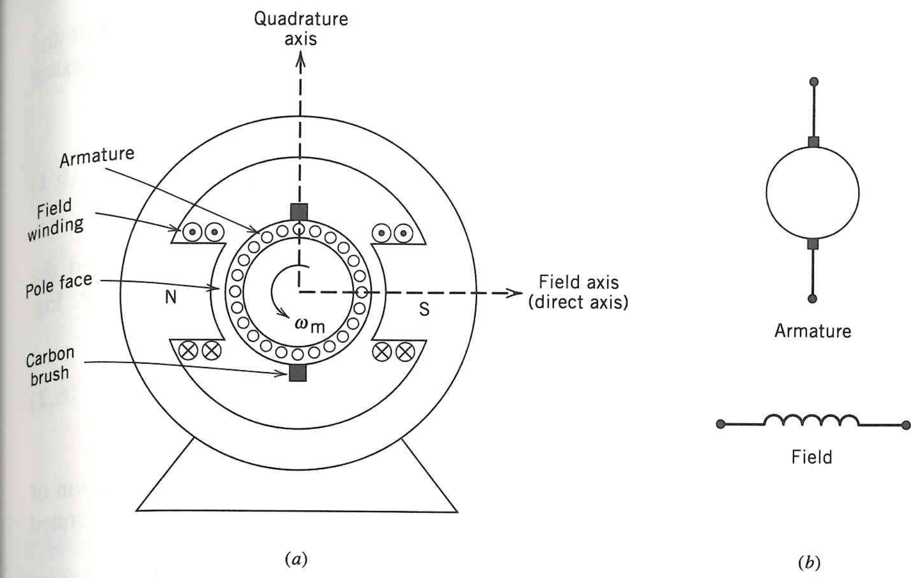


FIGURE 6.1 A two-pole DC machine: (*a*) schematic representation; (*b*) circuit representation.

periphery, the generated voltages of the armature turns reach their maxima at different times.

The commutator and brush combination converts the AC generated voltages to DC. The commutator is located on the same shaft as the armature and rotates together with the armature windings. The brushes are stationary and are located so that commutation occurs when the coil sides are in the neutral zone; that is, the potentials of the conductor loop that leaves the brush and the loop that comes in contact with the brush are the same.

The axis of the armature mmf is 90° from the axis of the field winding. Denoting the stator (field) winding and the rotor (armature) winding mmfs by F_s and F_r, respectively, these mmfs are shown in Fig. 6.2.

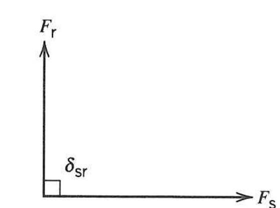


FIGURE 6.2 Stator and rotor mmf vector representation.

The electromagnetic torque T_e is produced by the interaction of the stator and rotor mmfs. Since they are in quadrature with each other, T_e may be expressed as follows:

$$T_e = -\frac{\pi\mu_0 r l}{g} F_s F_r \quad (6.1)$$

The rotor mmf F_r is a linear function of the armature current I_a, and the stator mmf F_s is similarly a linear function of the field current I_f. Hence, Eq. 6.1 may be written as follows:

$$T_e = K_T I_a I_f \quad (6.2)$$

where K_T = torque constant.

The DC induced voltage E_a appearing between the brushes is a function of the field current I_f and the speed of rotation ω_m of the machine. This generated voltage is given by

$$E_a = K_a' I_f \omega_m \quad (6.3)$$

where K_a' = voltage constant.

If the losses of the DC machine are neglected, from the energy conservation principle, the electrical power is equal to the mechanical power:

$$E_a I_a = \omega_m T_m \quad (6.4)$$

where

$E_a I_a$ = electrical power

$\omega_m T_m$ = mechanical power

At steady state, the mechanical torque T_m is equal to the electromagnetic torque T_e.

6.3 GENERATION OF UNIDIRECTIONAL VOLTAGE

A DC generator with two poles in the stator and a single conducting loop on the rotor is shown in Fig. 6.3.

As the rotor is rotated at an angular velocity ω_m, the armature flux linkages change and a voltage $e_{aa'}$ is induced between terminals a and a'. The expression for the voltage induced is given by Faraday's law as

$$e_{aa'} = -\frac{d\lambda}{dt} \quad (6.5)$$

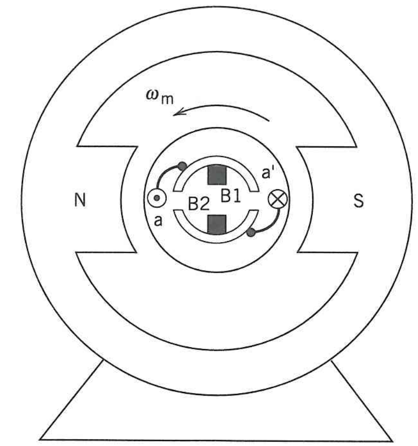


FIGURE 6.3 A two-pole DC generator.

This induced voltage is plotted against time in Fig. 6.4*b*, where at time $t = 0$ the conductors a and a' are as shown in Fig. 6.3. The plot of the flux $\phi_{aa'}$ and the plot of the rectified voltage (across brushes B1 and B2) are given in Figs. 6.4*a* and *c*, respectively.

It is seen from Fig. 6.4 that although the flux and the coil voltage are both sinusoidal functions of time, the voltage across the brushes is a unidirectional voltage.

Suppose that a second winding bb' is placed on the armature displaced from the aa' winding by 90°. Two new commutator segments are also added as illustrated in Fig. 6.5.

The induced voltages $e_{aa'}$ and $e_{bb'}$ across terminals aa' and bb', respectively, and the voltage across the brushes, e_{B1B2}, are plotted in Fig. 6.6.

It may be seen from Fig. 6.6*c* that the armature voltage is closer to a DC voltage for the generator of Fig. 6.4. Therefore, it may be concluded that by increasing the number of conducting loops on the armature and correspondingly increasing the number of commutator segments, the quality of the armature terminal voltage is greatly improved. In the limit, a pure DC voltage between brushes is obtained as shown in Fig. 6.7.

The generated voltage of a DC machine having p poles and Z conductors on the armature with a parallel paths between brushes is given in Eq. 5.48 and is repeated here as Eq. 6.6.

$$E_a = \frac{pZ}{2\pi a}\Phi_p \omega_m = K_a \Phi_p \omega_m \quad (6.6)$$

where $K_a = pZ/(2\pi a)$ = machine constant.

By substituting Eq. 6.6 into Eq. 6.4, the mechanical torque, which is also equal to the electromagnetic torque, is found as follows:

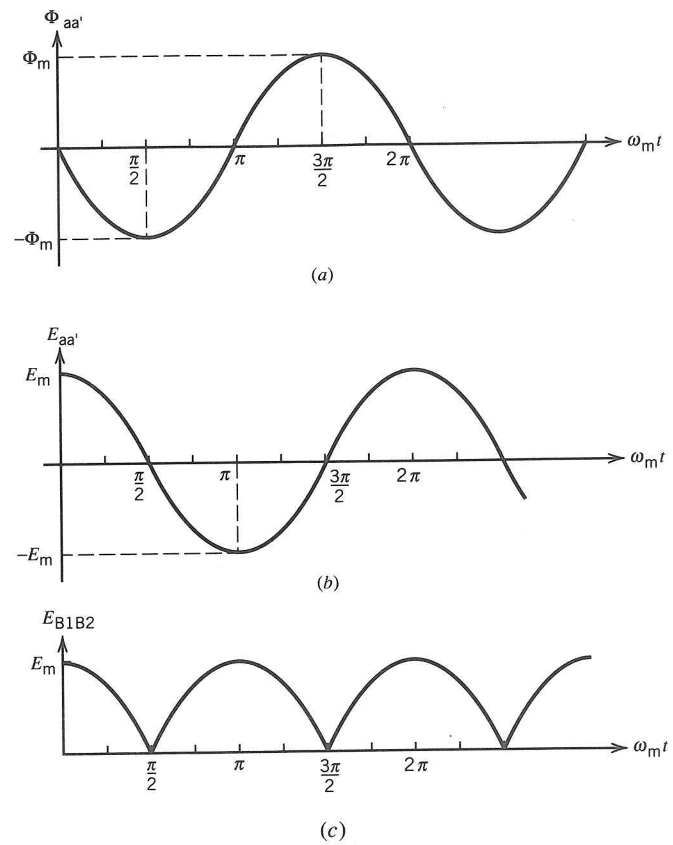


FIGURE 6.4 (*a*) Flux linkage of coil aa′; (*b*) induced voltage; (*c*) rectified voltage.

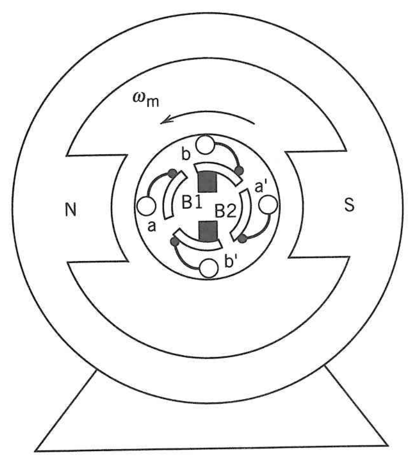


FIGURE 6.5 A two-pole, two-coil DC generator.

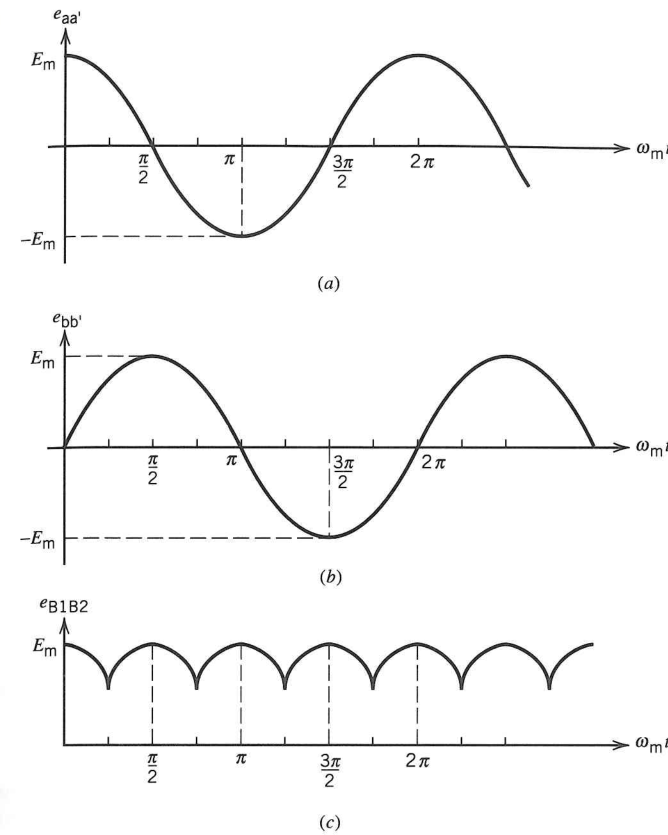


FIGURE 6.6 (*a*) Voltage of coil aa′; (*b*) voltage of coil bb′; (*c*) voltage across armature terminals (between brushes).

$$T_e = T_m = \frac{E_a I_a}{\omega_m} = K_a \Phi_p I_a \tag{6.7}$$

In the case of a generator, T_m is the input mechanical torque, which is converted to electrical power. For a motor, T_e is the developed electromagnetic torque, which is used to drive the mechanical load.

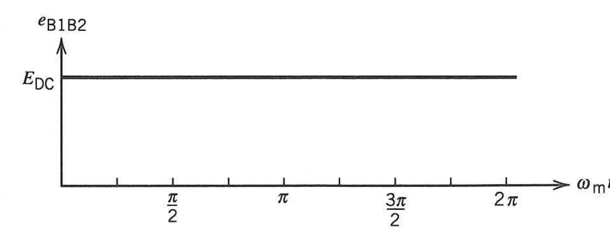


FIGURE 6.7 DC terminal voltage.

EXAMPLE 6.1

A six-pole DC machine has a flux per pole of 30 mWb. The armature has 53[illegible] conductors connected as a lap winding in which the number of parallel pat[illegible] a is equal to the number of poles p. The DC machine runs at 1050 revolutio[illegible] per minute (rpm), and it delivers a rated armature current of 225 A to a lo[illegible] connected to its terminals. Calculate the following:

- **a.** Machine constant K_a
- **b.** Generated voltage E_a
- **c.** Conductor current I_c
- **d.** Electromagnetic torque T_e
- **e.** Power developed, P_{dev}, by the armature

Solution

a. The machine constant is given by

$$K_a = pZ/(2\pi a) = (6)(536)/[(2\pi)(6)] = 85.31$$

b. The angular speed of the machine is

$$\omega_m = 2\pi n/60 = 2\pi(1050)/60 = 109.96 \text{ rad/s}$$

Thus, the generated voltage is found by using Eq. 6.6 as follows:

$$E_a = K_a \Phi_p \omega_m = (85.31)(0.030)(109.96) = 281.4 \text{ V}$$

c. For a lap winding, there are $a = 6$ parallel paths; therefore, the conduct[illegible] current is

$$I_c = I_{coil} = I_a/a = 225/6 = 37.5 \text{ A}$$

d. The electromagnetic torque is found by using Eq. 6.7 as follows:

$$T_e = K_a \Phi_p I_a = (85.31)(0.030)(225) = 575.84 \text{ N-m}$$

e. The power developed by the armature is

$$P_{dev} = T_e \omega_m = (575.84)(109.96) = 63.32 \text{ kW}$$
$$= E_a I_a = (281.4)(225) = 63.32 \text{ kW}$$

EXAMPLE 6.2

Repeat Example 6.1 if the armature is reconnected as a wave winding such that the rated conductor current remains the same. For a wave winding, there are two parallel paths; that is, $a = 2$.

Solution

a. The machine constant is

$$K_a = pZ/(2\pi a) = (6)(536)/[(2\pi)(2)] = 255.92$$

b. At the same angular speed $\omega_m = 109.96$ rad/s, the generated voltage is

$$E_a = K_a \Phi_p \omega_m = (255.92)(0.030)(109.96) = 844.2 \text{ V}$$

c. The rated conductor current is $I_c = 37.5$ A; thus, since a wave winding has $a = 2$ parallel paths, the armature current is

$$I_a = aI_c = 2(37.5) = 75 \text{ A}$$

d. The electromagnetic torque is

$$T_e = K_a \Phi_p I_a = (255.92)(0.030)(75) = 575.82 \text{ N-m}$$

e. The power developed is

$$P_{dev} = T_e \omega_m = (575.82)(109.96) = 63.32 \text{ kW}$$
$$= E_a I_a = (844.2)(75) = 63.32 \text{ kW}$$

DRILL PROBLEMS

D6.1 A four-pole DC machine has a flux per pole of 15 mWb. The armature has 75 coils with four turns per coil. The armature is connected as a wave winding that has $a = 2$ parallel paths. Find the generated voltage at a speed of 1050 rpm.

D6.2 A DC generator has six poles and is running at 1150 rpm. The armature has 120 slots with eight conductors per slot and is connected as a lap winding; that is, parallel paths = number of poles. The generated voltage is 230 V, and the armature current is 25 A.

a. Determine the required flux per pole.

b. Determine the electromagnetic torque developed.

6.4 TYPES OF DC MACHINES

DC machines are classified according to the electrical connections of the armature winding and the field windings. The operating characteristics of the specific DC machine being considered depend on the particular interconnection of the armature and field windings. There are generally four means of interconnection, giving rise to the following types of DC machines:

1. Shunt machine
2. Separately excited machine
3. Series machine
4. Compound machine

6.4.1 Shunt DC Machine

The armature and field windings are connected in parallel. The shunt field winding consists of several turns of small-diameter conductors, since the field current is normally a low current. The armature conductors are considerably larger because they are designed to carry rated current. The armature voltage and the field voltage are the same.

The interconnection for the DC shunt machine is illustrated in Fig. 6.8.

6.4.2 Separately Excited DC Machine

The armature and field windings are electrically separate from one another. Thus, the field winding is excited by a separate DC source.

The schematic representation of the separately excited DC machine is shown in Fig. 6.9.

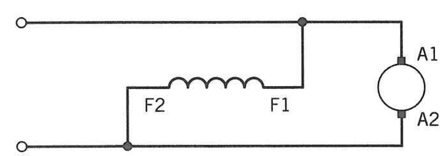


FIGURE 6.8 Shunt DC machine.

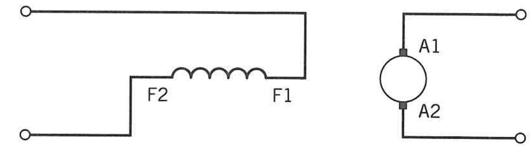


FIGURE 6.9 Separately excited DC machine.

6.4.3 Series DC Machine

The field winding and the armature are electrically connected in series. The series field winding consists of a few turns of large diameter conductors since it carries the same current as the armature.

The interconnection for the DC series machine is illustrated in Fig. 6.10.

6.4.4 Compound DC Machine

The compound DC machine has two field windings: one is connected in series with the armature, and the other is connected in parallel with the armature. The first is called the series field, and the second is called the shunt field. If the magnetic fluxes produced by both series field and shunt field windings are in the same direction, that is, additive, the machine is cumulative compound. If the magnetic fluxes are in opposition, the machine is differential compound. The cumulative and differential compound machines are illustrated in Fig. 6.11*a* and *b*, respectively.

A cumulative compound or differential compound machine may be connected either long-shunt compound or short-shunt compound. In a long-shunt compound machine, the series field is connected in series with the armature and the combination is in parallel with the shunt field. In the short-shunt compound machine, the shunt field is in parallel with the armature and the combination is connected in series with the series field. The long-shunt and short-shunt connections are shown in Fig. 6.12.

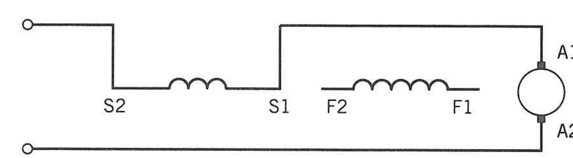


FIGURE 6.10 Series DC machine.

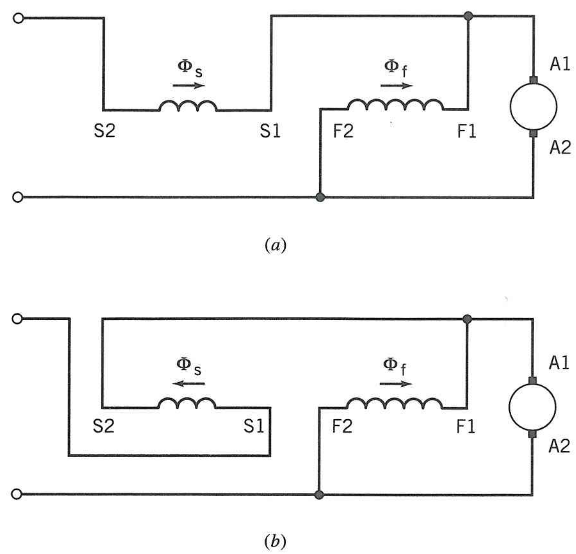


FIGURE 6.11 Compound DC machines: (*a*) cumulative compound; (*b*) differential compound.

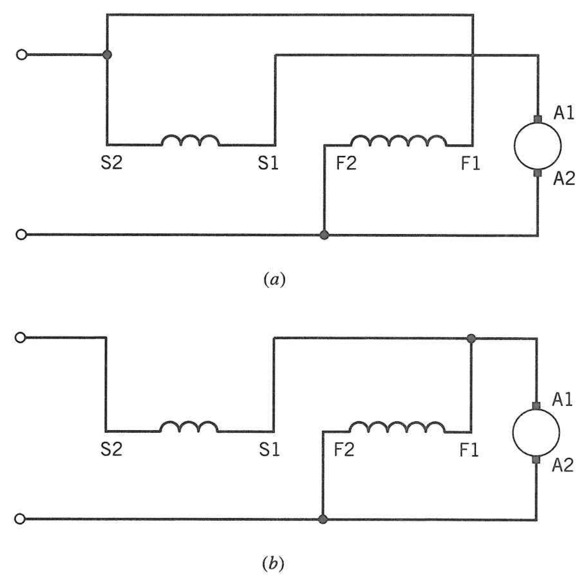


FIGURE 6.12 Compound DC machines: (*a*) long-shunt compound; (*b*) short-shunt compound.

6.5 DC MACHINE ANALYSIS

The armature and field windings of DC machines are modeled in steady state as shown in Fig. 6.13. The direction of the armature current I_a shown in the figure is for the case of a DC motor, and the torque T_e is the electromagnetic torque developed, which is available for driving a mechanical load. The voltage V_t is the applied terminal voltage.

In the case of a generator, the torque is the mechanical torque applied to the machine shaft, and the direction of the armature current is reversed. The voltage V_t is the output terminal voltage.

In the equivalent circuit of the DC machine shown, the mechanical losses, such as friction and windage losses, are not included. However, the electrical copper losses are modeled.

For shunt and compound machines, the voltage V_f applied to the field circuit is the same as the terminal voltage V_t. However, in the case of a separately excited generator, a separate voltage source is required to provide the excitation current I_f to the shunt field.

The generated or excitation voltage E_a is the emf induced in the armature of the machine. The expression for E_a derived in Section 6.3 may also be written as

$$E_a = K_a \Phi_p \omega_m = K'_a I_f \omega_m \tag{6.8}$$

Similarly, the electromagnetic torque may also be expressed as

$$T_e = K_a \Phi_p I_a = K'_a I_f I_a \tag{6.9}$$

In Equations 6.8 and 6.9 a linear relationship between Φ_p and I_f is assumed; that is, the saturation effect is neglected. Depending on whether a DC generator or DC motor is under consideration, the steady-state issues analyzed are different, as shown in Table 6.1.

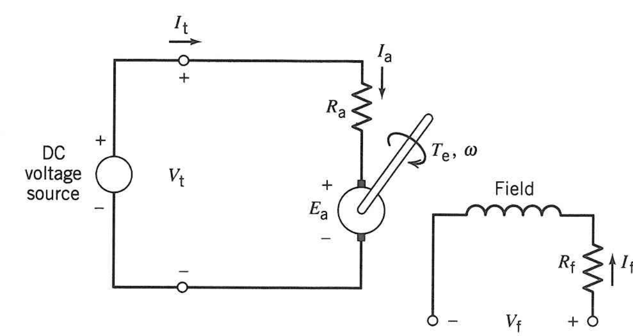


FIGURE 6.13 DC machine equivalent circuit.

Table 6.1 Scope of DC Machine Problems

Generator Problems	Motor Problems
• Speed fixed by prime mover	• Applied voltage fixed by source
• Issues:	• Issues:
Determine terminal voltage for a given load and fixed excitation current.	Determine speed for a given mechanical load and fixed excitation current.
Determine generated voltage for a given load and fixed terminal voltage.	Determine excitation voltage for a given mechanical load and a required speed.

EXAMPLE 6.3

A 12-kW, 240-V, 1200-rpm, separately excited DC generator has armature and field winding resistances of 0.20 Ω and 200 Ω, respectively. At no load, the terminal voltage is 240 V, the field current is 1.25 A, and the machine runs at 1200 rpm. When the generator delivers rated current to a load at 240 V, calculate

a. The generated voltage E_a

b. The field circuit voltage V_f

c. The developed torque T_e

Solution

a. At no load, the armature current $I_a = 0$; therefore, the generated voltage is equal to the terminal voltage.

$$E_a = V_t + I_a R_a = 240 + (0)(0.20) = 240 \text{ V}$$

The machine speed is

$$\omega_m = 2\pi n/60 = 2\pi(1200)/60 = 40\pi$$

The generated voltage may also be expressed in terms of the field current and the angular speed as given in Eq. 6.8. Thus,

$$E_a = K_a' I_f \omega_m = K_a'(1.25)(40\pi) = 240 \text{ V}$$

Solving for K_a' yields

$$K_a' = 1.528$$

At rated load, the armature current is

$$I_a = 12{,}000/240 = 50 \text{ A}$$

Therefore, the generated voltage is given by

$$E_a = V_t + I_a R_a = 240 + (50)(0.20) = 250 \text{ V}$$

b. Expressing the generated voltage in terms of the field current and the angular speed,

$$250 = (1.528)I_f(40\pi)$$

Solving for the field current yields

$$I_f = 1.30 \text{ A}$$

Thus, the field circuit voltage is found as follows:

$$V_f = I_f R_f = (1.30)(200) = 260 \text{ V}$$

c. The electromagnetic torque developed is obtained as follows:

$$T_e = E_a I_a/\omega_m = (250)(50)/(40\pi) = 99.5 \text{ N-m}$$

DRILL PROBLEMS

D6.3 A separately excited generator has a no-load voltage of 125 V at a field current of 2 A and a speed of 1780 rpm. Assume that the generator is operating on the straight-line portion of its saturation curve.

a. Calculate the generated voltage when the field current is increased to 2.5 A.

b. Calculate the generated voltage when the speed is reduced to 1650 rpm and the field current is increased to 2.75 A.

D6.4 A DC shunt generator has armature and field resistances of 0.2 Ω and 150 Ω, respectively. The generator supplies 10 kW to a load connected to its terminals at 230 V. Assuming that the total brush-contact voltage drop is 2 V, determine the induced voltage.

D6.5 A 100-kW, 600-V, 1200-rpm, long-shunt compound generator has a total brush voltage drop of 4 V, a series field winding resistance of 0.05 Ω, a shunt field winding resistance of 200 Ω, and an armature resistance of 0.1 Ω. The generator supplies rated output power to a load at rated voltage and rated speed. Calculate (a) the armature current and (b) the armature induced voltage.

6.6 DC GENERATOR PERFORMANCE

In this section, the performance characteristics of DC generators are described. Among these are the no-load or open-circuit characteristic, the terminal or load characteristic, voltage regulation, and efficiency.

6.6.1 No-Load Characteristics

Equations 6.8 and 6.9 assume a linear relationship between the flux Φ_p and the field current I_f. If the assumption of linearity is removed, these expressions have to be modified as follows:

$$E_a = K_a\Phi_p(I_f)\omega_m = K_a''\Phi_p(I_f)n \qquad (6.10)$$

$$T_e = K_a\Phi_p(I_f)I_a \qquad (6.11)$$

Equation 6.10 is referred to as the magnetization equation, and the corresponding curve is called the *saturation curve,* or *magnetization curve,* which is illustrated in Fig. 6.14. These curves are also referred to as the *no-load characteristics* of a separately excited generator.

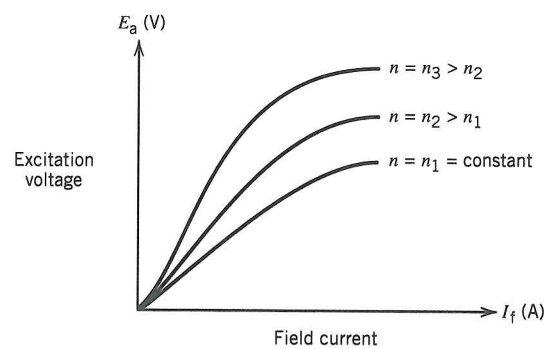


FIGURE 6.14 No-load characteristics of a separately excited generator.

6.6.2 Voltage Buildup in a Shunt Generator

In a shunt generator, the field winding is connected across the armature terminals. Thus, the generator itself provides its own field excitation. The equivalent circuit of the self-excited shunt generator is shown in Fig. 6.15.

When the generator is rotated by its prime mover, a small *residual voltage* E_{res} is generated because of the presence of residual flux in the magnetic field poles. This voltage is given by

$$E_{res} = K_a\Phi_{res}\omega_m \qquad (6.12)$$

The induced voltage E_{res} is essentially applied to the field circuit, and it causes a current I_f to flow in the field coils. The resultant mmf in the field coils produces more flux Φ_p in the poles, causing an increase in the generated voltage E_a, which increases the terminal voltage V_t. The higher V_t causes an increased I_f, further increasing the flux Φ_p, which increases E_a, and so forth. The final operating voltage is determined by the intersection of the field resistance line and the saturation curve. This voltage buildup process is depicted in Fig. 6.16. If the resistance of the field circuit is decreased, the resistance line is rotated clockwise, which results in a higher operating voltage, and vice versa.

Three factors affect the proper buildup of a shunt generator:

1. *Residual magnetism.* If there is no residual flux in the poles, there is no residual voltage; that is, if $\Phi_{res} = 0$, then $E_{res} = 0$, and the voltage will never build up.
2. *Critical resistance.* Normally, the shunt generator builds up to a voltage determined by the intersection of the field resistance line and the saturation curve. If the field resistance is greater than the *critical resistance,* the generator fails to build up and the voltage remains at the residual level. To solve this problem, the field resistance is reduced to a value less than the critical resistance.

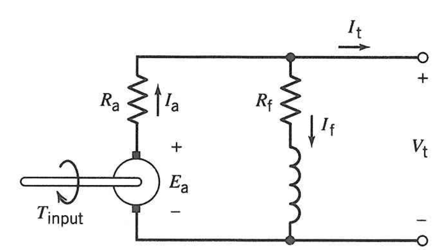


FIGURE 6.15 Equivalent circuit of a shunt generator.

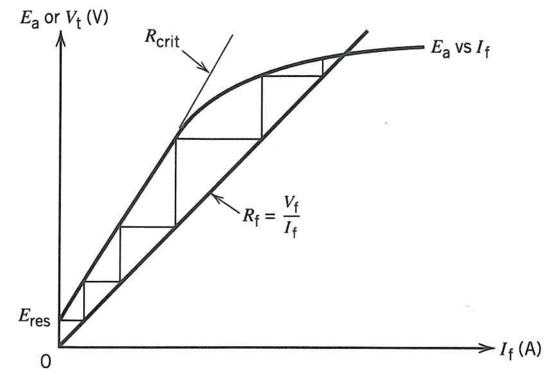


FIGURE 6.16 Buildup of voltage in a shunt generator.

3. *Relative polarity* of the field winding and the terminal voltage. For the shunt generator to build up properly, the current supplied to the field winding should produce flux that aids the residual flux. Otherwise, the flux produced by the field current will tend to neutralize the residual flux Φ_{res}, and the generator will not build up. When this happens, reversal of either the field connection or the direction of rotation will enable the generator to build up. Reversing both field connection and direction of rotation will cause the generator not to build up.

6.6.3 Terminal or Load Characteristics

The *terminal characteristic* of a device is a plot of the output, or terminal, quantities with respect to one another. For DC generators, the terminal characteristic is in the form of a plot of terminal voltage V_t versus load current I_L. Thus, it is also called the *load characteristic*.

For a shunt generator, the main causes of the decrease in terminal voltage as load is increased are

1. *Armature resistance drop*. This is an I_aR_a drop due to the armature resistance R_a.
2. *Brush contact drop*. The brushes are pressed on the commutator by mechanical springs. The nonideal contact, therefore, offers an electrical resistance and will cause a voltage drop when current flows through the brush. A constant value of 2 V is usually assumed for brush contact voltage drop.
3. *Armature reaction voltage drop*. When a load is connected to the terminals of the generator, a current will flow in the armature windings. This resulting armature mmf will produce its own magnetic field, which will affect the original magnetic field produced by the field poles. This interaction of the two fields is called *armature reaction*. Armature reaction is manifested in two ways.

   First, the uniform flux distribution under the pole faces is altered. In some areas under the pole faces, the armature magnetic field subtracts from the pole flux; in other areas, the armature field adds to the pole flux. This gives rise to commutation problems, as evidenced by arcing and sparking at the brushes.

   The second problem caused by armature reaction is flux weakening, resulting in a reduced generated voltage. In the areas of the pole faces where the armature magnetic field adds to the pole flux, because of saturation of the magnetic pole, there is only a small increase in flux. In the areas where the armature field subtracts from the field flux, there is a larger decrease in flux. Thus, there is a net decrease in the average flux under the entire pole face. This has the same effect as a reduction in the field magnetization; therefore, the effective or net field mmf may be expressed as

$$N_f I_f^{eff} = N_f I_f^{act} - (NI)^{ar} \tag{6.13}$$

   where

   N_f = number of turns in the shunt field winding
   I_f^{eff} = effective field current
   I_f^{act} = actual field current
   $(NI)^{ar}$ = demagnetizing mmf due to armature reaction

   The flux distortion and demagnetization caused by armature reaction are illustrated in Fig. 6.17.

4. *Reduced terminal voltage* in self-excited generators. The terminal voltage is reduced as a result of the preceding three causes. Since the field excitation is supplied by the generator itself, the reduced terminal voltage supplies a reduced field current. This causes a further reduction in the voltage.

The load characteristics of the different types of DC generators are shown in Fig. 6.18. The load characteristic of the separately excited generator is affected mainly by the first three causes in the preceding list; thus, it has a drooping characteristic.

The load characteristic of the self-excited shunt generator is similar to that of the separately excited generator. However, the terminal voltage is lower because of the fourth cause. Moreover, if the generator current is increased beyond a certain level, the terminal voltage collapses to zero.

In a series generator, the field excitation current is the same as the load current. Thus, the field flux and the resulting generated voltage will increase

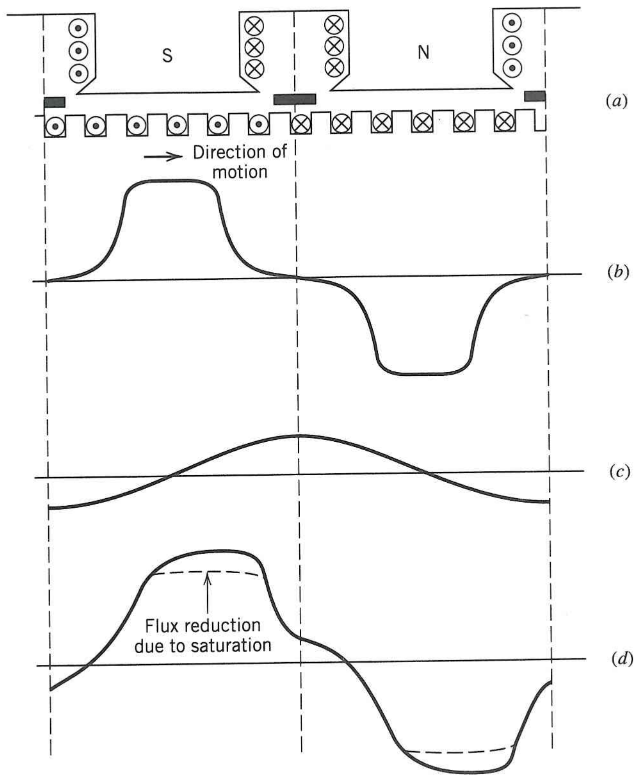


FIGURE 6.17 Flux distribution due to armature reaction: (*a*) armature and field winding mmfs; (*b*) uniform flux distribution due to field mmf; (*c*) flux distribution due to armature mmf; (*d*) resultant flux distribution.

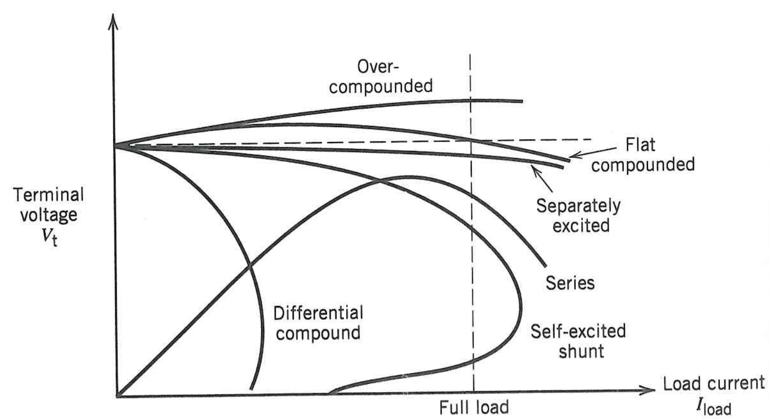


FIGURE 6.18 Load characteristics of DC generators.

with the load up to a certain point, beyond which the effects of saturation will cause the voltage of the series generator to collapse to zero.

In compound generators, the series and shunt fields are either in opposition or aiding each other. In a cumulative-compound generator, the series and shunt fields aid each other. The mmfs of either or both fields may be adjusted to produce a terminal voltage at full-load current that is less than the no-load voltage for an undercompounded generator, or equal to the no-load voltage for a flat-compounded generator, or greater than the no-load voltage for an overcompounded generator.

For a cumulative-compound generator, because of the presence of armature reaction, the effective or net field mmf is given by

$$N_f I_f^{eff} = N_f I_f^{act} + N_s I_s - (NI)^{ar} \tag{6.14}$$

where

N_s = number of turns in the series field winding

I_s = current through the series field winding

In a differential-compound generator, the series field opposes the shunt field, so that the terminal voltage drops a large amount for a small increase in load current. The differential-compound generator can be characterized as a constant-current generator.

6.6.4 Voltage Regulation

The terminal or load characteristics of the different types of DC generators are described in the previous section. A performance measure that gives essentially the same information about the generator is its voltage regulation. Voltage regulation (VR) is defined as follows:

$$\text{Voltage regulation (VR)} = \frac{V_{nl} - V_{fl}}{V_{fl}} \, 100\% \tag{6.15}$$

Voltage regulation gives an approximate description of the terminal characteristic. Positive voltage regulation implies a drooping characteristic, whereas negative voltage regulation implies a rising characteristic. Zero regulation implies a flat characteristic.

EXAMPLE 6.4

The magnetization curve of the 12-kW, 240-V, 1200-rpm, DC machine of Example 6.3 is shown in Fig. 6.19. The machine is operated as a separately