transformer is operating at rated voltage and no load when a three-phase fault occurs on its high-voltage side. Calculate

a. The short-circuit current

b. The short-circuit current supplied by each generator

c. The voltage at bus A

D11.3 A 2000-kVA, 13.2-kV synchronous motor is connected through a distribution feeder to a 2000-kVA, 13.2-kV synchronous generator. The subtransient reactance of each machine is equal to 20% based on its own ratings, and the feeder has a reactance of 8.5 Ω. The motor is initially drawing 1500 kW at 13 kV and 0.8 PF leading when a three-phase fault occurs at the motor terminals. Determine

a. The subtransient short-circuit current

b. The short-circuit current supplied by the generator

c. The short-circuit current contribution of the motor

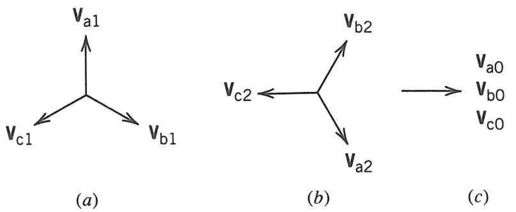
11.2.2 Symmetrical Components

The method of symmetrical components is used to solve power system problems involving unbalanced polyphase voltages and currents. It is analogous to the Fourier analysis of nonsinusoidal wave shapes wherein a nonsine wave is resolved into a number of sine waves of various frequencies. In symmetrical components, the unbalanced set of polyphase phasors is resolved into a number of balanced sets of phasors. After the unbalanced sets of voltage and current phasors are resolved into their symmetrical components, the power system may be solved using per-phase analysis.

A balanced, or symmetrical, set of phasors is defined as a set of phasors that have equal magnitudes and are separated by equal angles. Thus, the phasors in a three-phase balanced set are separated from each other by an angle of 120°.

In a three-phase system, an unbalanced set of phasors is resolved by using three sets of balanced phasors, namely positive sequence, negative sequence, and zero sequence. The positive (or abc) sequence consists of three phasors of equal magnitude and separated from each other by an angle of 120°. The negative (or cba) sequence also consists of three phasors of equal magnitude and separated from each other by an angle of 120°. The zero sequence also consists of three phasors of equal magnitude, but they are all in phase. These sequence components are illustrated in Fig. 11.1.

The a phase is considered as the principal phase, and its sequence components are used to represent the other phases. Thus, the sequence components of phases b and c are given by Eqs. 11.1, 11.2, and 11.3.



(*a*) (*b*) (*c*)

FIGURE 11.1 Sequence components of voltages.

$$\begin{aligned} \mathbf{V}_{b1} &= \mathbf{V}_{a1}e^{j240°} \\ \mathbf{V}_{c1} &= \mathbf{V}_{a1}e^{j120°} \end{aligned} \tag{11.1}$$

$$\begin{aligned} \mathbf{V}_{b2} &= \mathbf{V}_{a2}e^{j120°} \\ \mathbf{V}_{c2} &= \mathbf{V}_{a2}e^{j240°} \end{aligned} \tag{11.2}$$

$$\begin{aligned} \mathbf{V}_{b0} &= \mathbf{V}_{a0}e^{j360°} = \mathbf{V}_{a0} \\ \mathbf{V}_{c0} &= \mathbf{V}_{a0}e^{j360°} = \mathbf{V}_{a0} \end{aligned} \tag{11.3}$$

By introducing the operator $a = e^{j120°} = 1\underline{/120°}$, Eqs. 11.1 and 11.2 may be written as

$$\begin{aligned} \mathbf{V}_{b1} &= a^2\mathbf{V}_{a1} \\ \mathbf{V}_{c1} &= a\mathbf{V}_{a1} \end{aligned} \tag{11.4}$$

$$\begin{aligned} \mathbf{V}_{b2} &= a\mathbf{V}_{a2} \\ \mathbf{V}_{c2} &= a^2\mathbf{V}_{a2} \end{aligned} \tag{11.5}$$

The phase voltages are resolved into their sequence components and are expressed in terms of the sequence components of the principal phase a as follows:

$$\mathbf{V}_a = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} \tag{11.6}$$

$$\begin{aligned} \mathbf{V}_b &= \mathbf{V}_{b0} + \mathbf{V}_{b1} + \mathbf{V}_{b2} \\ &= \mathbf{V}_{a0} + a^2\mathbf{V}_{a1} + a\mathbf{V}_{a2} \end{aligned} \tag{11.7}$$

$$\begin{aligned} \mathbf{V}_c &= \mathbf{V}_{c0} + \mathbf{V}_{c1} + \mathbf{V}_{c2} \\ &= \mathbf{V}_{a0} + a\mathbf{V}_{a1} + a^2\mathbf{V}_{a2} \end{aligned} \tag{11.8}$$

In matrix form, Eqs. 11.6 to 11.8 may be expressed as

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \tag{11.9}$$

Adding Eqs. 11.6, 11.7, and 11.8 yields

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 3\mathbf{V}_{a0} + (1 + a^2 + a)\mathbf{V}_{a1} + (1 + a + a^2)\mathbf{V}_{a2} \qquad (11.10)$$

The quantity inside the parentheses on the right-hand side of Eq. 11.10 may be shown to reduce to zero; that is,

$$\begin{aligned} 1 + a + a^2 &= 1 + e^{j120^\circ} + e^{j240^\circ} \\ &= 1\underline{\angle 0^\circ} + 1\underline{\angle 120^\circ} + 1\underline{\angle 240^\circ} \\ &= 0.0 + j0.0 \end{aligned} \qquad (11.11)$$

Therefore, the zero-sequence component of the voltage of the principal phase a is found as

$$\mathbf{V}_{a0} = \tfrac{1}{3}(\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c) \qquad (11.12)$$

Multiplying Eqs. 11.7 and 11.8 by the operators a and a^2, respectively, and adding their products to Eq. 11.6 yields

$$\mathbf{V}_a + a\mathbf{V}_b + a^2\mathbf{V}_c = (1 + a + a^2)\mathbf{V}_{a0} + 3\mathbf{V}_{a1} + (1 + a + a^2)\mathbf{V}_{a2} \qquad (11.13)$$

By using Eq. 11.11, the positive-sequence component of the voltage is determined.

$$\mathbf{V}_{a1} = \tfrac{1}{3}(\mathbf{V}_a + a\mathbf{V}_b + a^2\mathbf{V}_c) \qquad (11.14)$$

Similarly, Eqs. 11.7 and 11.8 may be multiplied by the operators a^2 and a, respectively, and added to Eq. 11.6. Upon simplifying, the expression for the negative-sequence component of the voltage is obtained.

$$\mathbf{V}_{a2} = \tfrac{1}{3}(\mathbf{V}_a + a^2\mathbf{V}_b + a\mathbf{V}_c) \qquad (11.15)$$

Equations 11.13, 11.14, and 11.15 may be grouped together and written in matrix form as follows:

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} \qquad (11.16)$$

EXAMPLE 11.2

Find the sequence components of the unbalanced power system whose phase voltages are given by

$$\begin{aligned} \mathbf{V}_{an} &= 100\underline{\angle 0^\circ}\ \text{V} \\ \mathbf{V}_{bn} &= 80\underline{\angle -110^\circ}\ \text{V} \\ \mathbf{V}_{cn} &= 90\underline{\angle 130^\circ}\ \text{V} \end{aligned}$$

Solution By using Eqs. 11.12, 11.14, and 11.15, the sequence components of the voltage are computed as follows:

$$\begin{aligned} \mathbf{V}_{a0} &= \tfrac{1}{3}(\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn}) \\ &= \tfrac{1}{3}(100\underline{\angle 0^\circ} + 80\underline{\angle -110^\circ} + 90\underline{\angle 130^\circ}) = 5.35\underline{\angle -22.85^\circ}\ \text{V} \\ \mathbf{V}_{a1} &= \tfrac{1}{3}(\mathbf{V}_{an} + a\mathbf{V}_{bn} + a^2\mathbf{V}_{cn}) \\ &= \tfrac{1}{3}(100\underline{\angle 0^\circ} + 1\underline{\angle 120^\circ}\ 80\underline{\angle -110^\circ} + 1\underline{\angle 240^\circ}\ 90\underline{\angle 130^\circ}) \\ &= 89.68\underline{\angle 6.3^\circ}\ \text{V} \\ \mathbf{V}_{a2} &= \tfrac{1}{3}(\mathbf{V}_{an} + a^2\mathbf{V}_{bn} + a\mathbf{V}_{cn}) \\ &= \tfrac{1}{3}(100\underline{\angle 0^\circ} + 1\underline{\angle 240^\circ}\ 80\underline{\angle -110^\circ} + 1\underline{\angle 120^\circ}\ 90\underline{\angle 130^\circ}) \\ &= 9.77\underline{\angle -52.6^\circ}\ \text{V} \end{aligned}$$

The transformation matrix in Eq. 11.9 relating the phase voltages to their sequence components and the matrix used in Eq. 11.16 to obtain the sequence components from the original unbalanced voltages equally apply to a system of unbalanced three-phase currents. Thus, the phase currents are expressed in terms of their sequence components as follows:

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \qquad (11.17)$$

Similarly, the sequence components of the current are obtained from the original unbalanced phase currents by using the following equation:

$$\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} \qquad (11.18)$$

EXAMPLE 11.3

The sequence components of the current in a portion of a power system are given as

$$\mathbf{I}_{a1} = 2.5\angle{-90^\circ} \text{ pu}$$
$$\mathbf{I}_{a2} = 1.65\angle{90^\circ} \text{ pu}$$
$$\mathbf{I}_{a0} = 0.85\angle{90^\circ} \text{ pu}$$

Obtain the three phase currents.

Solution By using Eq. 11.17, the phase currents are found from the sequence components as follows:

$$\mathbf{I}_a = \mathbf{I}_{ao} + \mathbf{I}_{a1} + \mathbf{I}_{a2} = 0.85\angle{90^\circ} + 2.5\angle{-90^\circ} + 1.65\angle{90^\circ} = 0$$
$$\begin{aligned}\mathbf{I}_b &= \mathbf{I}_{ao} + a^2\mathbf{I}_{a1} + a\mathbf{I}_{a2}\\ &= 0.85\angle{90^\circ} + 1\angle{240^\circ}\ 2.5\angle{-90^\circ} + 1\angle{120^\circ}\ 1.65\angle{90^\circ}\\ &= 3.81\angle{160.5^\circ} \text{ pu}\end{aligned}$$
$$\begin{aligned}\mathbf{I}_c &= \mathbf{I}_{ao} + a\mathbf{I}_{a1} + a^2\mathbf{I}_{a2}\\ &= 0.85\angle{90^\circ} + 1\angle{120^\circ}\ 2.5\angle{-90^\circ} + 1\angle{240^\circ}\ 1.65\angle{90^\circ}\\ &= 3.81\angle{19.5^\circ} \text{ pu}\end{aligned}$$

DRILL PROBLEMS

D11.4 The phase current in a wye-connected, unbalanced load are

$$\mathbf{I}_a = 50 - j40 \text{ A}$$
$$\mathbf{I}_b = -30 - j20 \text{ A}$$
$$\mathbf{I}_c = -40 + j30 \text{ A}$$

Determine the sequence components of the currents.

D11.5 The line-to-line voltages across a three-phase, wye-connected load consisting of $Z = 100\angle{30^\circ}\ \Omega$ in each phase are as follows:

$$\mathbf{V}_{ab} = 205\angle{0^\circ} \text{ V}$$
$$\mathbf{V}_{bc} = 250\angle{-125^\circ} \text{ V}$$

$$\mathbf{V}_{ca} = 214\angle{107^\circ} \text{ V}$$

Determine the sequence components of the voltages.

D11.6 The sequence components of the current in phase a are

$$\mathbf{I}_{a1} = 6\angle{-15^\circ} \text{ pu}$$
$$\mathbf{I}_{a2} = 8\angle{225^\circ} \text{ pu}$$
$$\mathbf{I}_{a0} = 5\angle{-165^\circ} \text{ pu}$$

Determine the phase currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

11.2.3 Unsymmetrical Fault Analysis

Consider the portion of the power system shown in Fig. 11.2. The system is said to be symmetrical if it satisfies the following conditions:

1. Phase conductor impedances are equal, that is,

$$Z_{aa} = Z_{bb} = Z_{cc}$$

2. Mutual impedances between phase conductors are equal, that is,

$$Z_{ab} = Z_{bc} = Z_{ca}$$

3. Mutual impedances between phase conductors and the neutral wire are equal, that is,

$$Z_{an} = Z_{bn} = Z_{cn}$$

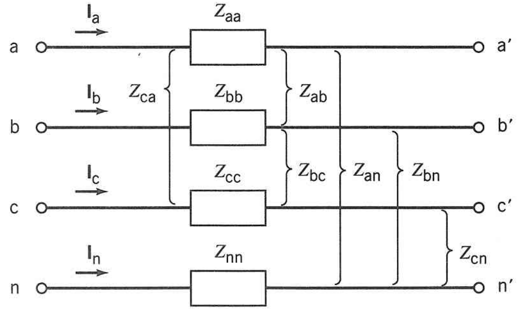


FIGURE 11.2 A portion of a three-phase power system.

The voltages on the right terminals (marked a′, b′, and c′) of the portion of the system shown in Fig. 11.2 are identified as $\mathbf{V}_a$, $\mathbf{V}_b$, and $\mathbf{V}_c$, and those on the left terminals (marked a, b, and c) are denoted as $\mathbf{E}_a$, $\mathbf{E}_b$, and $\mathbf{E}_c$. The line currents may be expressed in terms of their sequence components as in Eq. 11.17. If the three currents are added, their sum is given by

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 3\mathbf{I}_{a0} = -\mathbf{I}_n \tag{11.19}$$

A Kirchhoff's voltage equation may be written around each loop containing one phase with the neutral as return.

$$\begin{aligned}
\mathbf{E}_a &= \mathbf{I}_a Z_{aa} + \mathbf{I}_b Z_{ab} + \mathbf{I}_c Z_{ca} + \mathbf{I}_n Z_{an} + \mathbf{V}_a \\
&\quad - \mathbf{I}_a Z_{an} - \mathbf{I}_b Z_{bn} - \mathbf{I}_c Z_{cn} - \mathbf{I}_n Z_{nn} && (11.20) \\
\mathbf{E}_b &= \mathbf{I}_b Z_{bb} + \mathbf{I}_a Z_{ab} + \mathbf{I}_c Z_{bc} + \mathbf{I}_n Z_{bn} + \mathbf{V}_b \\
&\quad - \mathbf{I}_a Z_{an} - \mathbf{I}_b Z_{bn} - \mathbf{I}_c Z_{cn} - \mathbf{I}_n Z_{nn} && (11.21) \\
\mathbf{E}_c &= \mathbf{I}_c Z_{cc} + \mathbf{I}_a Z_{ca} + \mathbf{I}_b Z_{bc} + \mathbf{I}_n Z_{cn} + \mathbf{V}_c \\
&\quad - \mathbf{I}_a Z_{an} - \mathbf{I}_b Z_{bn} - \mathbf{I}_c Z_{cn} - \mathbf{I}_n Z_{nn} && (11.22)
\end{aligned}$$

Assuming that the power system is a symmetrical network and using the relation given in Eq. 11.19, Eqs. 11.20 to 11.22 may be expressed as

$$\begin{aligned}
\mathbf{E}_a &= \mathbf{I}_a Z_{aa} + (\mathbf{I}_b + \mathbf{I}_c) Z_{ab} + \mathbf{V}_a + \mathbf{I}_n(2Z_{an} - Z_{nn}) && (11.23) \\
\mathbf{E}_b &= \mathbf{I}_b Z_{aa} + (\mathbf{I}_a + \mathbf{I}_c) Z_{ab} + \mathbf{V}_b + \mathbf{I}_n(2Z_{an} - Z_{nn}) && (11.24) \\
\mathbf{E}_c &= \mathbf{I}_c Z_{aa} + (\mathbf{I}_a + \mathbf{I}_b) Z_{ab} + \mathbf{V}_c + \mathbf{I}_n(2Z_{an} - Z_{nn}) && (11.25)
\end{aligned}$$

The zero-sequence component of the voltage is found by adding Eqs. 11.23 to 11.25 and dividing the sum by 3. Thus,

$$\begin{aligned}
\mathbf{E}_{a0} &= \tfrac{1}{3}(\mathbf{E}_a + \mathbf{E}_b + \mathbf{E}_c) \\
&= \tfrac{1}{3}[(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)(Z_{aa} + 2Z_{ab}) + \mathbf{I}_n(6Z_{an} - 3Z_{nn}) + (\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c)] \\
&= \mathbf{I}_{a0}(Z_{aa} + 2Z_{ab} - 6Z_{an} + 3Z_{nn}) + \mathbf{V}_{a0} && (11.26)
\end{aligned}$$

The positive-sequence component of the voltage is obtained by multiplying Eq. 11.24 by the operator a, multiplying Eq. 11.25 by the operator a^2, and adding the two products to Eq. 11.23. The sum is divided by 3; hence,

$$\begin{aligned}
\mathbf{E}_{a1} &= \tfrac{1}{3}(\mathbf{E}_a + a\mathbf{E}_b + a^2\mathbf{E}_c) \\
&= \tfrac{1}{3}[(\mathbf{I}_a + a\mathbf{I}_b + a^2\mathbf{I}_c)(Z_{aa} - Z_{ab}) + (\mathbf{V}_a + a\mathbf{V}_b + a^2\mathbf{V}_c)] \\
&= \mathbf{I}_{a1}(Z_{aa} - Z_{ab}) + \mathbf{V}_{a1} && (11.27)
\end{aligned}$$

Similarly, the negative-sequence component is obtained by multiplying

Eq. 11.23, and dividing the sum by 3; thus,

$$\begin{aligned}
\mathbf{E}_{a2} &= \tfrac{1}{3}(\mathbf{E}_a + a^2\mathbf{E}_b + a\mathbf{E}_c) \\
&= \tfrac{1}{3}[(\mathbf{I}_a + a^2\mathbf{I}_b + a\mathbf{I}_c)(Z_{aa} - Z_{ab}) + (\mathbf{V}_a + a^2\mathbf{V}_b + a\mathbf{V}_c)] \\
&= \mathbf{I}_{a2}(Z_{aa} - Z_{ab}) + \mathbf{V}_{a2} && (11.28)
\end{aligned}$$

It is generally assumed that the voltages from the generators are of positive sequence only. Therefore, the zero-sequence voltage $\mathbf{E}_{a0}$ and the negative-sequence voltage $\mathbf{E}_{a2}$ on the source side are not present, and they can be set equal to zero in Eqs. 11.26 and 11.28, respectively. Thus, Eqs. 11.26 to 11.28 may be rewritten as

$$\begin{aligned}
\mathbf{V}_{a0} &= -\mathbf{I}_{a0}(Z_{aa} + 2Z_{ab} - 6Z_{an} + 3Z_{nn}) \\
&= -\mathbf{I}_{a0}Z_0 && (11.29) \\
\mathbf{V}_{a1} &= -\mathbf{I}_{a1}(Z_{aa} - Z_{ab}) + \mathbf{E}_{a1} \\
&= -\mathbf{I}_{a1}Z_1 + \mathbf{E}_{a1} && (11.30) \\
\mathbf{V}_{a2} &= -\mathbf{I}_{a2}(Z_{aa} - Z_{ab}) \\
&= -\mathbf{I}_{a2}Z_2 && (11.31)
\end{aligned}$$

The expression for the zero-sequence component of the voltage as given by Eq. 11.29 contains no positive- or negative-sequence terms. Also, the expression given by Eq. 11.30 contains only the positive-sequence components and no negative- or zero-sequence terms. Similarly, Eq. 11.31 contains only negative-sequence and no positive- or zero-sequence terms. In other words, Eqs. 11.29 to 11.31 are completely uncoupled.

It is also seen from Eq. 11.29 that the zero-sequence voltage $\mathbf{V}_{a0}$ depends only on the zero-sequence current $\mathbf{I}_{a0}$ and the impedance $(Z_{aa} + 2Z_{ab} - 6Z_{an} + 3Z_{nn})$, which is designated as Z_0 and is referred to as the *zero-sequence impedance*. Also, from Eq. 11.30, the positive-sequence voltage $\mathbf{V}_{a1}$ depends only on the positive-sequence current $\mathbf{I}_{a1}$ and the impedance $Z_1 = (Z_{aa} - Z_{ab})$, which is called the *positive-sequence impedance*. Furthermore, the negative-sequence voltage $\mathbf{V}_{a2}$ depends only on the negative-sequence current $\mathbf{I}_{a2}$ and the *negative-sequence impedance* $Z_2 = (Z_{aa} - Z_{ab})$.

In Eqs. 11.29 to 11.31, it is assumed that the voltage of the source $\mathbf{E}_{a1}$ and all the impedances are known; thus, there are six unknown variables to determine. These are the sequence components of the current and the sequence components of the voltage. There are exactly three equations (11.29–11.31) relating the current and voltage sequence components, and three more are needed for a unique solution. The additional three relationships are provided by the interconnection of the sequence networks for the various types of faults.

Sequence Networks and Sequence Impedances The three equations 11.29

shown in Fig. 11.3. These networks are called the *positive-sequence, negative-sequence,* and *zero-sequence networks*. They are constructed by using the positive-sequence, negative-sequence, and zero-sequence impedances of the various power system components.

The series impedance of a transmission line derived in Chapter 9 and also used in Section 10.1.3 represents the positive-sequence and negative-sequence impedances. The zero-sequence impedance is not the same as the positive- or negative-sequence impedance. The zero-sequence impedance is affected by several factors, including the characteristics of the earth as a return path and the type and number of ground wires used. A good discussion of the derivation of the zero-sequence impedance of a transmission line is given in Ref. 5, *Electrical Transmission and Distribution Reference Book*.

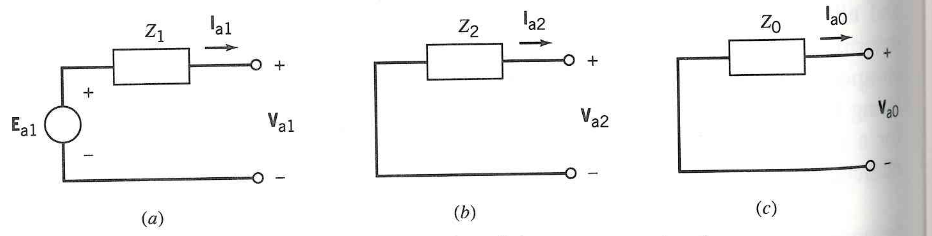
When the analysis involves calculation of the steady-state or sustained short-circuit current, the direct-axis synchronous reactance X_d of the synchronous machine is taken as the positive-sequence reactance. If subtransient or transient currents are required, X''_d or X'_d is used instead, respectively.

The rotating magnetic field due to positive-sequence currents in a synchronous machine rotates in the same direction as the rotor. However, the negative-sequence currents produce a rotating magnetic field at synchronous speed but in the opposite direction; thus, this negative-sequence field passes over the poles, field winding, and damper windings at twice synchronous speed, resulting in higher opposition and effectively lower reactance.

The zero-sequence currents, although in phase, are physically displaced 120° from each other because of the distribution of the windings; thus, the sum of the magnetic fields is equal to zero except for the effects of unbalances. Therefore, the zero-sequence reactance due to the small flux produced by the zero-sequence currents and slot and end connection leakage is still smaller than the negative-sequence reactance. Typical values of reactances of synchronous machines are given in Table 10 of Appendix A.

The impedance of a transformer to both positive- and negative-sequence currents is the same, and the equivalent circuits are those given in Section 10.1.3. The phase shift, if there is any, has the same magnitude but opposite direction. Phase shifts will be neglected in the following discussions.

The zero-sequence equivalent circuit of a three-phase transformer bank depends on the type of connection and the method of grounding. In a delta-connected winding, zero-sequence currents can flow, but because they are equal in each phase of the delta, they do not leave the terminals of the transformer; thus, it appears as an open circuit as viewed from the external circuit. If the neutral of a wye-connected winding is not connected to ground, the three phase currents add to zero and there is no zero-sequence current (i.e., $\mathbf{I}_{a0} = 0$); thus, it appears as an open circuit to the external circuit. When the neutral of a wye-connected winding is grounded, the sum of the three phase currents is equal to $3\mathbf{I}_{a0}$ flowing to or from ground. Since the zero-sequence network represents only one phase with current $\mathbf{I}_{a0}$ flowing, any ground impedance Z_g is included as $3Z_g$ to provide the proper neutral voltage. Some of the typical transformer connections and their zero-sequence equivalent circuits are shown in Fig. 11.4.



(a) (b) (c)

FIGURE 11.3 Representation of the sequence networks.

Single Line-to-Ground Fault A single line-to-ground (SLG) fault is the most commonly occurring unsymmetrical fault. It may be caused by a vehicular accident causing one of the phase conductors to fall and come in contact with the earth, or it may be caused by tree branches, or it could be caused by flashovers across dusty insulators during rainshowers. An SLG fault is illustrated in Fig. 11.5.

Assuming that prefault currents are negligible, the SLG fault is described by the following voltage and current relationships.

$$\mathbf{V}_a = 0 \tag{11.32}$$

$$\mathbf{I}_b = \mathbf{I}_c = 0 \tag{11.33}$$

The sequence components of the short-circuit current are found by using Eq. (11.18). Thus,

$$\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \mathbf{I}_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_a \\ \mathbf{I}_a \end{bmatrix} \tag{11.34}$$

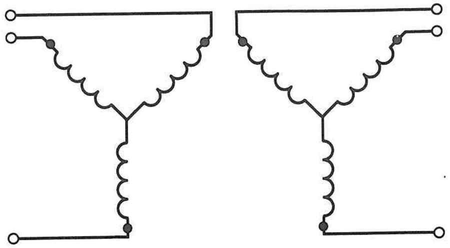
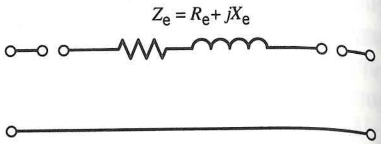
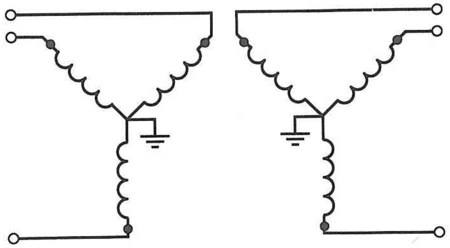
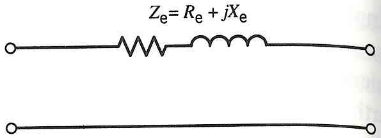
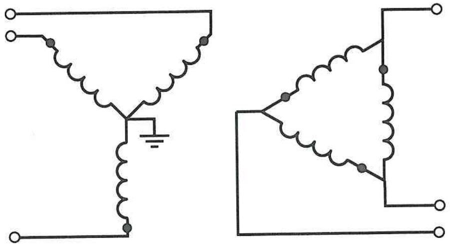
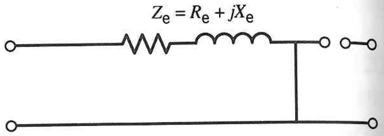
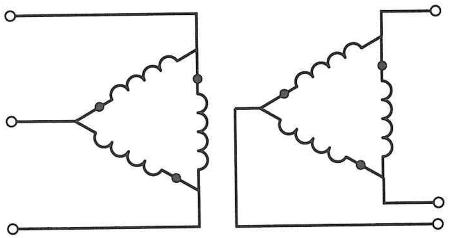
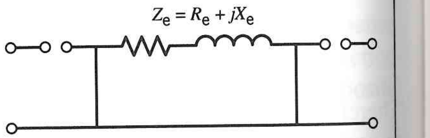
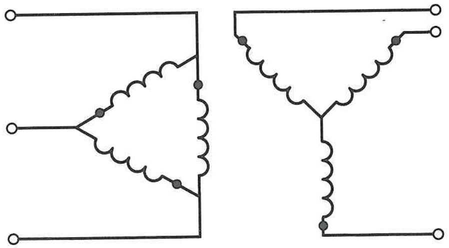
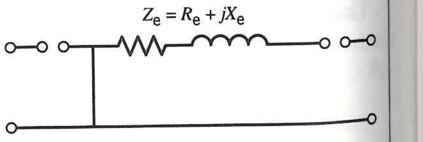
Therefore, it is seen that the sequence components of the current are all equal to each other; that is,

$$\mathbf{I}_{a0} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \tfrac{1}{3}\mathbf{I}_a \tag{11.35}$$

The voltage of the principal phase a may be expressed in terms of its sequence components as

$$\mathbf{V}_a = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = 0 \tag{11.36}$$

The expressions for the sequence components of the voltage are given by Eqs. 11.29 to 11.31. Substituting these expressions in Eq. 11.36 and rearranging

Connection Diagrams	Equivalent Circuits
	
	
	
	
	

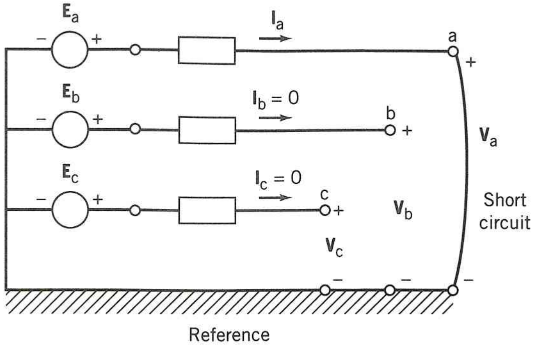


FIGURE 11.5 A single line-to-ground fault.

the terms yield

$$\mathbf{E}_{a1} - \mathbf{I}_{a1}(Z_1 + Z_2 + Z_0) = 0 \tag{11.37}$$

The expression for $\mathbf{I}_{a1}$ is obtained from Eq. 11.37, and using Eq. 11.35 provides the expressions for $\mathbf{I}_{a2}$ and $\mathbf{I}_{a0}$ as follows:

$$\begin{aligned} \mathbf{I}_{a1} &= \frac{\mathbf{E}_{a1}}{Z_1 + Z_2 + Z_0} \\ &= \mathbf{I}_{a2} \\ &= \mathbf{I}_{a0} \end{aligned} \tag{11.38}$$

The short-circuit current $\mathbf{I}_a$ is found as the sum of its sequence components; thus,

$$\begin{aligned} \mathbf{I}_a &= \mathbf{I}_{a0} + \mathbf{I}_{a1} + \mathbf{I}_{a2} \\ &= 3\frac{\mathbf{E}_{a1}}{Z_1 + Z_2 + Z_0} \end{aligned} \tag{11.39}$$

Equations 11.35 and 11.36 indicate that the sequence networks are to be interconnected in series for a single line-to-ground fault. This is illustrated in Fig. 11.6.

EXAMPLE 11.4

A three-phase generator is rated 150 MVA, 13.8 kV and has positive-, negative-, and zero-sequence reactances of 20%, 10%, and 5%, respectively. The generator is connected to a transmission line having positive- and negative-sequence reactances of 10% and a zero-sequence reactance of 30%. The

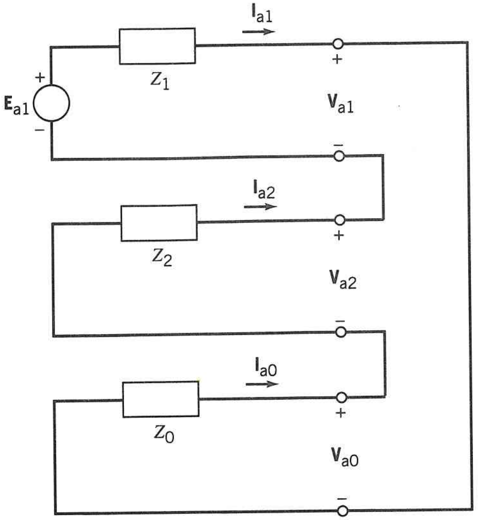


FIGURE 11.6 Interconnection of sequence networks for a single line-to-ground fault.

ator is operating at rated voltage and no-load conditions when a single line-to-ground short circuit occurs at the end of the line. Determine the phase currents at the fault and the phase voltages at the terminals of the generator. A one-line diagram of the power system is shown in Fig. 11.7.

Solution Assume that the fault is a phase-a-to-ground fault. The sequence networks are interconnected as shown in Fig. 11.8.

Since the generator is initially operating at no load and rated voltage, the generator voltage is taken as 1.0 per unit and used as reference. Thus,

$$\mathbf{E}_{a1} = 1.0\angle 0^\circ \text{ pu}$$

The sequence components of the current at the fault are computed as follows:

$$\begin{aligned}\mathbf{I}_{a1} &= (1.0\angle 0^\circ)/(j0.3 + j0.2 + j0.35) = (1.0\angle 0^\circ)/(j0.85)\\ &= 1.176\angle{-90^\circ} \text{ pu} = -\text{j}1.176 \text{ pu}\\ &= \mathbf{I}_{a0} = \mathbf{I}_{a2}\end{aligned}$$

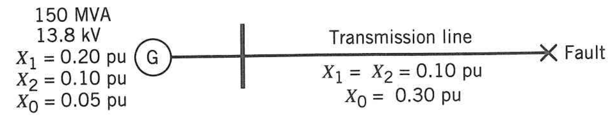


FIGURE 11.7 Power system for Example 11.4.

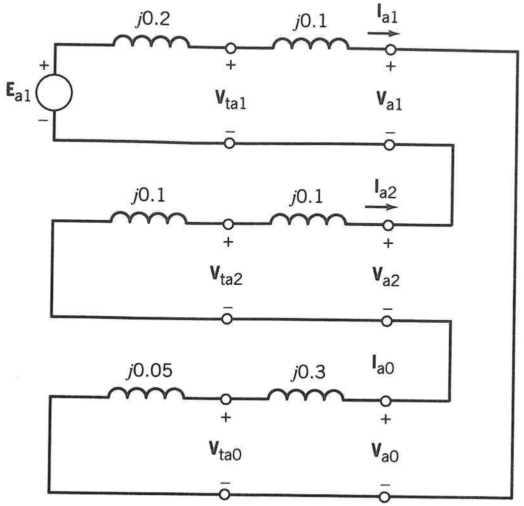


FIGURE 11.8 Interconnection of sequence networks for Example 11.4.

The phase currents are computed by using Eq. 11.17 as follows:

$$\begin{bmatrix}\mathbf{I}_a\\ \mathbf{I}_b\\ \mathbf{I}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{bmatrix}\begin{bmatrix}1.176\angle{-90^\circ}\\ 1.176\angle{-90^\circ}\\ 1.176\angle{-90^\circ}\end{bmatrix} = \begin{bmatrix}3.528\angle{-90^\circ}\\ 0.0\\ 0.0\end{bmatrix} \text{ pu}$$

The base current is computed as follows:

$$I_{\text{base}} = \frac{150{,}000}{13.8\sqrt{3}} = 6276 \text{ A}$$

The per-unit short-circuit current is multiplied by the base current to give the actual current in amperes. Thus,

$$I_a = (3.528)(6276) = 22{,}140 \text{ A}$$

The sequence components of the voltage at the terminals of the generator are computed as follows:

$$\begin{aligned}\mathbf{V}_{ta0} &= -j0.05\mathbf{I}_{a0} = -(j0.05)(-j1.176) = -0.0588 \text{ pu}\\ \mathbf{V}_{ta1} &= \mathbf{E}_{a1} - j0.20\mathbf{I}_{a1} = 1.0\angle 0^\circ - (j0.20)(-j1.176) = 0.7648 \text{ pu}\\ \mathbf{V}_{ta2} &= -j0.10\mathbf{I}_{a2} = -(j0.10)(-j1.176) = -0.1176 \text{ pu}\end{aligned}$$

The phase voltages at the generator terminals are computed by using Eq. 11.9 as follows:

$$\begin{bmatrix} \mathbf{V}_{ta} \\ \mathbf{V}_{tb} \\ \mathbf{V}_{tc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0588 \\ 0.7648 \\ -0.1176 \end{bmatrix} = \begin{bmatrix} 0.588\angle 0^\circ \\ 0.855\angle -117^\circ \\ 0.855\angle\ 117^\circ \end{bmatrix} \text{ pu}$$

The line-to-line voltages at the generator terminals are found by multiplying the per-unit values by the base voltage.

$$\begin{bmatrix} \mathbf{V}_{ta} \\ \mathbf{V}_{tb} \\ \mathbf{V}_{tc} \end{bmatrix} = \begin{bmatrix} 0.588\angle 0^\circ \\ 0.855\angle -117^\circ \\ 0.855\angle\ 117^\circ \end{bmatrix} (13.8 \text{ kV}) = \begin{bmatrix} 8.10\angle 0^\circ \\ 11.80\angle -117^\circ \\ 11.80\angle\ 117^\circ \end{bmatrix} \text{ kV}$$

Line-to-Line Fault A line-to-line (L-L) fault involves a short circuit between two phase conductors that are assumed to be phases b and c. Therefore, there is symmetry with respect to the principal phase a. An L-L fault is illustrated in Fig. 11.9.

Assuming that prefault currents are negligible, the voltage and current relationships describing the line-to-line fault are given by

$$\mathbf{V}_b = \mathbf{V}_c \tag{11.40}$$

$$\mathbf{I}_b = -\mathbf{I}_c; \qquad \mathbf{I}_a = 0 \tag{11.41}$$

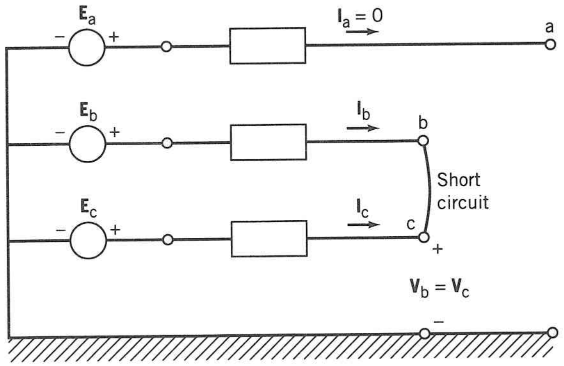


FIGURE 11.9 A line-to-line fault.

The sequence components of the short-circuit current are found by using Eq. 11.18. Thus,

$$\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (a - a^2)\mathbf{I}_b \\ (a^2 - a)\mathbf{I}_b \end{bmatrix} = \begin{bmatrix} 0 \\ j(\mathbf{I}_b/\sqrt{3}) \\ -j(\mathbf{I}_b/\sqrt{3}) \end{bmatrix} \tag{11.42}$$

Therefore, it is seen that the negative-sequence component of the current is equal to the negative of its positive-sequence component; that is,

$$\mathbf{I}_{a2} = -\mathbf{I}_{a1} \tag{11.43}$$

The voltages of the shorted phases, b and c, may be expressed in terms of the sequence components as follows:

$$\mathbf{V}_b = \mathbf{V}_{a0} + a^2\mathbf{V}_{a1} + a\mathbf{V}_{a2} \tag{11.44}$$

$$\mathbf{V}_c = \mathbf{V}_{a0} + a\mathbf{V}_{a1} + a^2\mathbf{V}_{a2} \tag{11.45}$$

Subtracting Eq. 11.45 from 11.44 gives

$$\mathbf{V}_b - \mathbf{V}_c = (a^2 - a)\mathbf{V}_{a1} + (a - a^2)\mathbf{V}_{a2} = 0 \tag{11.46}$$

Because $\mathbf{V}_b$ and $\mathbf{V}_c$ are equal, Eq. 11.46 reduces to zero and yields

$$\mathbf{V}_{a1} = \mathbf{V}_{a2} \tag{11.47}$$

Equations 11.43 and 11.47 indicate that the positive- and negative-sequence networks are to be interconnected in parallel for a line-to-line fault. This is illustrated in Fig. 11.10.

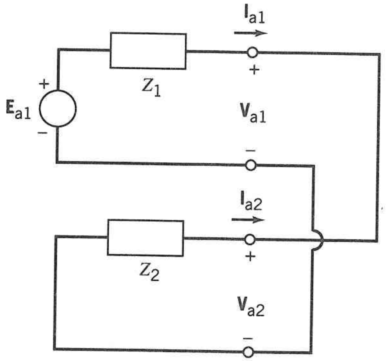


FIGURE 11.10 Interconnection of sequence networks for a line-to-line fault.

EXAMPLE 11.5

Repeat Example 11.4 assuming a phase-b-to-phase-c fault occurs at the end of the line in the given sample power system.

Solution The sequence networks are interconnected as shown in Fig. 11.11. The generator voltage is taken as reference phasor. Thus,

$$\mathbf{E}_{a1} = 1.0\underline{/0^\circ} \text{ pu}$$

The sequence components of the current at the fault are computed as follows:

$$\begin{aligned}
\mathbf{I}_{a1} &= (1.0\underline{/0^\circ})/(j0.3 + j0.2) = 2.0\underline{/-90^\circ} \text{ pu} = -j2.0 \text{ pu} \\
\mathbf{I}_{a2} &= -\mathbf{I}_{a1} = 2.0\underline{/90^\circ} \text{ pu} = j2.0 \text{ pu} \\
\mathbf{I}_{a0} &= 0
\end{aligned}$$

The phase currents are computed by using Eq. 11.17 as follows:

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2 \\ j2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.46 \\ 3.46 \end{bmatrix} \text{pu} = \begin{bmatrix} 0 \\ -21.74 \\ 21.74 \end{bmatrix} \text{kA}$$

The sequence components of the voltage at the terminals of the generator are computed as follows:

$$\begin{aligned}
\mathbf{V}_{ta0} &= 0 \\
\mathbf{V}_{ta1} &= \mathbf{E}_{a1} - j(0.20)\mathbf{I}_{a1} = 1.0\underline{/0^\circ} - (j0.20)(-j2.0) = 0.6\underline{/0^\circ} \text{ pu} \\
\mathbf{V}_{ta2} &= -j0.10\mathbf{I}_{a2} = -(j0.10)(j2.0) = 0.2\underline{/0^\circ} \text{ pu}
\end{aligned}$$

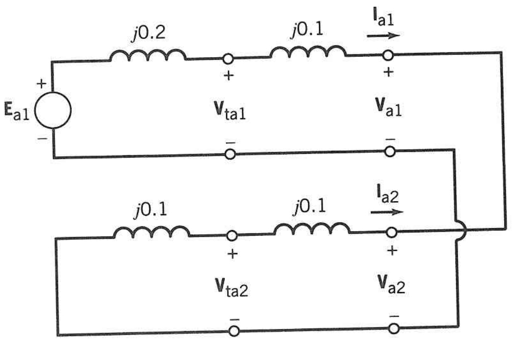


FIGURE 11.11 Interconnection of sequence networks for Example 11.5.

The phase voltages at the generator terminals are computed by using Eq. 11.9 as follows:

$$\begin{bmatrix} \mathbf{V}_{ta} \\ \mathbf{V}_{tb} \\ \mathbf{V}_{tc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.6 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.80\underline{/0^\circ} \\ 0.53\underline{/-139^\circ} \\ 0.53\underline{/\ 139^\circ} \end{bmatrix} \text{pu} = \begin{bmatrix} 11.0\underline{/0^\circ} \\ 7.3\underline{/-139^\circ} \\ 7.3\underline{/\ 139^\circ} \end{bmatrix} \text{kV}$$

Double Line-to-Ground Fault A double line-to-ground (2LG) fault involves a short circuit between two phase conductors b and c and ground. As with the line-to-line fault, there is symmetry with respect to the principal phase a. A 2LG fault is illustrated in Fig. 11.12.

For a 2LG fault with prefault currents assumed negligible, the voltage and current relationships are given by the following:

$$\mathbf{V}_b = \mathbf{V}_c = \mathbf{0} \tag{11.48}$$

$$\mathbf{I}_a = 0 \tag{11.49}$$

The sequence components of the voltage at the fault location are found by using Eq. 11.16. Thus,

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_a \\ \mathbf{V}_a \end{bmatrix} \tag{11.50}$$

Therefore, it is seen that the sequence components of the voltage at the fault are all equal; that is,

$$\mathbf{V}_{a1} = \mathbf{V}_{a2} = \mathbf{V}_{a0} \tag{11.51}$$

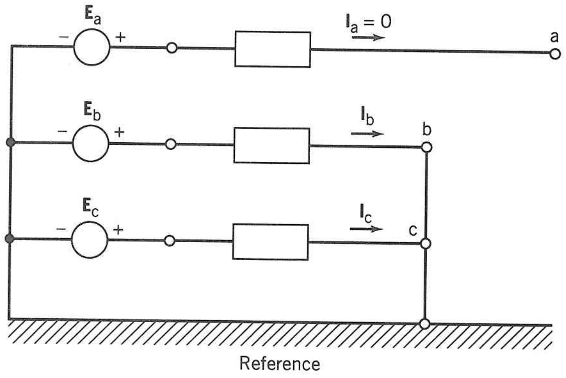


FIGURE 11.12 A double line-to-ground fault.

Neglecting prefault load currents, the current in phase a may be expressed as

$$\mathbf{I}_a = \mathbf{I}_{a0} + \mathbf{I}_{a1} + \mathbf{I}_{a2} = 0 \qquad (11.52)$$

Equations 11.51 and 11.52 indicate that the three sequence networks are to be interconnected in parallel for a double line-to-ground fault. This is illustrated in Fig. 11.13.

EXAMPLE 11.6

Repeat Example 11.4 assuming a double line-to-ground fault occurs at the end of the line in the given sample power system.

Solution The sequence networks are interconnected in parallel as shown in Fig. 11.14.

The generator voltage is taken as reference phasor. Thus,

$$\mathbf{E}_{a1} = 1.0\underline{/0°}\ \text{pu}$$

The sequence components of the current at the fault are computed as follows:

$$\begin{aligned}\mathbf{I}_{a1} &= \frac{1.0\underline{/0°}}{j0.3 + (j0.2)(j0.35)/(j0.2 + j0.35)} = \frac{1}{j0.427}\\ &= 2.34\underline{/-90°} = -j2.34\ \text{pu}\end{aligned}$$

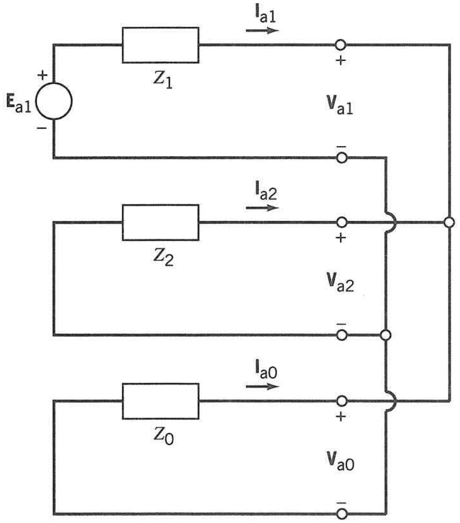


FIGURE 11.13 Interconnection of sequence networks for a double line-to-ground fault.

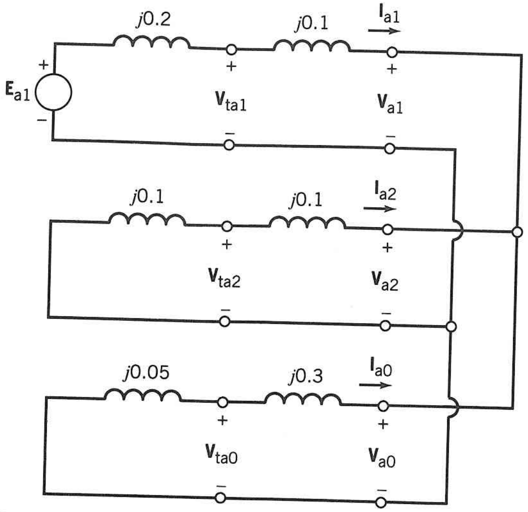


FIGURE 11.14 Interconnection of sequence networks for Example 11.6.

$$\mathbf{I}_{a2} = -\frac{j0.35}{j0.2 + j0.35}(2.34\underline{/-90°}) = 1.49\underline{/90°} = j1.49\ \text{pu}$$

$$\mathbf{I}_{a0} = -\frac{j0.20}{j0.2 + j0.35}(2.34\underline{/-90°}) = 0.85\underline{/90°} = j0.85\ \text{pu}$$

The phase currents are computed by using Eq. 11.17 as follows:

$$\begin{bmatrix}\mathbf{I}_a\\ \mathbf{I}_b\\ \mathbf{I}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{bmatrix}\begin{bmatrix}j0.85\\ -j2.34\\ j1.49\end{bmatrix} = \begin{bmatrix}0\\ 3.55\underline{/159°}\\ 3.55\underline{/\ 21°}\end{bmatrix}\ \text{pu}$$

$$= \begin{bmatrix}0\\ 22.28\underline{/159°}\\ 22.28\underline{/\ 21°}\end{bmatrix}\ \text{kA}$$

The sequence components of the voltage at the terminals of the generator are computed as follows:

$$\mathbf{V}_{ta0} = -j0.05\mathbf{I}_{a0} = -(j0.05)(0.85\underline{/90°}) = 0.043\ \text{pu}$$

$$\mathbf{V}_{ta1} = \mathbf{E}_{a1} - j0.20\mathbf{I}_{a1} = 1.0\underline{/0°} - (j0.20)(2.34\underline{/-90°}) = 0.532\ \text{pu}$$

$$\mathbf{V}_{ta2} = -j0.10\mathbf{I}_{a2} = -(j0.10)(1.49\underline{/90°}) = 0.149\ \text{pu}$$

The phase voltages are computed by using Eq. 11.9; that is,

$$\begin{bmatrix} \mathbf{V}_{ta} \\ \mathbf{V}_{tb} \\ \mathbf{V}_{tc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.043 \\ 0.532 \\ 0.149 \end{bmatrix} = \begin{bmatrix} 0.724\angle\underline{\ \ 0^\circ} \\ 0.446\angle\underline{-132^\circ} \\ 0.446\angle\underline{\ 132^\circ} \end{bmatrix} \text{pu}$$

$$= \begin{bmatrix} 10.0\angle\underline{\ \ 0^\circ} \\ 6.1\angle\underline{-132^\circ} \\ 6.1\angle\underline{\ 132^\circ} \end{bmatrix} \text{kV}$$

Three-Phase Fault A three-phase fault, although it is a symmetrical fault, may also be analyzed using the method of symmetrical components. Consider the three-phase fault illustrated in Fig. 11.15. Assuming the prefault currents are negligible, the voltage and current relationships describing this fault are

$$\mathbf{V}_a = \mathbf{V}_b = \mathbf{V}_c \tag{11.53}$$

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 3\mathbf{I}_{a0} = 0 \tag{11.54}$$

Equation 11.54 confirms that there is no zero-sequence current for a three-phase fault; that is, $\mathbf{I}_{a0}$ is identically equal to zero. The positive- and negative-sequence components of the voltage at the fault location are found as follows:

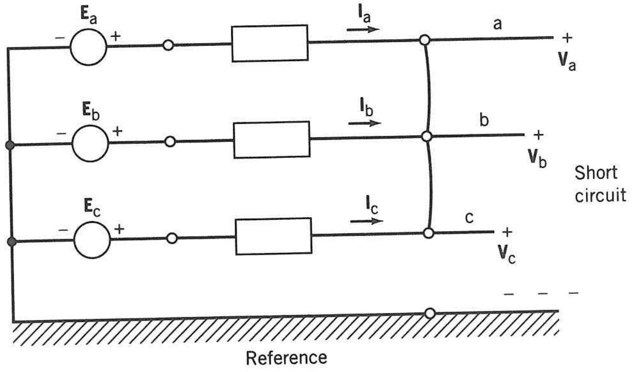
$$\mathbf{V}_{a1} = \tfrac{1}{3}(\mathbf{V}_a + a\mathbf{V}_b + a^2\mathbf{V}_c) = \tfrac{1}{3}(1 + a + a^2)\mathbf{V}_a = 0 \tag{11.55}$$

$$\mathbf{V}_{a2} = \tfrac{1}{3}(\mathbf{V}_a + a^2\mathbf{V}_b + a\mathbf{V}_c) = \tfrac{1}{3}(1 + a^2 + a)\mathbf{V}_a = 0 \tag{11.56}$$

Since the positive- and negative-sequence components of the voltage at the location of the fault are both equal to zero, the sequence components of the current are obtained from Eqs. 11.30 and 11.31 as follows:

$$\mathbf{I}_{a1} = \frac{\mathbf{E}_{a1}}{Z_1}$$

$$\mathbf{I}_{a2} = 0 \tag{11.57}$$



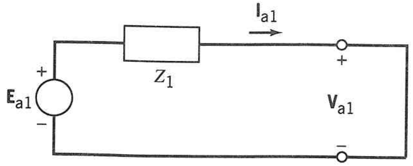


FIGURE 11.16 Sequence network for a three-phase fault.

It has been found that no zero- and negative-sequence currents are flowing during a three-phase fault. Therefore, the analysis of a three-phase fault involves only the positive-sequence network, which is shown in Fig. 11.16.

EXAMPLE 11.7

Repeat Example 11.4 assuming a three-phase fault occurs at the end of the line in the given sample power system.

Solution Only the positive-sequence network is used, and it is shown in Fig. 11.17.

At no load, the generator voltage is 1.0 per unit and is taken as reference. Thus,

$$\mathbf{E}_{a1} = 1.0\angle\underline{0^\circ} \text{ pu}$$

The sequence components of the current at the fault are computed as follows:

$$\mathbf{I}_{a1} = (1.0\angle\underline{0^\circ})/(j0.3) = 3.33\angle\underline{-90^\circ} \text{ pu} = -j3.33 \text{ pu}$$

$$\mathbf{I}_{a2} = 0$$

$$\mathbf{I}_{a0} = 0$$

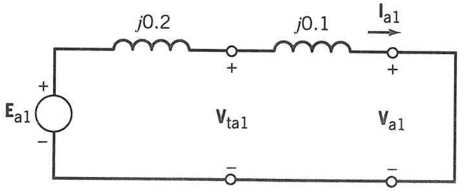


FIGURE 11.17 Sequence network for Example 11.7.

The phase currents are computed by using Eq. 11.17 as follows:

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.0 \\ -j3.33 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 3.33\angle{-90^\circ} \\ 3.33\angle{150^\circ} \\ 3.33\angle{30^\circ} \end{bmatrix} \text{pu} = \begin{bmatrix} 20.9\angle{-90^\circ} \\ 20.9\angle{150^\circ} \\ 20.9\angle{30^\circ} \end{bmatrix} \text{kA}$$

The sequence components of the voltage at the terminals of the generator are computed as follows:

$$\mathbf{V}_{ta0} = 0$$

$$\mathbf{V}_{ta1} = \mathbf{E}_{a1} - j0.20\mathbf{I}_{a1} = 1.0\angle{0^\circ} - (j0.20)(-j3.33) = 0.333 \text{ pu}$$

$$\mathbf{V}_{ta2} = 0$$

The phase voltages at the generator terminals are computed by using Eq. 11.9 as follows:

$$\begin{bmatrix} \mathbf{V}_{ta} \\ \mathbf{V}_{tb} \\ \mathbf{V}_{tc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.333 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.333\angle{0^\circ} \\ 0.333\angle{-120^\circ} \\ 0.333\angle{120^\circ} \end{bmatrix} \text{pu}$$

$$= \begin{bmatrix} 4.6\angle{0^\circ} \\ 4.6\angle{-120^\circ} \\ 4.6\angle{120^\circ} \end{bmatrix} \text{kV}$$

DRILL PROBLEMS

D11.7 A three-phase, 100-MVA, 12-kV, wye-connected synchronous generator has the following reactances: $X'' = X_2 = 20\%$ and $X_0 = 8\%$. The generator is connected to a 100-MVA, 12/115-kV, Δ/Y, three-phase transformer with a reactance of 10%. The neutrals of the generator and transformer windings are solidly grounded. The terminal voltage of the generator is 12 kV, and the transformer is at open circuit. A single line-to-ground fault occurs on the high-voltage side of the transformer.

a. Find the short-circuit current at the fault.

b. Find the short-circuit current supplied by the generator.

D11.8 Repeat Problem D11.7 if the fault is a line-to-line fault.

D11.9 Repeat Problem D11.7 if the fault is a double line-to-ground fault.

11.3 POWER SYSTEM PROTECTION

The operation of a power system is affected by disturbances that could be due to natural occurrences such as lightning, wind, trees, animals, and human errors or accidents. These disturbances could lead to abnormal system conditions such as short circuits, overloads, and open circuits.

Short circuits, which are also referred to as faults, are of the greatest concern because they could lead to damage to equipment or system elements and other operating problems including voltage drops, decrease in frequency, loss of synchronism, and complete system collapse. There is, therefore, a need for a device or a group of devices that is capable of recognizing a disturbance and acting automatically to alleviate any ill effects on the system element or on the operator. Such capability is provided by the protection system.

The protection system is designed to disconnect the faulted system element automatically when the short circuit currents are high enough to present a direct danger to the element or to the system as a whole. When the fault results in overloads or short-circuit currents that do not present any immediate danger, the protection system will initiate an alarm so that measures can be implemented to remedy the situation.

11.3.1 Components of Protection Systems

There are three principal components of a protection system:

1. Transducer
2. Protective relay
3. Circuit breaker

These components are described briefly in the following paragraphs.

Transducers The transducer serves as a sensor to detect abnormal system conditions and to transform the high values of short-circuit current and voltage to lower levels. The main sensors used are the current transformer (CT) and the potential transformer (PT).

The current transformer is designed to provide a standard continuous secondary current of 5 A. Standard CT ratios available include 50/5, 100/5, 150/5, 200/5, 250/5, 300/5, 400/5, 500/5, 600/5, 800/5, 900/5, 1000/5, 1200/5, 1500/5, and 2000/5. During fault conditions, the short-circuit currents could reach over 10 times normal for short periods of time without damaging the CT windings.

The current transformer has a primary winding that usually consists of one turn and a secondary winding of several turns. It is, therefore, unsafe to open-circuit the secondary of a CT whose primary is energized.

The potential transformer is designed to operate at a constant standard secondary voltage of 120 V. For low-voltage applications, the PT is just like any other two-winding voltage transformer. For primary voltages in the HV and EHV levels, a capacitor voltage-divider circuit is used together with the PT. The primary voltage is impressed across the series-connected capacitors. The PT is used to measure the voltage of a few kilovolts across the capacitor of the smaller capacitance value.

Protective Relays A protective relay is a device that processes the signals provided by the transducers, which may be in the form of a current, a voltage, or a combination of current and voltage. These signals arise as a result of a faulted condition such as a short circuit, defective equipment or lines, lightning strikes, or surges. The protective relay can initiate or permit the opening of various interrupting devices or sound an alarm. There are two main classifications of protective relays based on their construction: electromechanical and solid state.

The electromechanical relay develops an electromagnetic force or torque from the signal provided by the transducer; this force or torque is used to physically open, or close, a set of contacts to permit or initiate the tripping of circuit breakers or actuate an alarm.

The solid-state, or static, relay is energized by the same signals as in an electromechanical relay. However, there is no physical opening, or closing, of the relay contacts. Instead, the switching of the relay contacts is simulated by causing a solid-state device to change its status from conducting (closed position) to nonconducting (open position).

Electromechanical relays predate the solid-state relays. A majority of power system installations still use electromechanical relays. The improved reliability, versatility, and faster response (as low as one-quarter cycle) of solid-state relays have made them more attractive. Some electromechanical relays have been replaced by solid-state relays, and in newer installations a mixture of both types would usually be found.

Circuit Breakers A circuit breaker is a mechanical device used to energize and interrupt an electric circuit. It should be able to open and close quickly, maybe in the order of a few milliseconds. It should be able to carry the rated current continuously at the nominal voltage, and it must be able to withstand the large short-circuit current (called its momentary rating) that flows during the first cycle after a fault occurs. The circuit breaker must be able to interrupt a large short-circuit current called its interrupting rating. The momentary rating is about 1.6 times the interrupting rating because the former includes the effect of the DC component of the transient short-circuit current. The actual value of current interrupted by the circuit breaker depends on its speed, which could be 1/2, 3, 5, or 8 cycles.

When the current-carrying contacts of the circuit breaker are opened, an electric field appears across the contacts that ionizes the medium between them,

and an arc is established between the contacts. The circuit breaker must be able to extinguish, or interrupt, this arc as quickly as possible. The arc is made to take an elongated path, cooled, and finally extinguished when the AC current feeding the arc passes through its zero value. Sometimes, the arc is extinguished in air, oil, sulfur hexachloride (SF_6), or a vacuum.

11.3.2 Types of Protection Systems

The fundamental principle in designing protection systems is to divide the power system into zones that can be provided with the appropriate protection and can be disconnected or isolated as quickly as possible in order to minimize the effect on the rest of the power system. Each zone of protection is provided with two types of protection: primary and backup protection.

Primary or Main Protection Primary protection is provided to ensure fast and reliable tripping of the circuit breakers to clear faults occurring within the boundary of its own zone of protection. In general, primary protection is provided for each transmission line segment, major piece of equipment, and switchgear.

A separate zone of protection is established around each system element as shown in Fig. 11.18. There is overlapping of adjacent zones of protection around the circuit breakers. Thus, if a fault occurs within the overlap region, more breakers are tripped than is actually necessary. On the other hand, if there is no overlap, a fault that occurs in the region between two zones of protection will not be in either zone, and no protective relay will operate and no circuit breaker will be tripped.

Although each major piece of equipment and transmission line segment is provided with primary protection and the primary zones of protection are made

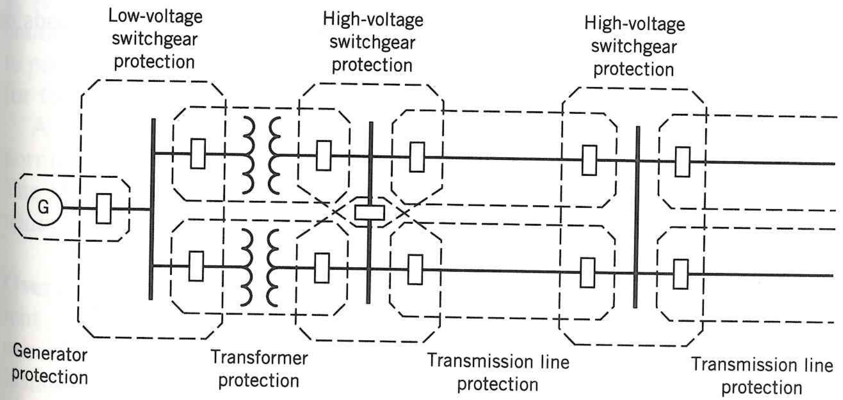


FIGURE 11.18 Zones of primary protection in a power system

to overlap, it is still possible for the protection to fail. Hence, there is a need for backup protection.

Backup Protection Backup protection is provided in case the primary protection fails to operate or is under repair or maintenance. The protective relays used for backup protection have longer time delays; that is, they are slower acting than the relays used for primary protection to give the latter the opportunity to perform their function.

The backup protection should be designed in such a way that the cause of failure of the primary protection is not going to cause the same failure in the backup protection. Therefore, the backup protection should be located at a different station from the primary protection.

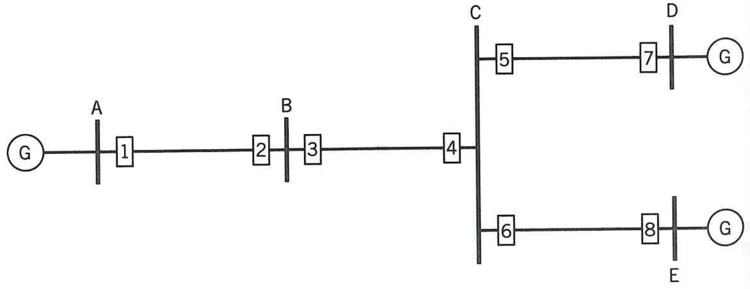
Consider a fault on line segment 3–4 of the power system shown in Fig. 11.19. If circuit breaker 3 fails to trip for some reason, the backup relay would trip circuit breaker 1; or, if the circuit breaker 4 fails to trip, the backup relays would trip circuit breakers 7 and 8. Because circuit breaker 1 is located differently from 3 and circuit breakers 7 and 8 are located differently from 4, the cause of failure of the primary protection would probably not cause failure of the backup protection, which might happen if backup protection were provided at 2 or 5 and 6 instead.

If a fault occurred at station C, primary protection would be provided by circuit breakers at 4, 5, and 6. Backup protection would be located at 3, 7, and 8.

11.3.3 Requirements of Protection Systems

In order to perform its functions properly, the protection system must have the following characteristics: speed, selectivity, reliability, and sensitivity.

Speed The speed of the protection system refers to the operating times of the protective relays. The potential damage to the faulted element depends on the length of time the short-circuit currents are allowed to flow. The speed of clearing, or isolating, the faulted system component also affects the stability of the whole system.



Protective relays may be characterized as instantaneous with an operating time of about 0.10 s, or as high speed with an operating time of less than 0.05 s. Solid-state, or static, relays can have operating times as low as one quarter of a cycle.

Selectivity Selectivity is the ability of the protection system to detect a fault, identify the point at which the fault occurred, and isolate the faulted circuit element by tripping the minimum number of circuit breakers. Selectivity of the protection system is obtained by proper coordination of the operating currents and time delays of the protective relays.

Reliability The reliability of the protection system is its ability to operate upon the occurrence of any fault for which it was designed to protect. In other words, the protection system should operate when it is supposed to and not operate when it is not supposed to.

Sensitivity Sensitivity refers to the characteristic of a protective relay that it operates reliably, when required, in response to a fault that produces the minimum short-circuit current flowing through the relay.

11.3.4 Types of Protective Relays

The protective relay is the device that responds to signals from the transducers by quickly initiating or allowing a control action to be implemented in order to prevent damage to the faulted equipment and to restore service as soon as possible. The operating characteristics of the more commonly used protective relays are described in this section.

A relay is said to *pick up* when it operates to open its normally closed (NC) contact or to close its normally open (NO) contact in response to a disturbance to produce a desired control action. The smallest value of the actuating quantity for the relay to operate is called its *pickup value*.

A relay is said to *reset* when it operates to close an open contact that is normally closed (NC) or to open a closed contact that is normally open (NO). The largest value of the actuating quantity for this to happen is called the *reset value*.

Overcurrent Relays The actuating quantity of an overcurrent relay is a current. The relay is designed to operate when the actuating quantity equals, or exceeds, its pickup value. An overcurrent relay can be either of two types: instantaneous or time-delay type. Both relay types are frequently provided in one relay case and are actuated by the same current; however, their individual pickup values can be adjusted separately. The pickup values may be adjusted by varying the tap settings in the input.

The instantaneous relay element has no intentional time delay, and it operates quickly from 1/2 to 3 cycles depending on the value of the fault current. A typical operating characteristic of this relay type is shown in Fig. 11.20.

The time-delay relay element is characterized by having an operating time that varies inversely as the fault current flowing through the relay. A typical inverse time characteristic is shown in Fig. 11.20. The time-delay characteristic may be shifted up or down by adjusting the time-dial setting so that the relay operates with a different time delay for the same value of fault current.

The difficulty in using overcurrent relays is that they are inherently nonselective. They detect overcurrents (faults) not only in their own zones of protection but also in adjacent zones. The selectivity can be improved by proper coordination of the relay pickup values and time-delay settings. As the electric load grows and the power system configuration changes, operating conditions and magnitudes of short-circuit currents will vary. The pickup values of the overcurrent relays have to be readjusted continually in response to these changes.

Overcurrent relays are popular especially on low-voltage circuits because of their low cost. They are also used in specific applications on high-voltage systems.

Directional Relays A directional relay is able to distinguish between current flowing in one direction and current flowing in the opposite direction. The relay responds to the phase angle difference between the actuating current and a reference current (or voltage) called the polarizing quantity.

Directional relaying is typically used in conjunction with some other relay, usually the overcurrent relay to improve its selectivity.

Differential Relays The operation of a differential relay is based on the vector difference of two or more similar electrical actuating quantities. The most common application is current differential relaying, in which the current entering and the current leaving the protected element are compared. If the difference exceeds the pickup value of the relay, it operates to trip the breakers to isolate the element.

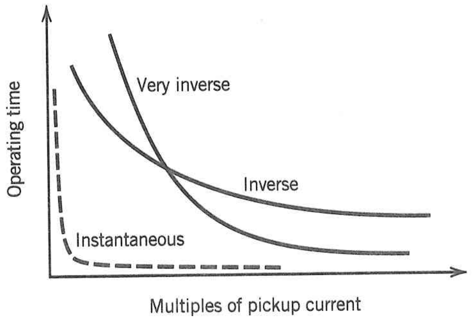


FIGURE 11.20 Time characteristics of overcurrent relays.

Typical differential relaying employing an overcurrent relay is shown in Fig. 11.21. Identical current transformers are placed at both ends of the protected element, and the CT secondaries are connected in parallel with an overcurrent relay. The directions of current flows shown in Fig. 11.21 are those corresponding to normal load conditions or to a fault external to the protected element. Thus, it is seen that the CT secondary currents merely circulate between the CTs, and no current flows in the overcurrent relay.

Suppose a fault occurs on the protected element as shown in Fig. 11.22. The short-circuit currents flow into the fault, and the CT secondary currents no longer circulate. The vectorial sum of the CT secondary currents flows through the overcurrent relay and causes the relay to disconnect the element from the system.

Even though the current transformers used for the differential relay are identical, the secondary currents may not be identical because of CT transformation inaccuracies. Thus, the secondary currents will no longer merely circulate for normal load conditions or for external faults. The differential current that will flow through the overcurrent relay may be sufficient for the relay to pick up and cause false tripping of the circuit breakers.

Percentage-Differential Relays The difficulty encountered in differential relaying due to CT errors is eased by the use of a percentage-differential relay. This type of relay has an operating coil and two restraining coils. The operating current is proportional to $(I_A - I_B)$ and must exceed a certain percentage of the restraining current, which is proportional to $\frac{1}{2}(I_A + I_B)$, before the relay will operate.

Distance Relays In a distance relay, a voltage and a current are balanced against each other and the relay responds to the ratio of the voltage to the current, which is the impedance of the transmission line from the relay location

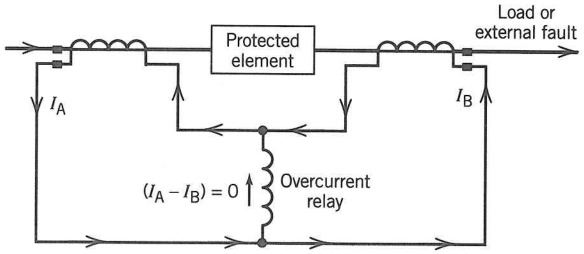


FIGURE 11.21 Differential relaying.

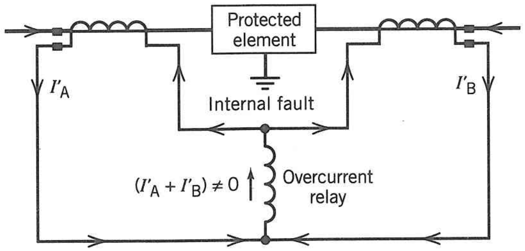


FIGURE 11.22 Fault currents in a differential relay.

to the point of interest. The impedance may be used to measure distance along a transmission line, hence the name distance relay.

This distance relay is useful because it is able to differentiate between a fault and normal operating conditions and to differentiate between faults in a specific area and a fault in a different part of the system. The operation of the distance relay is limited to a certain range of pickup values of impedance. The distance relay picks up whenever the measured impedance is less than or equal to the selected pickup value of impedance.

There are several types of distance relays, including impedance, reactance, and mho relays. The mho relay has an inherent directional characteristic; that is, it responds to or "sees" faults in only one direction. On the other hand, the impedance and the reactance relays see faults in all directions. Thus, a directional relay is commonly used together with the impedance relay, and a mho relay is used as a starting unit for the reactance relay.

Pilot Relays Pilot relaying is a means of communicating information from the end of a protected line to the protective relays at both line terminals. The relays determine whether a fault is internal or external to the protected line. The communication channel, or pilot, is used to transmit this information between line terminals. If the fault is internal to the protected line, all the circuit breakers at the terminals of the line are tripped in high speed. If the fault is external to the protected line, the tripping of the circuit breakers is prevented, or blocked.

Three types of pilots are commonly used for protective relaying: wire, power line carrier, and microwave pilot.

A wire pilot consists of a twisted pair of copper wires of the telephone line type. It may be leased from the telephone company, or it may be owned by the electric utility.

The power line carrier is the most commonly used pilot for protective relaying. In this type of pilot, a low-voltage, high-frequency current (30 to 300 kHz) is transmitted along one phase of the high-voltage power line to a receiver at the other end of the protected line. Line traps, located at both line terminals, serve to contain a carrier signal inside the zone of the protected line.

The microwave pilot is an ultra-high-frequency radio system operating above 900 MHz. In this pilot, transmitters and receivers operate the same way as in power line carrier pilot; however, line traps are replaced by a line-of-sight antenna.

11.3.5 Applications of Protection Systems

The applications of the different types of protection systems for the protection of various types of equipment and transmission lines are described in this section. These discussions are confined to protection for the high-voltage, bulk-power system components.

Generator Protection The protection system provided to the synchronous generator must be able to detect any abnormal condition immediately and act quickly to prevent damage to the generator and minimize the effect on the power system. Synchronous generators are provided with protection against various disturbances, including short circuits in the stator windings, loss of field excitation, stator and rotor overheating, and overspeed. Short-circuit protection of the stator windings is of primary concern and is discussed in this section.

When a short circuit occurs between stator windings of a synchronous generator, or between a stator winding and ground, the protection system should quickly trip the main circuit breaker to disconnect the machine from the rest of the system and at the same time disconnect the field winding from the exciter.

The best stator winding short-circuit protection is provided by a percentage-differential relay, which is shown in Fig. 11.23. It may be noted that the

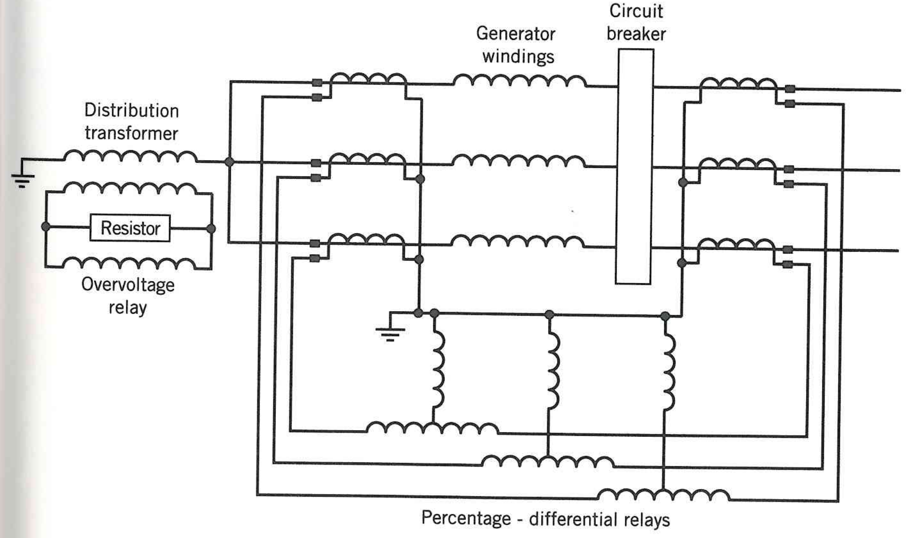


FIGURE 11.23 Generator protection.

relaying protects against a three-phase fault, or line-to-line fault, in the stator windings.

When the synchronous generator neutral is grounded through a high resistance, the short-circuit current for a single line-to-ground fault is much less than the short-circuit currents for faults involving the phase windings. Thus, the ground fault current may not be detected by the differential relaying protection. An overvoltage relay connected across the grounding resistor would be able to detect the increased voltage across the resistor in the presence of a ground fault, and the overvoltage relay will operate.

Transformer Protection The protection system provided for a power transformer depends primarily on its capacity and its voltage rating. Thus, for small transformers with capacities up to about 2 MVA, power fuses are deemed to be adequate. For larger transformers, with capacities greater than 10 MVA, percentage-differential relays with harmonic restraint are recommended. The one-line diagram of the protection of a three-winding transformer using a three-winding percentage-differential relay is illustrated in Fig. 11.24.

The differential relaying protection must satisfy two basic requirements:

1. The protection must not operate for normal load conditions and faults external to the transformer.
2. The relays must operate to trip the circuit breakers for an internal fault that is severe enough to cause direct damage to the transformer.

Three-phase transformers with Y-Δ–connected windings require further consideration. The primary and secondary currents of such transformers normally differ not only in magnitude but also in phase angles because of the inherent phase shifts in Y-Δ or Δ-Y connections. The current transformers must, therefore, be connected in such a manner that the CT secondary line currents as seen by the protective relays are equal under normal operating conditions or for external faults. The correct magnitude relationship is obtained by the proper choice of CT ratios and, if necessary, the use of an autotransformer in the CT secondary circuit. The correct phase-angle relationship is obtained by connecting the CTs on the wye-connected side of the transformer in delta and the CTs on the delta-connected side in wye. In this way, the CT connections are able to compensate for the phase shift introduced by the Y-Δ or Δ-Y connection. The design of the protection for a Y-Δ transformer using percentage-differential relays is illustrated in Fig. 11.25 and the following example.

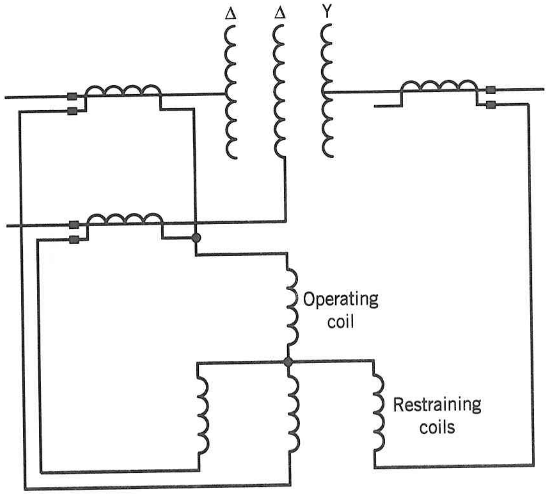


FIGURE 11.24 Three-winding transformer protection.

EXAMPLE 11.8

Design the protection of a three-phase, 50-MVA, 230/34.5-kV power transformer using available standard CT ratios. The high-voltage side is Y connected

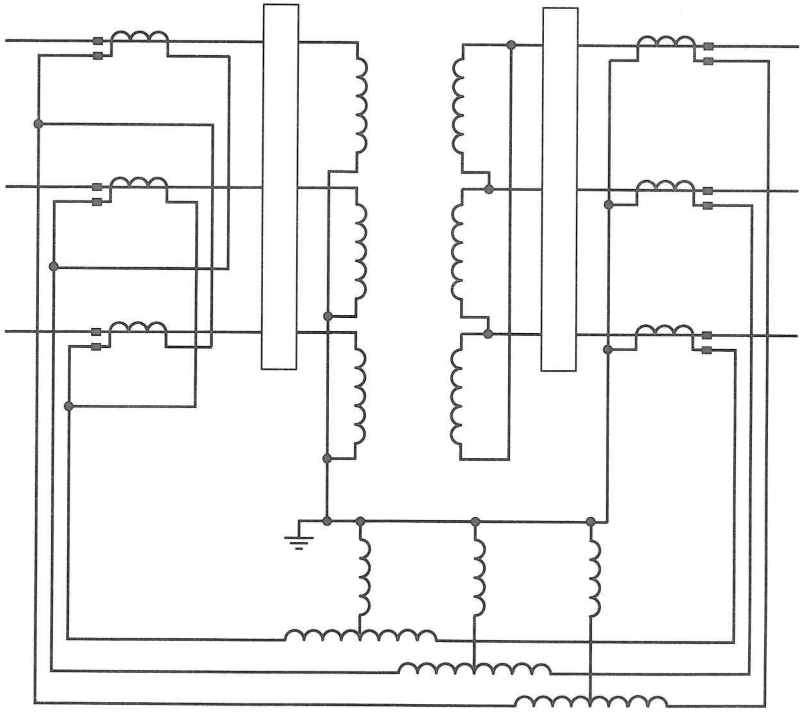


FIGURE 11.25 Y-Δ transformer protection.

and the low-voltage side is Δ connected. Specify the CT ratios, and show the three-phase wiring diagram indicating the CT polarities. Determine the currents in the transformer and the CTs. Specify the rating of an autotransformer, if one is needed.

Solution When the transformer is carrying rated load, the line currents on the high-voltage side and low-voltage side are

$$I_{\mathrm{HV}} = \frac{50{,}000}{\sqrt{3}(230)} = 125.5 \text{ A}$$

$$I_{\mathrm{LV}} = \frac{50{,}000}{\sqrt{3}(34.5)} = 836.7 \text{ A}$$

The CTs on the low-voltage side are connected in wye, and the CT ratio selected for this side is 900/5. The current in the leads flowing to the percentage-differential relay on this side is equal to the CT secondary current and is given by

$$I_{\mathrm{LV\ lead}} = 836.7\left(\frac{5}{900}\right) = 4.65 \text{ A}$$

The current in the leads to the relay from the low-voltage side must be balanced by an equal current in the leads connected to the Δ-connected CTs on the high-voltage side. This requires a CT secondary current equal to

$$I_{\mathrm{CT\ sec}} = \frac{4.65}{\sqrt{3}} = 2.68 \text{ A}$$

To obtain a CT secondary current of 2.68 A, the CT ratio of the high-voltage CTs is chosen as

$$\text{CT ratio} = \frac{125.5}{2.68} = 46.8$$

The nearest available standard CT ratio is 250/5. If this CT ratio is selected, the CT secondary currents will actually be

$$I_{\mathrm{CT\ sec}} = 125.5\left(\frac{5}{250}\right) = 2.51 \text{ A}$$

Therefore, the currents in the leads to the Δ-connected CTs from the percentage-differential relays will be

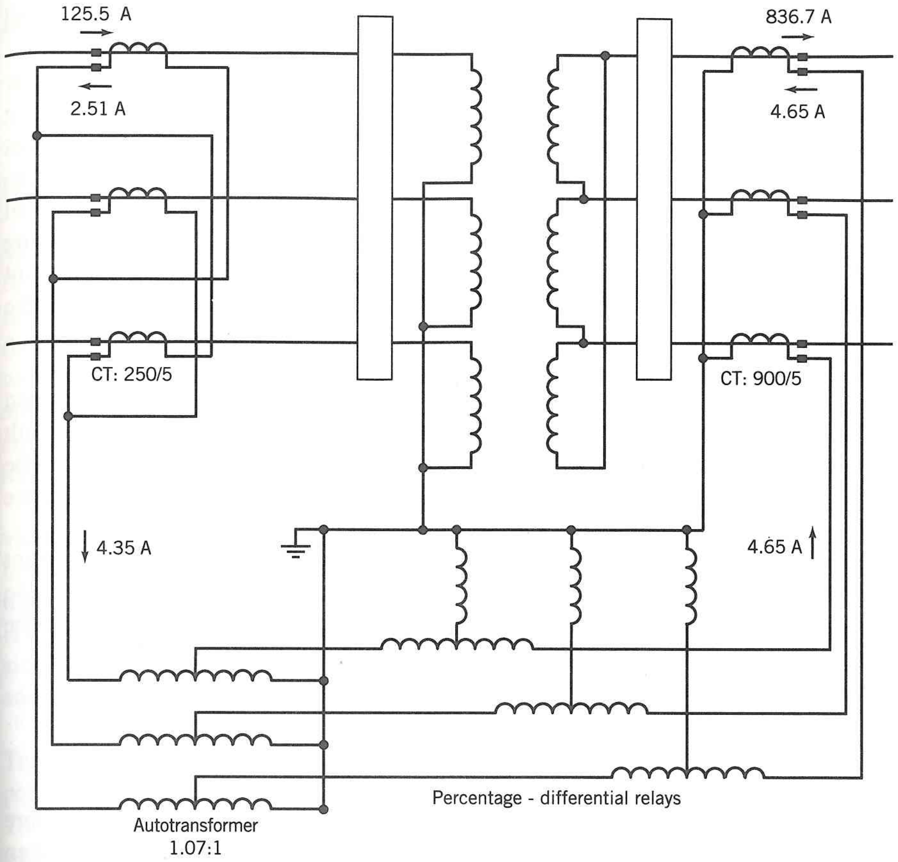


FIGURE 11.26 Y-Δ transformer protection of Example 11.8.

It is seen that the currents in the leads on both sides of the percentage-differential relay are not balanced. This condition cannot just be ignored because it could lead to improper tripping of the circuit breaker for an external fault. This problem can be solved by using an autotransformer as shown in Fig. 11.26. The autotransformer should have a turns ratio of

$$N_{\mathrm{autotransformer}} = \frac{4.65}{4.35} = 1.07$$

In the design of the transformer protection of Example 11.8, the magnetizing current of the transformer has been assumed to be negligible. This is a reasonable assumption during normal operating conditions because the magnetizing current is a small percentage of the rated load current. However, when a transformer is being energized, it may draw a large magnetizing inrush current in the primary, causing an unbalance in current, and the differential relay will interpret this an internal fault and will pick up to trip the circuit breakers.

To prevent the protection system from operating and tripping the transformer during its energization, percentage-differential relaying with harmonic restraint is recommended. This is based on the fact that the magnetizing inrush current has high harmonic content, whereas the fault current consists mainly of fundamental frequency sinusoid. Thus, the current supplied to the restraining coil consists of the fundamental and harmonic components of the normal restraining current of $(I_A + I_B)/2$, plus another signal proportional to the harmonic content of the differential current $(I_A - I_B)/2$. Only the fundamental frequency of the differential current is supplied to the operating coil of the relay.

Bus Protection The sum of the currents flowing through the lines connected to the same bus is equal to zero for normal operating conditions or for a fault external to the bus. Hence, differential relaying using overcurrent relays can be used to provide protection against bus faults. This is illustrated in the one-line diagram shown in Fig. 11.27.

All of the current transformers are connected in parallel and an overcurrent relay is connected across their output. When there is no fault on the bus, the line currents are indicated by the solid arrows, and the current through the overcurrent relay is given by

$$I_R = \frac{1}{\text{CT ratio}}(I_1 + I_2 - I_3 - I_4) = 0$$

Therefore, no current flows through the relay. On the other hand, when there is a fault at the bus, currents I_3 and I_4 reverse directions as shown by the broken arrows; thus, the current through the relay is now given by

$$I_R = \frac{1}{\text{CT ratio}}(I_1 + I_2 + I_3 + I_4) > 0$$

Hence, the overcurrent relay picks up to trip all circuit breakers.

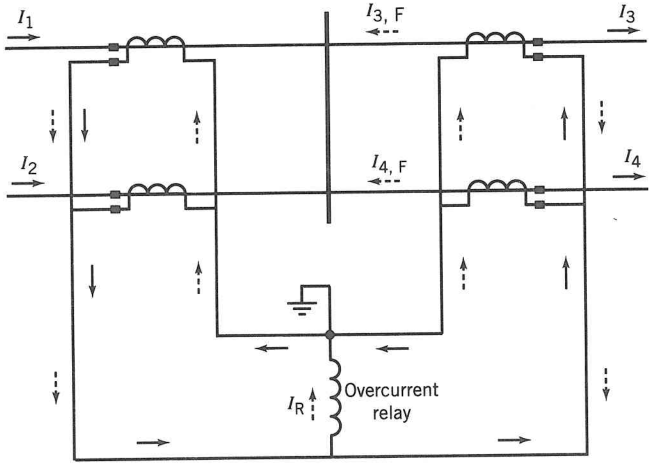


FIGURE 11.27 Bus protection using an overcurrent relay.

Transmission Line Protection Transmission lines provide the means for bringing the electric energy from the generating plants to the major substations, from these substations to the load center substations, and ultimately to the individual consumers. Transmission lines travel over wide-open spaces, as well as thickly populated metropolitan areas. Because of these long distances and extensive exposure to nature and human accidents, most of the short circuits that occur in power systems are on overhead transmission and distribution lines.

For the low-voltage distribution circuits, the protection system usually provided consists of circuit reclosers and power fuses that act as relays and circuit breakers combined. For the protection of medium-voltage and high-voltage transmission lines, separate relays and circuit breakers are employed. Since HV and EHV transmission lines are the major means of bulk-power transmission, the protection of these transmission lines is designed to be more reliable and more selective, and it is also more expensive.

Transmission Line Protection by Overcurrent Relays Consider that the portion of the power system to be protected is radial as shown in Fig. 11.28. The generator connected to bus 1 represents the rest of the power system, and it supplies loads at buses 1, 2, 3, 4, and 5 through four transmission lines. Since the short-circuit current comes from the left side of each line, it is sufficient to provide only one circuit breaker at the sending end for each line. So that service disruption is minimized, for a fault on line 4–5, only circuit breaker CB_4 should be tripped; for a fault on the line 3–4, only breaker CB_3 should be opened; for a fault on the line 2–3, only CB_2 should be tripped; and so on.

The short-circuit current due to a fault on any of the lines depends on the fault location and the type of fault. Since the total impedance increases with the distance from the generator to the fault, the short-circuit current is inversely proportional to this distance.

Overcurrent relays are used mainly to provide protection for subtransmission and distribution lines. Two forms of overcurrent protection are

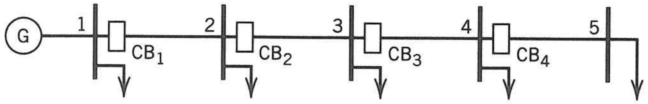


FIGURE 11.28 A radial power system.

provided: primary protection for the line itself and backup protection for an adjacent line. Two types of overcurrent relay units are used: time overcurrent relay and instantaneous relay.

The time overcurrent relays at each of the four buses 1, 2, 3, and 4 provide primary protection for their own line segment and provide remote backup protection to adjacent line downstream from the relay location. Thus, the relay at bus 1 provides primary protection for line 1–2 and also provides backup protection for line 2–3; the relay at bus 2 provides primary protection for line 2–3 and backup protection for line 3–4; and so forth.

When the relay at bus 1 provides backup protection for line 2–3, it must be adjusted to be selective with the primary relaying at bus 2. The relay at bus 2 is expected to operate first for a fault on line 2–3 before the relay at bus 1 operates. The operating times of the relays at the different buses are shown in Fig. 11.29. Thus, for the fault shown, the relay at bus 4 picks up to trip circuit breaker CB_4. If CB_4 fails to open for any reason, the fault current persists, and after a certain time delay the relay at bus 3 picks up to trip CB_3. The relay at bus 2 is selective with the relay at bus 3; thus, if both CB_3 and CB_4 failed to open, the relay at bus 2 would pick up and trip CB_2 after a longer time delay.

For setting the pickup values and the selectivity clearances between the time overcurrent relays for backup protection, there are four criteria to consider:

1. The relay must be able to pick up for the minimum short-circuit currents for which the relay is designed to protect.
2. The relay should not pick up for normal load conditions. A common practice is to set the minimum pickup value equal to twice the peak load current in order to carry emergency peak loads and pick up cold load.
3. The relay nearer the source providing backup protection for the downstream relay must pick up for one third of the minimum current seen by the latter.

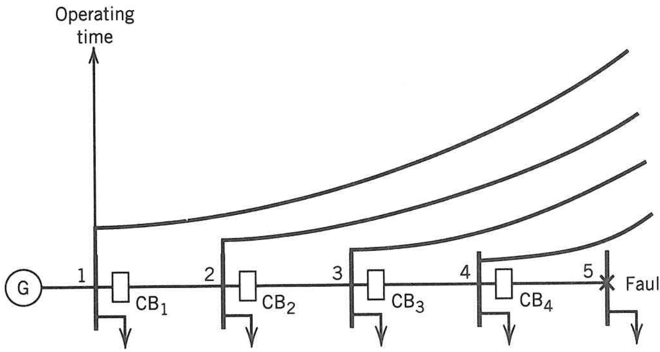


FIGURE 11.29 Overcurrent protection of a radial power system.

4. The relay nearer the source providing backup protection for the downstream relay should pick up for the maximum current seen by the latter but no sooner than (a selectivity clearance of) at least 0.3 s.

The instantaneous overcurrent relay unit provides primary protection for its own protected line segment to supplement the time-delay overcurrent relay unit. The instantaneous unit operates quickly with no intentional time delay, so it should be adjusted such that it does not operate for a fault on neighboring lines. The pickup value is usually adjusted so that the instantaneous relay is able to detect a fault occurring at a distance of up to 80% of the line segment. The operating times of the instantaneous relay and the time-delay units are shown in Fig. 11.30.

The complete protection system for a line consists of three overcurrent relays for phase fault protection and one overcurrent relay for ground fault protection. This protection system is shown in Fig. 11.31.

Transmission Line Protection by Distance Relays To improve reliability, the generating plants, transmission lines, and substations of a high-voltage power system are interconnected to form a network. Overcurrent relay protection can no longer be used because coordinating the relays becomes a difficult, if not impossible, task.

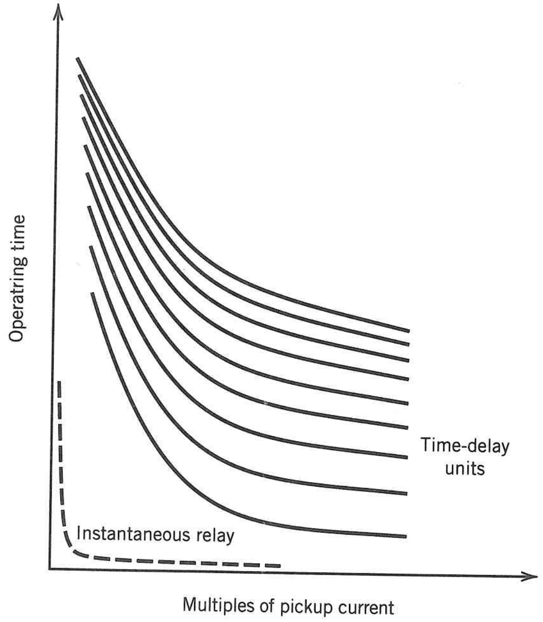


FIGURE 11.30 Typical time characteristics of an overcurrent relay.

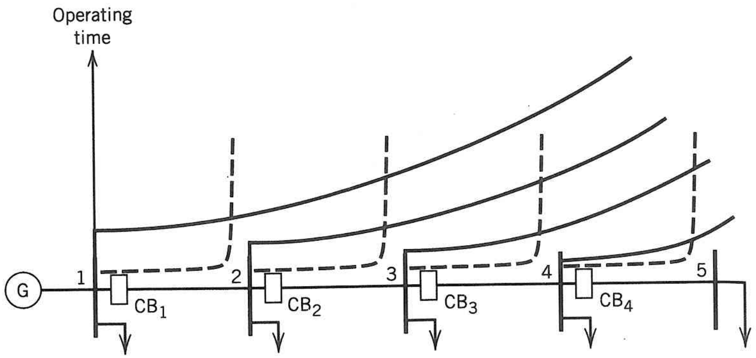


FIGURE 11.31 Complete overcurrent protection of a radial system.

Protection of transmission lines connected as a network can be provided by distance relays. These distance relays provide phase fault protection for the line, while an overcurrent relay provides ground fault protection.

Distance relays provide primary protection for a line section and backup protection for an adjacent line. The value of impedance at the farthest fault location for which a distance relay picks up is called its "reach." One distance relay contains three distance relay units responding to three independently adjustable pickup impedance values (or zones of protection) with three independently adjustable time delays.

The primary impedance corresponding to a particular fault location, or relay unit reach, is converted to a secondary value that is used to adjust the phase or ground distance relay. This secondary impedance value is given by

$$Z_{\text{sec}} = Z_{\text{pri}}\left(\frac{\text{CT ratio}}{\text{PT ratio}}\right)$$

The first-zone, or high-speed zone, unit of the distance relay has a reach of up to 80% to 90% of the length of the transmission line. This relay unit operates with no time delay.

The second-zone unit of the distance relay provides protection for the rest of the line beyond the reach of the first-zone unit. This second-zone unit has a reach of at least 20% of the adjoining line section. Because this unit "sees" faults in the adjoining line, it must be selective with the first-zone unit of the adjacent line. Therefore, this second-zone unit is provided with a time delay of about 0.2 s to 0.5 s.

The third-zone unit of the distance relay provides backup protection for the rest of the adjoining line section. This unit is adjusted to reach beyond the end of the adjoining line to ensure that backup protection is provided for the full line section. The time delay provided for this unit is usually about 0.4 s to 1.0 s. The protection of a portion of a power system by using distance relays

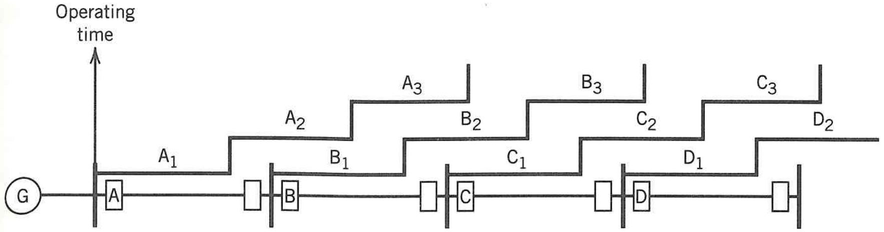


FIGURE 11.32 Line protection by distance relays.

*11.4 TRANSIENT STABILITY

Power system stability refers to the ability of the various synchronous machines in the system to remain in synchronism, or stay in step, with each other following a disturbance. Stability may be classified as steady-state, dynamic, or transient stability.

Steady-state stability refers to the ability of the various machines to regain and maintain synchronism after a small and slow disturbance, such as a gradual change in load. Transient stability is stability after a sudden large disturbance such as a fault, loss of a generator, a switching operation, and a sudden load change. Dynamic stability is the case between steady-state and transient stability, and the period of study is much longer so that the effects of regulators and governors may be included.

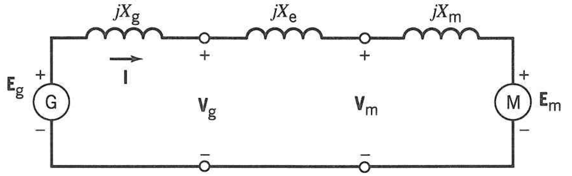
Consider the two-machine power system shown in Fig. 11.33. Each machine may be represented as a constant emf in series with a synchronous reactance and negligible resistance. Assume that the machine on the left acts as a generator and the other machine acts as a motor. The emfs may be expressed in polar form as follows:

$$\mathbf{E}_{\text{g}} = E_{\text{g}}\angle\delta$$
$$\mathbf{E}_{\text{m}} = E_{\text{m}}\angle 0^\circ \qquad (11.58)$$

The current flowing in the circuit is given by

$$\mathbf{I} = \frac{\mathbf{E}_{\text{g}} - \mathbf{E}_{\text{m}}}{jX_{\text{T}}} = \frac{E_{\text{g}}\angle\delta - E_{\text{m}}\angle 0^\circ}{jX_{\text{T}}} \qquad (11.59)$$

where $X_{\text{T}} = X_{\text{g}} + X_{\text{e}} + X_{\text{m}}$.



The real power delivered by the generator to the motor is found as follows:

$$
\begin{aligned}
P &= \mathrm{Re}[\mathbf{E}_{\mathrm{g}}\mathbf{I}^{*}] \\
&= \mathrm{Re}\left[(E_{\mathrm{g}}\angle\delta)\left(\frac{E_{\mathrm{g}}\angle\delta - E_{\mathrm{m}}\angle 0^{\circ}}{jX_{\mathrm{T}}}\right)^{*}\right] \\
&= \mathrm{Re}\left[\frac{E_{\mathrm{g}}^{2}}{X_{\mathrm{T}}}\angle 90^{\circ} - \frac{E_{\mathrm{g}}E_{\mathrm{m}}}{X_{\mathrm{T}}}\angle\delta + 90^{\circ}\right] \\
&= \frac{E_{\mathrm{g}}E_{\mathrm{m}}}{X_{\mathrm{T}}}\sin\delta \qquad (11.60)
\end{aligned}
$$

Thus, it is seen that the power supplied by the generator to the motor varies with the sine of the phase angle difference of the two emfs, which is also the displacement angle between the two rotors.

The maximum power P_{max} that can be transmitted at steady state from the generator to the motor with the total reactance X_{T} is found from Eq. 11.60 for the case $\delta = 90^{\circ}$. That is,

$$P_{\mathrm{max}} = \frac{E_{\mathrm{g}}E_{\mathrm{m}}}{X_{\mathrm{T}}} \qquad (11.61)$$

P_{max} is called the steady-state stability limit. Its value can be raised only by increasing either of the two internal voltages by adjusting their respective excitations or by decreasing the series reactance of the transmission line connecting the two machines.

The plot of the transmitted power versus the displacement angle is called the power-angle curve, and it is illustrated in Fig. 11.34. The graph indicates that the system is stable over the operating region $-90^{\circ} < \delta < 90^{\circ}$, where the derivative of P with respect to δ is positive, that is, $dP/d\delta > 0$; this condition implies that an increase in displacement angle results in an increase in transmitted power.

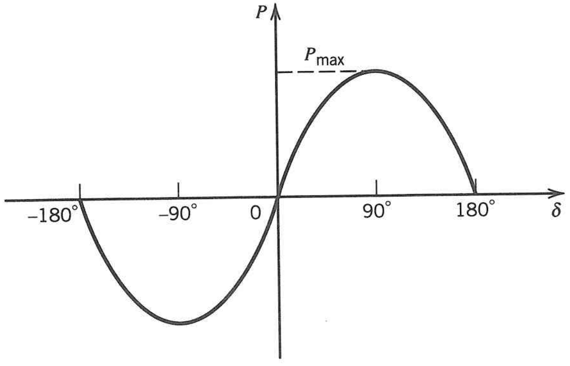


FIGURE 11.34 Power-angle curve.

Consider the two-machine system operating at steady state at the point A on the power-angle curve of Fig. 11.35. Assume that the generator is delivering electrical power P_{o} at an angle δ_{o} to the motor, which is driving a mechanical load connected to its shaft.

Case 1. Suppose that a small increment of shaft load is added to the motor. This net torque tends to retard the motor and its speed decreases, causing an increase in δ. The input power increases correspondingly until equilibrium is obtained at a new operating point B, higher than A.

Case 2. Suppose that motor load is increased gradually until point C of maximum power is reached. If additional load is applied to the motor, δ increases as before and goes beyond 90°. This time, however, instead of an increase in the input power, the input power decreases, which further increases the net retarding torque. This torque retards the motor even faster until it pulls out of step.

Case 3. Suppose a large increment of load is suddenly applied to the motor. The deficiency in input will be supplied temporarily by the decrease in kinetic energy. The motor will slow down, increasing δ and the power input. Assuming that the new load is less than P_{max}, δ will increase to a new value such that the motor input equals the motor load. When this is reached, the motor is still running slow, thus increasing δ beyond its proper value and producing an accelerating torque that increases the motor speed. However, when the motor regains normal speed, δ may have gone beyond point D so that the motor input is less than the load. This leads to motor pull-out.

Case 4. Assume for this case that the sudden incremental load is not too great. The motor will regain its normal speed before δ becomes too large. The motor speed will continue to increase because of the net accelerating torque

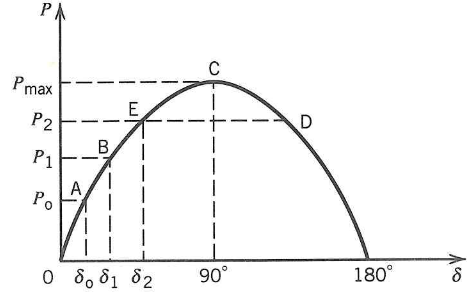


FIGURE 11.35 Two machine system operating states.

and will become greater than normal. When this happens, δ will decrease and again approach its proper value. These oscillations will later die out because of damping torques, and the motor will come to a stable operating condition at point E.

The upper limit to the sudden increment in load that the rotor can carry without pulling out of step is called the transient stability limit. This is always lower than the steady-state limit and can have different values depending on the nature and magnitude of the disturbance.

*11.4.1 Swing Equation

The motion of a synchronous machine is governed by Newton's law of rotation, which states that the product of the moment of inertia times the angular acceleration is equal to the net accelerating torque. Mathematically, this may be expressed as follows:

$$J\alpha = T_a = T_m - T_e \tag{11.62}$$

Equation 11.62 may also be written in terms of the angular position as follows:

$$J\,\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \tag{11.63}$$

where

J = moment of inertia of the rotor

T_a = net accelerating torque or algebraic sum of all torques acting on the machine

T_m = shaft torque corrected for the rotational losses including friction and windage and core losses

T_e = electromagnetic torque

By convention, the values of T_m and T_e are taken as positive for generator action and negative for motor action.

For stability studies, it is necessary to find an expression for the angular position of the machine rotor as a function of time t. However, because the displacement angle and relative speed are of greater interest, it is more convenient to measure angular position and angular velocity with respect to a synchronously rotating reference frame with a synchronous speed of ω_{sm}. Thus, the rotor position may be described by the following:

$$\theta_m = \omega_{sm}t + \delta_m \tag{11.64}$$

The derivatives of θ_m may be expressed as

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt} \tag{11.65}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \tag{11.66}$$

Substituting Eq. 11.66 into Eq. 11.63 yields

$$J\,\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \tag{11.67}$$

Multiplying Eq. 11.67 by the angular velocity of the rotor transforms the torque equation into a power equation. Thus,

$$J\omega_m\,\frac{d^2\delta}{dt^2} = \omega_m T_a = \omega_m T_m - \omega_m T_e \tag{11.68}$$

Replacing $\omega_m T$ by P and $J\omega_m$ by M, the so-called swing equation is obtained. The swing equation describes how the machine rotor moves, or swings, with respect to the synchronously rotating reference frame in the presence of a disturbance, that is, when the net accelerating power is not zero.

$$M\,\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \tag{11.69}$$

where

$M = J\omega$ = inertia constant

$P_a = P_m - P_e$ = net accelerating power

$P_m = \omega T_m$ = shaft power input corrected for the rotational losses

$P_e = \omega T_e$ = electrical power output corrected for the electrical losses

It may be noted that the inertia constant was taken equal to the product of the moment of inertia J and the angular velocity ω_m, which actually varies during a disturbance. Provided the machine does not lose synchronism, however, the variation in ω_m is quite small. Thus, M is usually treated as a constant.

Another constant, which is often used because its range of values for particular types of rotating machines is quite narrow, is the so-called normalized inertia constant H. It is related to M as follows:

$$H = \frac{1}{2}\frac{M\omega_{sm}}{S_{\text{rated}}}\ \text{MJ/MVA} \tag{11.70}$$

Solving for M from Eq. 11.70 and substituting into 11.69 yields the swing equation expressed in per unit. Thus,

$$\frac{2H}{\omega_{\mathrm{sm}}}\frac{d^2\delta_{\mathrm{m}}}{dt^2} = \frac{P_{\mathrm{a}}}{S_{\mathrm{rated}}} = \frac{P_{\mathrm{m}} - P_{\mathrm{e}}}{S_{\mathrm{rated}}} \tag{11.71}$$

It may be noted that the angle δ_{m} and angular velocity ω_{m} in Eq. 11.71 are expressed in mechanical radians and mechanical radians per second, respectively. For a synchronous generator with p poles, the electrical power angle and radian frequency are related to the corresponding mechanical variables as follows:

$$\begin{aligned}\delta(t) &= \frac{p}{2}\delta_{\mathrm{m}}(t)\\ \omega(t) &= \frac{p}{2}\omega_{\mathrm{m}}(t)\end{aligned} \tag{11.72}$$

Similarly, the synchronous electrical radian frequency is related to synchronous angular velocity as follows:

$$\omega_{\mathrm{s}} = \frac{p}{2}\omega_{\mathrm{sm}} \tag{11.73}$$

Therefore, the per-unit swing equation of Eq. 11.71 may be expressed in electrical units and takes the form of Eq. 11.74.

$$\frac{2H}{\omega_{\mathrm{s}}}\frac{d^2\delta}{dt^2} = P_{\mathrm{a}} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{11.74}$$

Depending on the unit of the angle δ, Eq. 11.74 takes the form of either Eq. 11.75 or Eq. 11.76. Thus, the per-unit swing equation takes the form

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{\mathrm{a}} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{11.75}$$

when δ is in electrical radians, or

$$\frac{H}{180 f}\frac{d^2\delta}{dt^2} = P_{\mathrm{a}} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{11.76}$$

when δ is in electrical degrees.

When a disturbance occurs, an unbalance in the power input and power output ensues, producing a net accelerating torque. The solution of the swing equation in the form of the differential equation of (11.75) or (11.76) is appropriately called the swing curve $\delta(t)$.

*11.4.2 Single Machine-to-Infinite Bus

Consider the system shown in Fig. 11.36, which consists of a single machine connected to an infinite bus through an external reactance. The synchronous machine is driven by a constant-speed prime mover.

The electrical power output of the synchronous generator may be found in the same manner as in Eq. 11.60 and may be expressed as follows:

$$P_{\mathrm{e}} = P_{\max}\sin\delta \tag{11.77}$$

Substituting the expression for the electrical power output of the generator and rearranging the terms in the swing equation yield the following:

$$\frac{2H}{\omega_{\mathrm{s}}}\frac{d^2\delta}{dt^2} + P_{\max}\sin\delta = P_{\mathrm{m}} \tag{11.78}$$

It is seen that the resulting differential equation is nonlinear. Except for a few cases, it may not be possible to solve the equation analytically to obtain a closed form of the solution $\delta(t)$. It is, therefore, usually necessary to use a numerical technique to solve the differential equation (11.75) or (11.76).

EXAMPLE 11.9

A 500-MVA, 20-kV, 60-Hz, four-pole synchronous generator is connected to an infinite bus through a purely reactive network. The generator has an inertia constant H = 6.0 MJ/MVA and is delivering power of 1.0 per unit to the infinite bus at steady state. The maximum power that can be delivered is 2.5 per unit. A fault occurs that reduces the generator output power to zero.

- **a.** Find the angular acceleration.
- **b.** Find the speed in rev/min at the end of 15 cycles.
- **c.** Find the change in the angle δ at the end of 15 cycles.

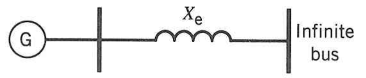


FIGURE 11.36 Single machine-to-infinite bus system.

Solution

a. Because the generator is initially operating at steady state, the mechanical input power is equal to the electrical power output prior to the fault. That is,

$$P_m = P_e = 1.0 \text{ pu}$$

Therefore, the accelerating torque is found by using Eq. 11.76.

$$\begin{aligned}\alpha &= \frac{d^2\delta}{dt^2} = \frac{180f}{H}(P_m - P_e) \\ &= \frac{(180)(60)}{6.0}(1.0 - 0.0) = 1800 \text{ elec. degrees/s}^2\end{aligned}$$

For a four-pole machine,

$$\begin{aligned}\alpha = \frac{d^2\delta}{dt^2} &= \frac{180f}{H}(P_m - P_e) = \frac{(180)(60)}{6.0}(1.0 - 0.0) \\ &= 1800 \text{ elec. degrees/s}^2 \\ &= \left(\frac{2}{p}\right)1800 = 900 \text{ mech. degrees/s}^2 \\ &= 900\left(\frac{60 \text{ s/min}}{360°/\text{rev}}\right) = 150 \text{ rpm/s}\end{aligned}$$

b. A 15-cycle interval is equivalent to a time interval of

$$t = (15)(1/60) = 0.25 \text{ s}$$

The synchronous speed of the machine is found as follows:

$$\omega_{sm} = 120f/p = (120)(60)/4 = 1800 \text{ rpm}$$

Integrating the angular acceleration α expressed in rpm/s starting from $t = 0$ up to a final time $t = 0.25$ s yields

$$\omega_m = \omega_{sm} + \alpha t = 1800 + (150)(0.25) = 1837.5 \text{ rpm}$$

c. The machine is initially operating at the angle δ_0, which is found from the following expression:

Therefore, the initial angle is

$$\delta_0 = \sin^{-1}(P_0/P_{\max}) = \sin^{-1}(1.0/2.5) = 23.58°$$

Integrating the angular acceleration expressed in electrical degrees/s^2 twice from $t = 0$ up to the final time of $t = 0.25$ s gives the value of the angle. That is,

$$\delta = \delta_0 + \tfrac{1}{2}\alpha t^2 = 23.58° + \tfrac{1}{2}(1800)(0.25)^2 = 79.83°$$

DRILL PROBLEMS

D11.10 A three-phase, 350-MVA, 13.8-kV, 60-Hz, four-pole, synchronous generator has an inertia constant of $H = 6$ MJ/MVA. The machine is connected to an infinite bus and is operating at steady state. Determine the kinetic energy stored in the rotor at synchronous speed.

D11.11 The prime mover of the generator of Problem D11.10 is supplying an input power (net of the rotational losses) of 500,000 hp. The electrical power developed suddenly decreases to 280 MW.

a. Determine the angular acceleration.

b. If the acceleration computed in part (a) remains constant for a period of 20 cycles, find the change in the rotor angle δ in electrical degrees and the speed in rpm at the end of 20 cycles.

D11.12 The generator of Problem D11.10 is initially operating at steady state and is delivering rated MVA at 0.8 power factor lagging. An external fault suddenly occurs that reduces the generator power output by 60%. Neglect the machine losses, and assume that the power input to the shaft remains constant. Find the accelerating torque in newton-meters when the fault occurs.

D11.13 For the generator of Problem D11.10, the mechanical power input P_m is initially 0.8 pu and is assumed to remain constant. The maximum power $P_{\max}$ that the generator can develop is 2.25 pu. An external fault suddenly occurs that reduces the generator output power to zero. Find the change in angle δ at the end of 30 cycles.

*11.4.3 Multimachine Systems

Consider two finite synchronous machines swinging with respect to each other.

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{a1} = P_{m1} - P_{e1} \tag{11.79}$$

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{a2} = P_{m2} - P_{e2} \tag{11.80}$$

Subscripts 1 and 2 pertain to machines 1 and 2, respectively. P_{m1} and P_{m2} are the mechanical power inputs from the respective prime movers of the two machines. P_{e1} and P_{e2} are the electrical power outputs of the two machines, which may be expressed in terms of the power flow equations of Section 10.2 as follows:

$$P_{e1} = V_1V_2Y_{12}\cos(\delta_1 - \delta_2 - \theta_{12}) + V_1^2Y_{11}\cos\theta_{11} \tag{11.81}$$

$$P_{e2} = V_2V_1Y_{21}\cos(\delta_2 - \delta_1 - \theta_{21}) + V_2^2Y_{22}\cos\theta_{22} \tag{11.82}$$

It is seen, therefore, that the swing equations are nonlinear differential equations that are coupled through the electrical power that each synchronous machine delivers. The nonlinearities involve trigonometric functions, and the coupling is dependent on the electrical network connecting the two machines.

In a multimachine system, the electrical outputs of the synchronous machines are functions of the angular positions and, possibly, the angular velocities of all of the synchronous machines in the system. For example, if there were four machines in the system, the swing equations are given by the following:

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}(\delta_1, \delta_2, \delta_3, \delta_4, \omega_1, \omega_2, \omega_3, \omega_4) \tag{11.83}$$

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}(\delta_1, \delta_2, \delta_3, \delta_4, \omega_1, \omega_2, \omega_3, \omega_4) \tag{11.84}$$

$$M_3 \frac{d^2\delta_3}{dt^2} = P_{m3} - P_{e3}(\delta_1, \delta_2, \delta_3, \delta_4, \omega_1, \omega_2, \omega_3, \omega_4) \tag{11.85}$$

$$M_4 \frac{d^2\delta_4}{dt^2} = P_{m4} - P_{e4}(\delta_1, \delta_2, \delta_3, \delta_4, \omega_1, \omega_2, \omega_3, \omega_4) \tag{11.86}$$

Equations 11.83 to 11.86 constitute a system of nonlinear differential equations whose solution requires any of the known numerical integration techniques such as the Runge–Kutta method and various predictor-corrector methods.

* 11.4.4 Computer Solutions

Earlier stability studies analyzed the behavior of a single machine–infinite bus system that involved the simulation of a few ordinary differential equations.

Subsequent studies included a simple representation of the prime mover in which a constant torque is assumed to be applied to the generator shaft. The exciter, the speed governor, and saturation effects were not modeled.

Present transient stability programs allow representation of some of the synchronous machines in greater detail. In the vicinity of the fault, it is a common practice to use a two-axis generator model and to include the excitation and speed-governing systems. These models provide an accurate description of the postfault conditions for a longer time span. Where the simple models may be adequate for "first swing" calculations of about 1.0 s, the more detailed models may be valid for as long as 5.0 s.

Model Formulation The interactions among the various models that are usually included in a modern transient stability program are depicted in Fig. 11.37.

The electrical network assigns the exciter a voltage magnitude that it tries to maintain. The exciter supplies the voltage V_f to the generator field winding. A supplementary signal V_s may be included as part of the input to the exciter to provide positive damping. This input signal may be derived from the angular frequency deviation ω and other auxiliary signals.

The generator is coupled to the electrical network through its bus phasor voltage $\mathbf{V}$ and stator phasor current $\mathbf{I}$. The dynamic motion of the machine rotor is given in terms of the swing angle δ, the angular frequency deviation ω, the mechanical input power P_m, and the electrical output power P_e. Mathematically, this is expressed as follows:

$$\frac{d\delta}{dt} = \omega \tag{11.87}$$

$$\frac{d\omega}{dt} = \frac{1}{M}(P_m - P_e - D\omega) \tag{11.88}$$

where

$M = J\omega =$ inertia constant
$J =$ generator inertia
$D =$ damping coefficient

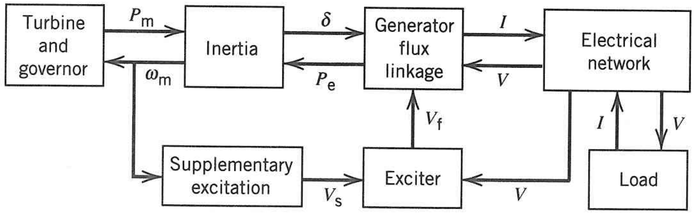


FIGURE 11.37 System models in modern stability studies.

The electrical power delivered by the generator to the external network is given by

$$P_e = \mathrm{Re}[\mathbf{V}\mathbf{I}^*] \tag{11.89}$$

where $\mathbf{V}$ and $\mathbf{I}$ are the generator terminal voltage and current phasors, respectively.

The stator current is decomposed into components along the direct and quadrature axes, $\mathbf{I}_d$ and $\mathbf{I}_q$, respectively. The direct axis is taken along the centerline of the machine pole and the quadrature axis is perpendicular to the direct axis. Thus, the stator current may be expressed as

$$\mathbf{I} = \mathbf{I}_d + \mathbf{I}_q \tag{11.90}$$

The excitation voltage of the synchronous generator at steady state is found as follows:

$$\begin{aligned}\mathbf{E}_a &= \mathbf{V} + jX_q\mathbf{I}_q + jX_d\mathbf{I}_d \\ &= \mathbf{E}_q + j(X_d - X_q)\mathbf{I}_d\end{aligned} \tag{11.91}$$

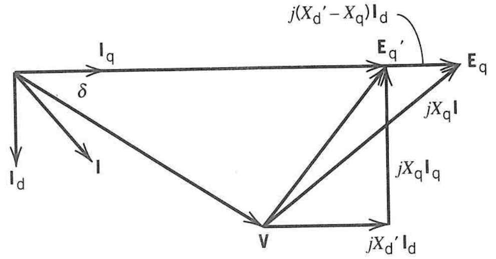
where

$$\mathbf{E}_q = \mathbf{V} + jX_q\mathbf{I} \tag{11.92}$$

During transient conditions, the d-axis transient reactance X'_d is used instead of the synchronous reactance X_d. The q-axis reactance X'_q is the same as X_q because the field is on the direct axis only. Although X_d is normally greater than X_q, X_q is also normally larger than X'_d. The phasor diagram during transient conditions is shown in Fig. 11.38 with $\mathbf{E}_q$ as reference.

From the phasor diagram, the voltage $\mathbf{E}'_q$ behind transient reactance is given by

$$\mathbf{E}'_q = \mathbf{E}_q + j(X'_d - X_q)\mathbf{I}_d \tag{11.93}$$



$\mathbf{E}'_q$ is the voltage proportional to the resultant field flux linkages due to the interaction between the field and armature magnetic fields. Since the field flux linkages cannot change instantaneously after a disturbance, $\mathbf{E}'_q$ also does not change instantaneously. If X'_d is assumed to be equal to X_q, then $\mathbf{E}'_q$ reduces to $\mathbf{E}_q$ and can be calculated as follows:

$$\mathbf{E}'_q = \mathbf{V} + jX'_d\mathbf{I} \tag{11.94}$$

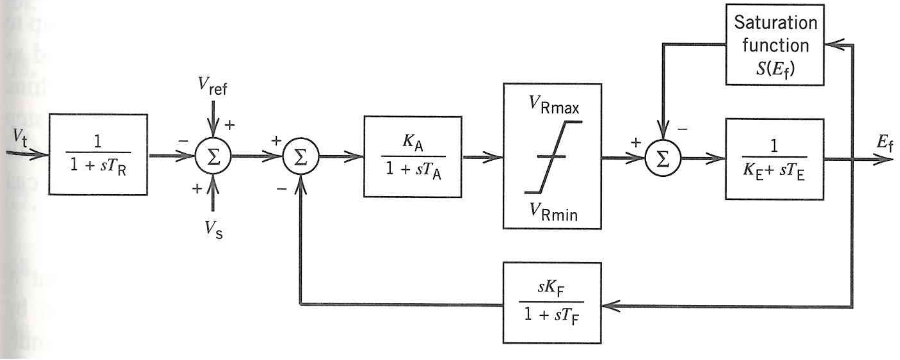
The synchronous generator is thus modeled as a constant-voltage source in series with its transient reactance.

In a typical transient stability program, the exciters of the synchronous machines being studied may be represented in varying complexities. The block diagram of what has come to be known as the IEEE type 1 excitation control system is shown in Fig. 11.39.

The input to the exciter control system is the generator terminal voltage V_t. The first transfer function represents the regulator input filtering; since its time constant T_R is very small, it may be neglected. At the first summing point, the terminal voltage is compared with the reference voltage V_{ref}, and to this error are added the supplementary signals V_s. The error voltage serves as the input to the voltage regulator, which is represented by a transfer function with a gain K_A and a time constant T_A. Maximum and minimum limits are imposed on the regulator output.

The exciter is modeled as a transfer function $1/(K_E + sT_E)$. The saturation function $S(E_f)$, representing the increase in field excitation requirements, may be modeled if it is desired to include the effects of saturation. The transfer function $K_F s/(1 + sT_F)$ in the feedback circuit from the exciter output to the second summing point provides the major loop damping.

The network equations of power flow are used to describe the performance of the electrical network for transient stability studies. The expression for the phasor voltage at bus k was derived in Section 10.2 and is repeated here as Eq. 11.95.



$$\mathbf{V}_k = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{\mathbf{V}_k^*} - \sum_{m \neq k} Y_{km}\mathbf{V}_m\right] \tag{11.95}$$

Power system loads are represented as either static impedance (static admittance) to ground, or constant power (constant current) at fixed power factor, or a combination of these types. Values of static-impedance and constant-current models are obtained from the scheduled bus loads and the bus voltages found in a base case power flow solution.

Summarizing, the mathematical expressions describing the power system may be divided into two categories:

1. Differential equations involving the vector of state variables $\mathbf{X}$ representing machine rotor angles and angular velocities, vector of bus voltages $\mathbf{V}$, and vector of bus currents $\mathbf{I}$.

$$\mathbf{F}(\dot{\mathbf{X}}, \mathbf{X}, \mathbf{V}, \mathbf{I}) = \mathbf{0} \tag{11.96}$$

2. Algebraic power flow equations.

$$\mathbf{G}(\mathbf{X}, \mathbf{V}, \mathbf{I}) = \mathbf{0} \tag{11.97}$$

Runge–Kutta Integration Method The usual procedure for solving transient stability problems is to integrate the differential equations (11.96) for time step n using the voltage and current values from time step $(n-1)$. The Runge–Kutta method is commonly used for the numerical integration of nonlinear differential equations. Once the integration step is done, the new values of the voltages are used in the solution of the algebraic equations (11.97) to compute the values of the bus voltages and currents for time step n. The process of alternating integration and network solution is repeated as time progresses to step $(n+1)$. A fixed time increment of 0.01 s is frequently used in the integration.

In this Runge–Kutta computational method, the generators are effectively treated independently of each other during the integration step and are merely coupled to each other during the solution of the algebraic network equations. The solution of Eq. 11.96 involves the solution of a set of at least two and up to several differential equations for each generator. These are typically solved as a block; that is, integration of the machine equations proceeds for one machine at a time. It may be noted that there is a time skew between the integration step and the network solution step; that is, integration is performed using values of the other parameters one time step behind. For small integration steps, this can still give acceptable results.

Predictor-Corrector Integration Method The time skew problem that is characteristic of the Runge–Kutta integration method may be minimized by solving the differential and algebraic equations simultaneously. Such a simultaneous solution is performed using predictor-corrector methods. Larger integration steps are allowed, and integration step sizes are easily varied. When the power system is quiescent, long step sizes are used; and when it is rapidly changing, much shorter step sizes are automatically chosen. This results in a minimum number of time steps.

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PROBLEMS

11.1 A three-phase synchronous generator is connected to a step-up three-phase transformer T_1, which is connected to a 60-km-long transmission line. At the far end of the line, a step-down transformer bank T_2 is connected. The secondary of T_2 supplies two motor loads M_1 and M_2. The ratings of the various types of equipment are

Generator: 10 MVA; 12 kV; $X = 20\%$; wye
T_1: 5 MVA; 12/69 kV; $X = 10\%$; delta-wye
T_2: 5 MVA; 69/4.16 kV; $X = 10\%$; wye-delta
M_1: 2000 kVA; 4.16 kV; $X = 20\%$; wye
M_2: 1000 kVA; 4.16 kV; $X = 20\%$; wye
Transmission line: $X = 0.5\ \Omega/\text{km}$

a. Draw the per-phase equivalent circuit of the system showing all reactances in per unit and the base voltages used at various parts of the network. Choose the generator ratings as bases in the generator circuit.

b. For a three-phase fault on the low-voltage terminals of transformer T_2, calculate the short-circuit current in amperes supplied by the generator assuming that all internal voltages are $1.0\angle 0^\circ$ pu.

11.2 A three-phase, wye-connected synchronous generator is rated 350 MVA, 13.2 kV, with reactances of $X''_d = 14\%$, $X'_d = 24\%$, and $X_d = 120\%$. The generator is connected to a step-up transformer rated 400 MVA, 13.2/115 kV, $X = 8\%$. The generator is operating at no load and rated voltage when a three-phase short circuit occurs on the high-voltage terminals of the transformer. Find the following currents expressed in per unit and in amperes.

a. The subtransient short-circuit current at the fault

b. The transient short-circuit current supplied by the generator

c. The steady-state short-circuit current at the fault

11.3 A synchronous generator is rated 300 MVA, 12 kV, 60 Hz. It is wye connected, and its neutral is solidly grounded. The machine reactances are $X''_d = X_2 = 0.15$ pu and $X_0 = 0.05$ pu. The generator is operating at rated voltage at no load when a fault occurs at the generator terminals.

a. Find the ratio of the short-circuit current for a single line-to-ground fault to the short-circuit current for a three-phase fault.

b. Find the ratio of the short-circuit current for a line-to-line fault to the short-circuit current for a three-phase fault.

c. Find the ratio of the short-circuit current for a double line-to-ground fault to the short-circuit current for a three-phase fault.

11.4 An inductive reactance is to be inserted between the neutral of the generator of Problem 11.3 and ground in order to limit the short-circuit current for a single line-to-ground fault to that for a three-phase fault.

a. Determine the required value of reactance in ohms.

b. With the inductive reactance inserted between the neutral of the generator and ground, find the ratio of the short-circuit current for a line-to-line fault to the short-circuit current for a three-phase fault on the terminals of the generator.

c. With the inductive reactance computed in part (a) inserted between the neutral of the generator and ground, find the ratio of the short-circuit current for a double line-to-ground fault to the short-circuit current for a three-phase fault on the terminals of the generator.

11.5 A resistance is to be inserted between the neutral of the generator of Problem 11.3 and ground in order to limit the short-circuit current for a single line-to-ground fault to that for a three-phase fault.

a. Determine the required value of resistance in ohms.

b. With the resistance inserted between the neutral of the generator and ground, find the ratio of the short-circuit current for a line-to-line fault to the short-circuit current for a three-phase fault on the terminals of the generator.

c. With the resistance computed in part (a) inserted between the neutral of the generator and ground, find the ratio of the short-circuit current for a double line-to-ground fault to the short-circuit current for a three-phase fault on the terminals of the generator.

11.6 The power system shown in the one-line diagram of Fig. 11.40 has two synchronous generators each rated 100 MVA, 13.2 kV. The generator reactances are $X'' = X_2 = 20\%$ and $X_0 = 4\%$. The two transformers are each rated 100 MVA, 13.2/115 kV, with a reactance of 10%. The reactances of the transmission line are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. The system is operating at no load and rated voltage.

a. Find the short-circuit current for a double line-to-ground fault at bus 3.

b. Repeat part (a) for a single line-to-ground fault.

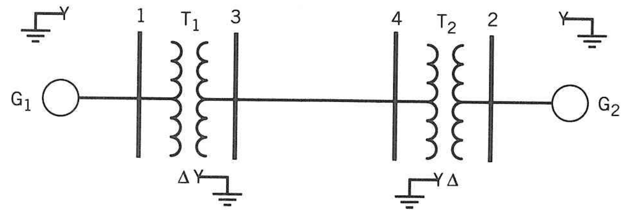


FIGURE 11.40 Power system of Problem 11.6.

11.7 Two generators G_1 and G_2 are connected through transformers T_1 and T_2 to a high-voltage bus to which a high-voltage transmission line is also connected. The line is open at the far end, wherein a three-phase fault occurs. The prefault voltage at the end of the transmission line is 345 kV. The equipment ratings and the per-unit reactances based on their respective ratings are as follows:

G_1: wye, 600 MVA, 12 kV, $X''_d = X_2 = 0.10$, $X_0 = 0.05$
G_2: wye, 400 MVA, 12 kV, $X''_d = X_2 = 0.13, X_0 = 0.07$

T_1: 600 MVA, 12/345 kV, Δ/Y, $X = 0.16$
T_2: 400 MVA, 12/345 kV, Y/Y, $X = 0.14$
Line: $X_1 = X_2 = 0.20$, $X_0 = 0.50$ on a base of 1000 MVA, 345 kV

a. Assume all wye-winding neutrals are solidly grounded. Determine the fault current and the contributions of each generator to the fault current.

b. Repeat part (a) for a single line-to-ground fault.

c. Repeat part (a) for a line-to-line fault.

d. Repeat part (a) for a double line-to-ground fault.

11.8 A 69-kV transmission line is supplied through a transformer by a 13.2-kV generator. The high-voltage side of the transformer is wye connected with its neutral solidly grounded, and the generator side is connected in delta. The positive-sequence reactances of the generator and transmission line are 20 Ω and 50 Ω, respectively, and the negative-sequence reactance of the generator is 15 Ω. The zero-sequence reactances of the generator, and transmission line are 10 Ω and 150 Ω, respectively. The reactance of the transformer is 10 Ω referred to the generator side.

a. Calculate the short-circuit current for a single line-to-ground fault at the receiving end of the line.

b. Repeat part (a) for a double line-to-ground fault.

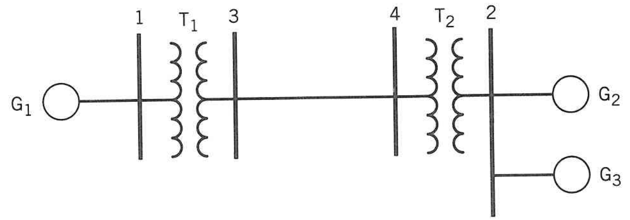
11.9 A single line-to-ground fault occurs at the far end of a radial transmission line with the following sequence impedances:

$$Z_1 = (0.5 + j1.50)\ \text{pu}$$
$$Z_2 = (0.5 + j1.50)\ \text{pu}$$
$$Z_0 = (1.0 + j3.00)\ \text{pu}$$

The source is assumed to be an infinite bus of voltage 1.0 pu. Determine the fault current.

11.10 A four-bus power system is shown in Fig. 11.41. The ratings of the various types of equipment are given as follows:

G_1: 300 MVA, 12 kV, $X'' = 0.20$ per unit
G_2: 500 MVA, 20 kV, $X'' = 0.17$ per unit
G_3: 750 MVA, 20 kV, $X'' = 0.15$ per unit



T_1: 300 MVA, 12/230 kV, Δ-Y, $X = 0.10$ per unit
T_2: 500 MVA, 230/20 kV, Y-Δ, $X = 0.08$ per unit
Transmission line: 230-kV, $X = 75\ \Omega$

A three-phase short circuit occurs at bus 3, where the prefault voltage is 230 kV. Neglect prefault load currents.

a. Draw the impedance diagram in per unit using a power base of 1000 MVA and a voltage base of 20 kV for generator G_3.

b. Determine the fault current in amperes.

c. Calculate the contributions to the fault current from generator G_1 and from the transmission line.

d. Compute the fault current supplied by generators G_2 and G_3.

e. Determine the voltage at bus 2.

11.11 For the power system of Fig. 11.19, specify the location of the backup protection so that the cause of failure in the primary protection of the following system components will not cause the same failure in the backup.

a. Line 5–7

b. Line 6–8

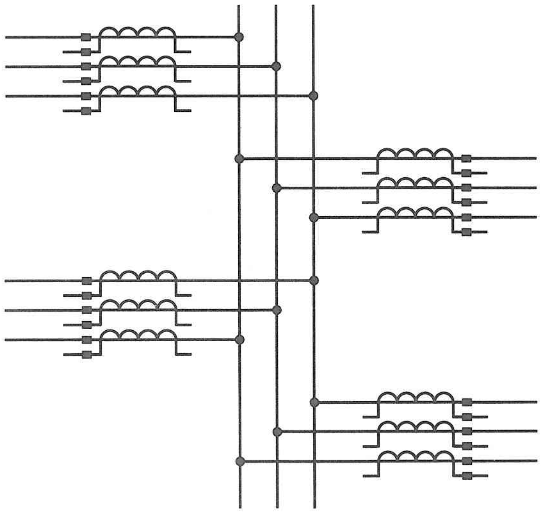
c. Station B.

11.12 Consider the power system of Fig. 11.19. In response to a fault, the protection system tripped breakers 3, 7, and 8. Determine all possible locations of the fault. Determine what single failure had occurred, if any.

11.13 Repeat Problem 11.12 if the breakers tripped are 4, 6, and 7.

11.14 Repeat Problem 11.12 if the breakers tripped are 4, 6, 7, and 8.

11.15 For the three-phase high-voltage bus shown in Fig. 11.42, sketch the developed three-phase wiring diagram for the bus differential protection using overcurrent relays.



11.16 A three-phase step-down transformer bank is rated 10 MVA and 69/13.8 kV. The high-voltage side is wye connected, and the low-voltage side is delta connected. Sketch the developed three-phase wiring diagram for the protection of the transformer bank using percentage-differential relays. Show all CT ratings, connections, and polarities. Also show the values of the currents in the lines, leads, relay windings, and transformer windings. Indicate the connections and ratings of any autotransformer that may be needed.

11.17 Repeat Problem 11.16 for the protection of a three-phase power transformer rated 100 MVA and 230/69 kV. Assume that the transformer windings are wye connected in both the primary and secondary sides.

11.18 Repeat Problem 11.17 when the transformer windings are connected in wye at the primary and in delta at the secondary.

11.19 A portion of a power system is shown in Fig. 11.43. Stations A and B have distance relays that are each adjusted for a first-zone reach of 100 ohms and a third-zone reach of 125 ohms. A fault occurs at F.

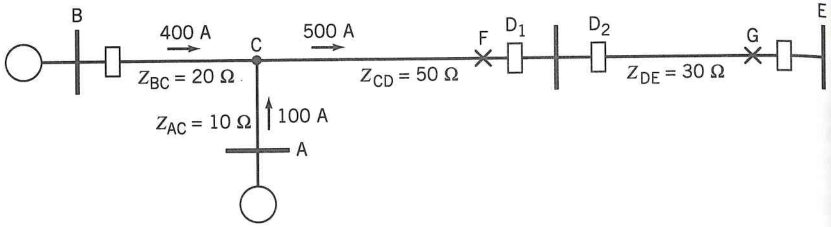


FIGURE 11.43 Portion of power system of Problem 11.19.

a. What is the impedance seen by the relay at A and by the relay at B?

b. Can the relay at A see the fault before the breaker at B has tripped?

c. Can the relay at A see the fault after the breaker at B has tripped?

11.20 Repeat Problem 11.19 if the fault occurs at G and the breaker D2 fails to open. Assume that the fault current magnitudes are the same as those shown in Fig. 11.43.

11.21 A three-phase, 75-MVA, 13.2-kV, 60-Hz, eight-pole, synchronous generator has an inertia constant of $H = 4$ MJ/MVA. The machine is connected to an infinite bus, and it is operating at steady state. The prime mover is initially supplying an input power of 80,000 hp. The electric power output suddenly decreases to 40 MW.

a. Calculate the angular acceleration.

b. If the acceleration computed in part (a) remains constant for a period of 20 cycles, find the change in the rotor angle δ in electrical degrees and the speed in rpm at the end of 20 cycles.

11.22 A three-phase, 60-Hz synchronous generator has an inertia constant $H = 6$ MJ/MVA. It is connected to an infinite bus through a purely reactive network, and it delivers a real power of 1.0 pu. The maximum power that the generator could deliver

is 2.5 per unit. A fault occurs at the terminals of the generator, and the output power is reduced to zero. The fault is cleared 0.30 s after it occurs. Determine the angle δ after the fault is cleared.

11.23 A three-phase synchronous generator has an inertia constant $H = 5$ MJ/MVA. It is connected to an infinite bus through a purely reactive network, and it delivers a real power of 0.8 pu. A fault occurs at the terminals of the generator, and the output power is reduced to zero. The maximum power that the generator could deliver is 2.0 pu. The fault is cleared 0.5 s after it occurs. Determine the angle δ after the fault is cleared.

11.24 The generator shown in Fig. 11.44 is delivering 1.0 pu current at 0.8 PF lagging to the infinite bus. The infinite-bus voltage is 1.0 pu. A fault occurs at point F. The fault is cleared by opening the circuit breakers at the ends of the faulted line section. Determine the prefault, on-fault, and postfault power-angle equations.

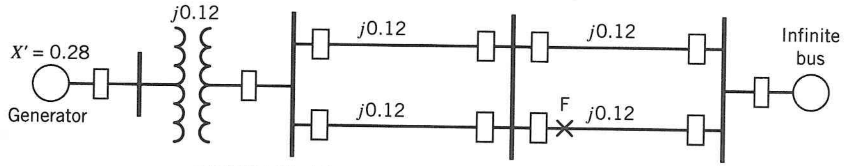


FIGURE 11.44 Power system of Problem 11.24.

11.25 A synchronous generator is connected to an infinite bus through two parallel transmission lines, each with a reactance of 0.5 pu. The excitation voltage of the generator is 1.2 pu, and its transient reactance is 0.20 pu. The generator is delivering a real power of 1.10 pu when a three-phase fault suddenly occurs near the end of one of the transmission lines where it connects to the bus. Determine the angle δ at the end of 10 cycles. Assume $H = 4$ pu-s.

11.26 A three-phase, 300-MVA, 13.8-kV, 60-Hz, 16-pole synchronous generator has an inertia constant $H = 2.5$ pu-s. The generator is initially operating at steady state with $P_m = P_e = 1.0$ pu and $\delta = 20°$. A three-phase fault occurs at the generator terminals. Determine the power angle eight cycles after the short occurs. Assume that P_m remains constant at 1.0 pu.

11.27 A hydroelectric power plant has two three-phase synchronous generators with the following ratings:

Unit 1: 400 MVA, 13.8 kV, 0.80 PF, 32 poles, $H_1 = 2.0$ pu-s
Unit 2: 200 MVA, 13.8 kV, 0.80 PF, 16 poles, $H_2 = 2.5$ pu-s

Choose a power base of 100 MVA.

a. Write the swing equation for each generator.

b. Derive the single equivalent machine swing equation.

11.28 A three-phase synchronous generator is connected to an infinite bus through a transformer and parallel transmission lines. The transformer reactance is 0.10 pu, and each transmission line has a reactance of 0.20 pu. The generator has an inertia constant $H = 3.0$ pu-s, and its transient reactance is 0.25. Assume that the mechanical power input P_m remains constant. The generator delivers a real power of 1.0 pu at 0.95 PF lagging to the infinite bus.

a. Determine the excitation voltage of the generator.

b. Write the expression for the electrical power delivered by the generator as a function of its power angle δ.

11.29 The generator of Problem 11.28 is initially operating at the given steady-state condition. A three-phase short circuit occurs on the bus interconnecting the transformer and the transmission lines. The fault is cleared after five cycles. Assume that none of the circuit breakers opened during the fault. Determine the angle δ after the fault is cleared.

11.30 The block diagram of an IEEE type 2 Excitation System is shown in Fig. 11.45. Derive the state equations in matrix form describing the dynamic performance of this excitation system. Assume that there is no exciter saturation.

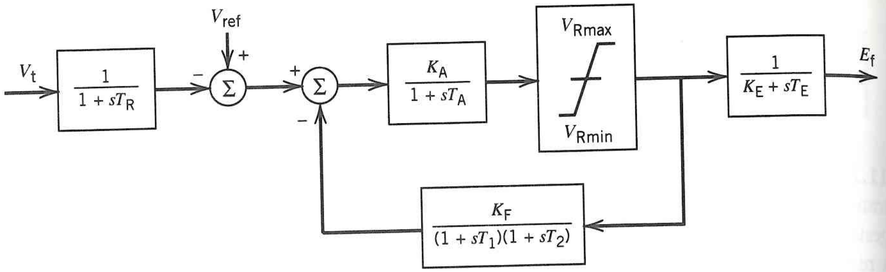


FIGURE 11.45 IEEE type 2 excitation system.

Appendices

Appendix A

Table of Conductor Characteristics

The following tables contain data on the electrical characteristics of various types of conductors, including copper, ACSR, hollow copper, Copperweld-copper, and Copperweld conductors. Other tables include skin effects and spacing factors for inductive reactances and capacitive reactances. Table 10 gives typical values of the different reactances of synchronous machines.

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Table 1 Characteristics of Copper Conductors, Hard Drawn, 97.3 Percent Conductivity

Size of Conductor		Number of Strands	Diameter of Individual Strands Inches	Outside Diameter Inches	Breaking Strength Pounds	Weight Pounds per Mile	Approx. Current Carrying Capacity* Amps	Geometric Mean Radius at 60 Cycles Feet	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor Per Mile At 1 Ft. Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor Per Mile At 1 Ft Spacing		
									25°C. (77°F.)				50°C. (122°F.)									
Circular Mils	A.W.G. or B. & S.								d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
1 000 000	...	37	0.1644	1.151	43 830	16 300	1 300	0.0368	0.0585	0.0594	0.0620	0.0634	0.0640	0.0648	0.0672	0.0685	0.1666	0.333	0.400	0.216	0.1081	0.0901
900 000	...	37	0.1560	1.092	39 510	14 670	1 220	0.0349	0.0650	0.0658	0.0682	0.0695	0.0711	0.0718	0.0740	0.0752	0.1693	0.339	0.406	0.220	0.1100	0.0916
800 000	...	37	0.1470	1.029	35 120	13 040	1 130	0.0329	0.0731	0.0739	0.0760	0.0772	0.0800	0.0806	0.0826	0.0837	0.1722	0.344	0.413	0.224	0.1121	0.0934
750 000	...	37	0.1424	0.997	33 400	12 230	1 090	0.0319	0.0780	0.0787	0.0807	0.0818	0.0853	0.0859	0.0878	0.0888	0.1739	0.348	0.417	0.226	0.1132	0.0943
700 000	...	37	0.1375	0.963	31 170	11 410	1 040	0.0308	0.0836	0.0842	0.0861	0.0871	0.0914	0.0920	0.0937	0.0947	0.1759	0.352	0.422	0.229	0.1145	0.0954
600 000	...	37	0.1273	0.891	27 020	9 781	940	0.0285	0.0975	0.0981	0.0997	0.1006	0.1066	0.1071	0.1086	0.1095	0.1799	0.360	0.432	0.235	0.1173	0.0977
500 000	...	37	0.1162	0.814	22 510	8 151	840	0.0260	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1845	0.369	0.443	0.241	0.1205	0.1004
500 000	...	19	0.1622	0.811	21 590	8 151	840	0.0256	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1853	0.371	0.445	0.241	0.1206	0.1005
450 000	...	19	0.1539	0.770	19 750	7 336	780	0.0243	0.1300	0.1304	0.1316	0.1323	0.1422	0.1426	0.1437	0.1443	0.1879	0.376	0.451	0.245	0.1224	0.1020
400 000	...	19	0.1451	0.726	17 560	6 521	730	0.0229	0.1462	0.1466	0.1477	0.1484	0.1600	0.1603	0.1613	0.1619	0.1909	0.382	0.458	0.249	0.1245	0.1038
350 000	...	19	0.1357	0.679	15 590	5 706	670	0.0214	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1943	0.389	0.466	0.254	0.1269	0.1058
350 000	...	12	0.1708	0.710	15 140	5 706	670	0.0225	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1918	0.384	0.460	0.251	0.1253	0.1044
300 000	...	19	0.1257	0.629	13 510	4 891	610	0.01987	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1982	0.396	0.476	0.259	0.1296	0.1080
300 000	...	12	0.1581	0.657	13 170	4 891	610	0.0208	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1957	0.392	0.470	0.256	0.1281	0.1068
250 000	...	19	0.1147	0.574	11 360	4 076	540	0.01813	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.203	0.406	0.487	0.266	0.1329	0.1108
250 000	....	12	0.1443	0.600	11 130	4 076	540	0.01902	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.200	0.401	0.481	0.263	0.1313	0.1094
211 600	4/0	19	0.1055	0.528	9 617	3 450	480	0.01668	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.207	0.414	0.497	0.272	0.1359	0.1132
211 600	4/0	12	0.1328	0.552	9 483	3 450	490	0.01750	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.205	0.409	0.491	0.269	0.1343	0.1119
211 600	4/0	7	0.1739	0.522	9 154	3 450	480	0.01579	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.210	0.420	0.503	0.273	0.1363	0.1136
167 800	3/0	12	0.1183	0.492	7 556	2 736	420	0.01559	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.210	0.421	0.505	0.277	0.1384	0.1153
167 800	3/0	7	0.1548	0.464	7 366	2 736	420	0.01404	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.216	0.431	0.518	0.281	0.1405	0.1171
133 100	2/0	7	0.1379	0.414	5 926	2 170	360	0.01252	0.440	0.440	0.440	0.440	0.481	0.481	0.481	0.481	0.222	0.443	0.532	0.289	0.1445	0.1205
105 500	1/0	7	0.1228	0.368	4 752	1 720	310	0.01113	0.555	0.555	0.555	0.555	0.606	0.607	0.607	0.607	0.227	0.455	0.546	0.298	0.1488	0.1240
83 690	1	7	0.1093	0.328	3 804	1 364	270	0.00992	0.699	0.699	0.699	0.699	0.765				0.233	0.467	0.560	0.306	0.1528	0.1274
83 690	1	3	0.1670	0.360	3 620	1 351	270	0.01016	0.692	0.692	0.692	0.692	0.757				0.232	0.464	0.557	0.299	0.1495	0.1246
66 370	2	7	0.0974	0.292	3 045	1 082	230	0.00883	0.881	0.882	0.882	0.882	0.964				0.239	0.478	0.574	0.314	0.1570	0.1308
66 370	2	3	0.1487	0.320	2 913	1 071	240	0.00903	0.873				0.955				0.238	0.476	0.571	0.307	0.1537	0.1281
66 370	2	1	......	0.258	3 003	1 061	220	0.00836	0.864				0.945				0.242	0.484	0.581	0.323	0.1614	0.1345
52 630	3	7	0.0867	0.260	2 433	858	200	0.00787	1.112				1.216				0.245	0.490	0.588	0.322	0.1611	0.1343
52 630	3	3	0.1325	0.285	2 359	850	200	0.00805	1.101		Same as d-c		1.204		Same as d-c		0.244	0.488	0.585	0.316	0.1578	0.1315
52 630	3	1	......	0.229	2 439	841	190	0.00745	1.090				1.192				0.248	0.496	0.595	0.331	0.1656	0.1380
41 740	4	3	0.1180	0.254	1 879	674	180	0.00717	1.388				1.518				0.250	0.499	0.599	0.324	0.1619	0.1349
41 740	4	1	......	0.204	1 970	667	170	0.00663	1.374				1.503				0.254	0.507	0.609	0.339	0.1697	0.1415
33 100	5	3	0.1050	0.226	1 505	534	150	0.00638	1.750				1.914				0.256	0.511	0.613	0.332	0.1661	0.1384
33 100	5	1	......	0.1819	1 591	529	140	0.00590	1.733				1.895				0.260	0.519	0.623	0.348	0.1738	0.1449
26 250	6	3	0.0935	0.201	1 205	424	130	0.00568	2.21				2.41				0.262	0.523	0.628	0.341	0.1703	0.1419
26 250	6	1	......	0.1620	1 280	420	120	0.00526	2.18				2.39				0.265	0.531	0.637	0.356	0.1779	0.1483
20 820	7	1	......	0.1443	1 030	333	110	0.00468	2.75				3.01				0.271	0.542	0.651	0.364	0.1821	0.1517
16 510	8	1	......	0.1285	826	264	90	0.00417	3.47				3.80				0.277	0.554	0.665	0.372	0.1862	0.1552

* For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles.

Table 2.A Characteristics of Aluminum Cable Steel Reinforced

rcular Mils or A.W.G. Aluminum	Aluminum			Steel		Outside Diameter Inches	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity† Amps	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor per Mile at 1 Ft. Spacing All Currents			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Ft. Spacing		
	Strands	Layers	Strand Dia. Inches	Strands	Strand Dia. Inches							25°C. (77°F.) Small Currents				50°C. (122°F.) Current Approx. 75% Capacity‡									
												d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.1495	0.299	0.359	0.1953	0.0977	0.0814
510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.1508	0.302	0.362	0.1971	0.0986	0.0821
431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.1522	0.304	0.365	0.1991	0.0996	0.0830
351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.1536	0.307	0.369	0.201	0.1006	0.0838
272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.1551	0.310	0.372	0.203	0.1016	0.0847
192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.1568	0.314	0.376	0.206	0.1028	0.0857
113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.1585	0.317	0.380	0.208	0.1040	0.0867
033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.1603	0.321	0.385	0.211	0.1053	0.0878
954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.1624	0.325	0.390	0.214	0.1068	0.0890
900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.1639	0.328	0.393	0.216	0.1078	0.0898
874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.1646	0.329	0.395	0.217	0.1083	0.0903
795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.1670	0.334	0.401	0.220	0.1100	0.0917
795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1660	0.332	0.399	0.219	0.1095	0.0912
795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1637	0.327	0.393	0.217	0.1085	0.0904
715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.1697	0.339	0.407	0.224	0.1119	0.0932
715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1687	0.337	0.405	0.223	0.1114	0.0928
715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1664	0.333	0.399	0.221	0.1104	0.0920
666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.1715	0.343	0.412	0.226	0.1132	0.0943
636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.1726	0.345	0.414	0.228	0.1140	0.0950
636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1718	0.344	0.412	0.227	0.1135	0.0946
636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1693	0.339	0.406	0.225	0.1125	0.0937
605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.1739	0.348	0.417	0.230	0.1149	0.0957
605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.1730	0.346	0.415	0.229	0.1144	0.0953
556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1751	0.350	0.420	0.232	0.1159	0.0965
556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1728	0.346	0.415	0.230	0.1149	0.0957
500 000	30	2	0.1291	7	0.1291	0.904	314 500	24 400	4 122	0.0311	690	0.187	0.187	0.187	0.187	0.206				0.1754	0.351	0.421	0.234	0.1167	0.0973
477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.1790	0.358	0.430	0.237	0.1186	0.0988
477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.1766	0.353	0.424	0.235	0.1176	0.0980
397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.1836	0.367	0.441	0.244	0.1219	0.1015
397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235		Same as d-c		0.259		Same as d-c		0.1812	0.362	0.435	0.242	0.1208	0.1006
336 400	26	2	0.1138	7	0.0885	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.1872	0.376	0.451	0.250	0.1248	0.1039
336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.1855	0.371	0.445	0.248	0.1238	0.1032
300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.1908	0.382	0.458	0.254	0.1269	0.1057
300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.1883	0.377	0.452	0.252	0.1258	0.1049
266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.1936	0.387	0.465	0.258	0.1289	0.1074

										For Current Approx. 75% Capacity ‡										Single Layer Conductors								
																				Small Currents			Current Approx. 75% Capacity‡					
																				25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles			
266 800	6	1	0.2109	7	0.0703	0.633	3/0	9 645	1 802	0.00684	460	0.351	0.351	0.351	0.352	0.386	0.430	0.510	0.552	0.194	0.388	0.466	0.252	0.504	0.605	0.259	0.1294	0.1079
4/0	6	1	0.1878	1	0.1878	0.563	2/0	8 420	1 542	0.00814	340	0.441	0.442	0.444	0.445	0.485	0.514	0.567	0.592	0.218	0.437	0.524	0.242	0.484	0.581	0.267	0.1336	0.1113
3/0	6	1	0.1672	1	0.1672	0.502	1/0	6 675	1 223	0.00600	300	0.556	0.557	0.559	0.560	0.612	0.642	0.697	0.723	0.225	0.450	0.540	0.259	0.517	0.621	0.275	0.1377	0.1147
2/0	6	1	0.1490	1	0.1490	0.447	1	5 345	970	0.00510	270	0.702	0.702	0.704	0.706	0.773	0.806	0.866	0.895	0.231	0.462	0.554	0.267	0.534	0.641	0.284	0.1418	0.1182
1/0	6	1	0.1327	1	0.1327	0.398	2	4 280	769	0.00446	230	0.885	0.885	0.887	0.888	0.974	1.01	1.08	1.12	0.237	0.473	0.568	0.273	0.547	0.656	0.292	0.1460	0.1216
1	6	1	0.1182	1	0.1182	0.355	3	3 480	610	0.00418	200	1.12	1.12	1.12	1.12	1.23	1.27	1.34	1.38	0.242	0.483	0.580	0.277	0.554	0.665	0.300	0.1500	0.1250
2	6	1	0.1052	1	0.1052	0.316	4	2 790	484	0.00418	180	1.41	1.41	1.41	1.41	1.55	1.59	1.66	1.69	0.247	0.493	0.592	0.277	0.554	0.665	0.308	0.1542	0.1285
2	7	1	0.0974	1	0.1299	0.325	4	3 525	566	0.00504	180	1.41	1.41	1.41	1.41	1.55	1.59	1.62	1.65	0.247	0.493	0.592	0.267	0.535	0.642	0.306	0.1532	0.1276
3	6	1	0.0937	1	0.0937	0.281	5	2 250	384	0.00430	160	1.78	1.78	1.78	1.78	1.95	1.98	2.04	2.07	0.252	0.503	0.604	0.275	0.551	0.661	0.317	0.1583	0.1320
4	6	1	0.0834	1	0.0834	0.250	6	1 830	304	0.00437	140	2.24	2.24	2.24	2.24	2.47	2.50	2.54	2.57	0.257	0.514	0.611	0.274	0.549	0.659	0.325	0.1627	0.1355
4	7	1	0.0772	1	0.1029	0.257	6	2 288	356	0.00452	140	2.24	2.24	2.24	2.24	2.47	2.50	2.53	2.55	0.257	0.515	0.618	0.273	0.545	0.655	0.323	0.1615	0.1346
5	6	1	0.0743	1	0.0743	0.223	7	1 460	241	0.00416	120	2.82	2.82	2.82	2.82	3.10	3.12	3.16	3.18	0.262	0.525	0.630	0.279	0.557	0.665	0.333	0.1666	0.1388
6	6	1	0.0661	1	0.0661	0.198	8	1 170	191	0.00394	100	3.56	3.56	3.56	3.56	3.92	3.94	3.97	3.98	0.268	0.536	0.643	0.281	0.561	0.673	0.342	0.1708	0.1423

Based on copper 97 percent, aluminum 61 percent conductivity.
For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles.
"Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps." and is approximately the current which will produce 50°C. conductor temp. (25°C. rise) with 25°C. air temp., wind 1.4 miles per hour.

ble 2.B Characteristics of "Expanded" Aluminum Cable Steel Reinforced

lar G.	Aluminum			Steel		Filler Section				Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor per Mile at 1 Ft. Spacing All Currents			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Ft. Spacing		
						Aluminum		Paper								25°C. (77°F.) Small Currents				50°C. (122°F.) Current Approx. 75% Capacity									
	Strands	Layers	Strand Dia. Inches	Strands	Strand Dia. Inches	Strands	Strand Dia. Inches	Strands	Layers							d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
000	54	2	0.1255	19	0.0834	4	0.1182	23	2	1.38	534 000	35 371	7 200	(1)	(1)	(1)				(1)				(1)	(1)	(1)	(1)	(1)	(1)
000	54	2	0.1409	19	0.0921	4	0.1353	24	2	1.55	724 000	41 900	9 070																
000	66	2	0.1350	19	0.100	4	0.184	18	2	1.75	840 000	49 278	11 340																

Electrical Characteristics not available until laboratory measurements are completed.

Table 3.A Characteristics of Anaconda Hollow Copper Conductors

Design Number	Size of Conductor Circular Mils or A.W.G.	Wires		Outside Diameter Inches	Breaking Strain Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps‡	r_a Resistance Ohms per Conductor per Mile				x_a Inductive Reactance Ohms per Conductor per Mile at 1 Ft. Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Ft. Spacing		
									25°C. (77°F.)		50°C. (122°F.)							
		Number	Diameter Inches						d-c 25 cycles	50 cycles 60 cycles	d-c 25 cycles	50 cycles 60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
966	890 500	28	0.1610	1.650	36 000	15 085	0.0612	1395	0.0671	0.0676	0.0734	0.0739	0.1412	0.282	0.339	0.1907	0.0953	0.0794
96R1	750 000	42	0.1296	1.155	34 200	12 345	0.0408	1160	0.0786	0.0791	0.0860	0.0865	0.1617	0.323	0.388	0.216	0.1080	0.0900
939	650 000	50	0.1097	1.126	29 500	10 761	0.0406	1060	0.0909	0.0915	0.0994	0.1001	0.1621	0.324	0.389	0.218	0.1089	0.0908
360R1	600 000	50	0.1053	1.007	27 500	9 905	0.0387	1020	0.0984	0.0991	0.1077	0.1084	0.1644	0.329	0.395	0.221	0.1105	0.0921
938	550 000	50	0.1009	1.036	25 200	9 103	0.0373	960	0.1076	0.1081	0.1177	0.1183	0.1663	0.333	0.399	0.224	0.1119	0.0932
4R5	510 000	50	0.0970	1.000	22 700	8 485	0.0360	910	0.1173	0.1178	0.1283	0.1289	0.1681	0.336	0.404	0.226	0.1131	0.0943
892R3	500 000	18	0.1558	1.080	21 400	8 263	0.0394	900	0.1178	0.1184	0.1289	0.1296	0.1630	0.326	0.391	0.221	0.1104	0.0920
933	450 000	21	0.1353	1.074	19 300	7 476	0.0398	850	0.1319	0.1324	0.1443	0.1448	0.1630	0.326	0.391	0.221	0.1106	0.0922
924	400 000	21	0.1227	1.014	17 200	6 642	0.0376	810	0.1485	0.1491	0.1624	0.1631	0.1658	0.332	0.398	0.225	0.1126	0.0939
925R1	380 500	22	0.1211	1.003	16 300	6 331	0.0373	780	0.1565	0.1572	0.1712	0.1719	0.1663	0.333	0.399	0.226	0.1130	0.0942
565R1	350 000	21	0.1196	0.950	15 100	5 813	0.0353	750	0.1695	0.1700	0.1854	0.1860	0.1691	0.338	0.406	0.230	0.1150	0.0958
936	350 000	15	0.1444	0.860	15 400	5 776	0.0311	740	0.1690	0.1695	0.1849	0.1854	0.1754	0.351	0.421	0.237	0.1185	0.0988
378R1	350 000	30	0.1059	0.736	16 100	5 739	0.0253	700	0.1685	0.1690	0.1843	0.1849	0.1860	0.372	0.446	0.248	0.1241	0.1034
954	321 000	22	0.1113	0.920	13 850	5 343	0.0340	700	0.1851	0.1856	0.202	0.203	0.1710	0.342	0.410	0.232	0.1161	0.0968
935	300 000	18	0.1205	0.839	13 100	4 984	0.0307	670	0.1980	0.1985	0.216	0.217	0.1761	0.352	0.423	0.239	0.1194	0.0995
903R1	300 000	15	0.1338	0.797	13 200	4 953	0.0289	660	0.1969	0.1975	0.215	0.216	0.1793	0.359	0.430	0.242	0.1212	0.1010
178R2	300 000	12	0.1507	0.750	13 050	4 937	0.0266	650	0.1964	0.1969	0.215	0.216	0.1833	0.367	0.440	0.247	0.1234	0.1028
926	250 000	18	0.1100	0.766	10 950	4 155	0.0279	600	0.238	0.239	0.260	0.261	0.1810	0.362	0.434	0.245	0.1226	0.1022
915R1	250 000	15	0.1214	0.725	11 000	4 145	0.0266	590	0.237	0.238	0.259	0.260	0.1834	0.367	0.440	0.249	0.1246	0.1038
24R1	250 000	12	0.1368	0.683	11 000	4 133	0.0245	580	0.237	0.238	0.259	0.260	0.1876	0.375	0.450	0.253	0.1267	0.1066
923	4/0	18	0.1005	0.700	9 300	3 521	0.0255	530	0.281	0.282	0.307	0.308	0.1855	0.371	0.445	0.252	0.1258	0.1049
922	4/0	15	0.1109	0.663	9 300	3 510	0.0238	520	0.281	0.282	0.307	0.308	0.1889	0.378	0.453	0.256	0.1278	0.1065
50R2	4/0	14	0.1152	0.650	9 300	3 510	0.0234	520	0.280	0.281	0.306	0.307	0.1898	0.380	0.455	0.257	0.1285	0.1071
158R1	3/0	16	0.0961	0.606	7 500	2 785	0.0221	460	0.354	0.355	0.387	0.388	0.1928	0.386	0.463	0.262	0.1310	0.1091
495R1	3/0	15	0.0996	0.595	7 600	2 785	0.0214	460	0.353	0.354	0.386	0.387	0.1943	0.389	0.466	0.263	0.1316	0.1097
570R2	3/0	12	0.1123	0.560	7 600	2 772	0.0201	450	0.352	0.353	0.385	0.386	0.1976	0.395	0.474	0.268	0.1338	0.1115
909R2	2/0	15	0.0880	0.530	5 950	2 213	0.0191	370	0.446	0.446	0.487	0.487	0.200	0.400	0.481	0.271	0.1357	0.1131
412R2	2/0	14	0.0913	0.515	6 000	2 207	0.0184	370	0.446	0.446	0.487	0.487	0.202	0.404	0.485	0.274	0.1368	0.1140
937	2/0	13	0.0950	0.505	6 000	2 203	0.0181	370	0.446	0.446	0.487	0.487	0.203	0.406	0.487	0.275	0.1375	0.1146
930	125 600	14	0.0885	0.500	5 650	2 083	0.0180	360	0.473	0.473	0.517	0.517	0.203	0.406	0.487	0.276	0.1378	0.1149
934	121 300	15	0.0836	0.500	5 400	2 015	0.0179	350	0.491	0.491	0.537	0.537	0.203	0.407	0.488	0.276	0.1378	0.1149
901	119 400	12	0.0936	0.470	5 300	1 979	0.0165	340	0.507	0.507	0.555	0.555	0.207	0.415	0.498	0.280	0.1400	0.1167

‡For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles, average tarnished surface.

Table 3.B Characteristics of General Cable Type HH Hollow Copper Conductors

Conductor Size Circular Mils or A.W.G.	Outside⁽¹⁾ Diameter Inches	Wall Thickness Inches	Weight Pounds per Mile	Breaking Strength Pounds	Geometric Mean Radius Feet	Approx. Current Carrying Capacity⁽²⁾ Amps	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor per Mile at 1 Foot Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Foot Spacing		
							25°C. (77°F.)				50°C. (122°F.)									
							d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
1 000 000	2.103	0.150*	16 160	43 190	0.0833	1620	0.0576	0.0576	0.0577	0.0577	0.0630	0.0630	0.0631	0.0631	0.1257	0.251	0.302	0.1734	0.0867	0.0722
950 000	2.035	0.147*	15 350	41 030	0.0805	1565	0.0606	0.0606	0.0607	0.0607	0.0663	0.0664	0.0664	0.0664	0.1274	0.255	0.306	0.1757	0.0879	0.0732
900 000	1.966	0.144*	14 540	38 870	0.0778	1505	0.0640	0.0640	0.0641	0.0641	0.0700	0.0701	0.0701	0.0701	0.1291	0.258	0.310	0.1782	0.0891	0.0742
850 000	1.901	0.140*	13 730	36 710	0.0751	1450	0.0677	0.0678	0.0678	0.0678	0.0741	0.0742	0.0742	0.0742	0.1309	0.262	0.314	0.1805	0.0903	0.0752
800 000	1.820	0.137*	12 920	34 550	0.0722	1390	0.0720	0.0720	0.0720	0.0721	0.0788	0.0788	0.0788	0.0788	0.1329	0.266	0.319	0.1833	0.0917	0.0764
790 000	1.650	0.131†	12 760	34 120	0.0646	1335	0.0729	0.0729	0.0730	0.0730	0.0797	0.0798	0.0799	0.0799	0.1385	0.277	0.332	0.1906	0.0953	0.0794
750 000	1.750	0.133*	12 120	32 390	0.0691	1325	0.0768	0.0768	0.0768	0.0769	0.0840	0.0840	0.0841	0.0841	0.1351	0.270	0.324	0.1864	0.0932	0.0777
700 000	1.686	0.130*	11 310	30 230	0.0665	1265	0.0822	0.0823	0.0823	0.0823	0.0900	0.0900	0.0901	0.0901	0.1370	0.274	0.329	0.1891	0.0945	0.0788
650 000	1.610	0.126*	10 500	28 070	0.0635	1200	0.0886	0.0886	0.0886	0.0887	0.0969	0.0970	0.0970	0.0970	0.1394	0.279	0.335	0.1924	0.0962	0.0802
600 000	1.558	0.123*	9 692	25 910	0.0615	1140	0.0959	0.0960	0.0960	0.0960	0.1050	0.1051	0.1051	0.1051	0.1410	0.282	0.338	0.1947	0.0974	0.0811
550 000	1.478	0.119*	8 884	23 750	0.0583	1075	0.1047	0.1048	0.1048	0.1048	0.1146	0.1146	0.1147	0.1147	0.1437	0.287	0.345	0.1985	0.0992	0.0827
512 000	1.400	0.115*	8 270	22 110	0.0551	1020	0.1124	0.1125	0.1125	0.1125	0.1230	0.1230	0.1231	0.1231	0.1466	0.293	0.352	0.202	0.1012	0.0843
500 000	1.390	0.115*	8 076	21 590	0.0547	1005	0.1151	0.1151	0.1152	0.1152	0.1259	0.1260	0.1260	0.1260	0.1469	0.294	0.353	0.203	0.1014	0.0845
500 000	1.268	0.109†	8 074	21 590	0.0494	978	0.1151	0.1152	0.1152	0.1152	0.1259	0.1260	0.1260	0.1261	0.1521	0.304	0.365	0.209	0.1047	0.0872
500 000	1.100	0.130†	8 068	21 590	0.0420	937	0.1150	0.1151	0.1152	0.1153	0.1258	0.1259	0.1260	0.1260	0.1603	0.321	0.385	0.219	0.1098	0.0915
500 000	1.020	0.144†	8 063	21 590	0.0384	915	0.1150	0.1150	0.1152	0.1152	0.1258	0.1259	0.1260	0.1261	0.1648	0.330	0.396	0.225	0.1124	0.0937
450 000	1.317	0.111*	7 268	19 430	0.0518	939	0.1279	0.1280	0.1280	0.1280	0.1400	0.1401	0.1401	0.1401	0.1496	0.299	0.359	0.207	0.1033	0.0861
450 000	1.188	0.105†	7 266	19 430	0.0462	910	0.1278	0.1279	0.1279	0.1280	0.1399	0.1400	0.1400	0.1401	0.1554	0.311	0.373	0.214	0.1070	0.0892
400 000	1.218	0.106*	6 460	17 270	0.0478	864	0.1439	0.1440	0.1440	0.1440	0.1575	0.1576	0.1576	0.1576	0.1537	0.307	0.369	0.212	0.1061	0.0884
400 000	1.103	0.100†	6 458	17 270	0.0428	838	0.1438	0.1439	0.1439	0.1440	0.1574	0.1575	0.1575	0.1576	0.1593	0.319	0.382	0.219	0.1097	0.0914
350 000	1.128	0.102*	5 653	15 110	0.0443	790	0.1644	0.1645	0.1645	0.1645	0.1799	0.1800	0.1800	0.1800	0.1576	0.315	0.378	0.218	0.1089	0.0907
350 000	1.014	0.096†	5 650	15 110	0.0393	764	0.1644	0.1645	0.1645	0.1646	0.1799	0.1800	0.1800	0.1801	0.1637	0.328	0.393	0.225	0.1127	0.0939
300 000	1.020	0.096*	4 845	12 950	0.0399	709	0.1918	0.1919	0.1919	0.1919	0.210	0.210	0.210	0.210	0.1628	0.326	0.391	0.225	0.1124	0.0937
300 000	0.919	0.091†	4 843	12 950	0.0355	687	0.1917	0.1918	0.1918	0.1919	0.210	0.210	0.210	0.210	0.1688	0.338	0.405	0.232	0.1162	0.0968
250 000	0.914	0.091*	4 037	10 790	0.0357	626	0.230	0.230	0.230	0.230	0 252	0.252	0.252	0.252	0.1685	0.337	0.404	0.233	0.1163	0.0970
250 000	0.818	0.086†	4 036	10 790	0.0315	606	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1748	0.350	0.420	0.241	0.1203	0.1002
250 000	0.766	0.094†	4 034	10 790	0.0292	594	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1787	0.357	0.429	0.245	0.1226	0.1022
214 500	0.650	0.098†	3 459	9 265	0.0243	524	0.268	0.268	0.268	0.268	0.293	0.293	0.293	0.294	0.1879	0.376	0.451	0.257	0.1285	0.1071
4/0	0.733	0.082†	3 415	9 140	0.0281	539	0.272	0.272	0.272	0.272	0.297	0.297	0.298	0.298	0.1806	0.361	0.433	0.248	0.1242	0.1035
3/0	0.608	0.080†	2 707	7 240	0.0230	454	0.343	0.343	0.343	0.343	0.375	0.375	0.375	0.375	0.1907	0.381	0.458	0.262	0.1309	0.1091
2/0	0.500	0.080†	2 146	5 750	0.0186	382	0.432	0.432	0.432	0.432	0.472	0.473	0.473	0.473	0.201	0.403	0.483	0.276	0.1378	0.1149

Notes: *Thickness at edges of interlocked segments. †Thickness uniform throughout.
(1) Conductors of smaller diameter for given cross-sectional area also available; in the naught sizes, some additional diameter expansion is possible.
(2) For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles.

Table 4.A Characteristics of Copperweld-Copper Conductors

Nominal Designation	Size of Conductor: Number and Diameter of Wires		Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Rated Breaking Load Lbs.	Weight Lbs. per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity at 60 Cycles Amps*	r_a Resistance Ohms per Conductor per Mile at 25°C.(77°F.) Small Currents				r_a Resistance Ohms per Conductor per Mile at 50°C. (122°F.) Current Approx. 75% of Capacity**				x_a Inductive Reactance Ohms per Conductor per Mile One ft. Spacing Average Currents			x_a' Capacitive Reactance Megohms per Conductor per Mile One ft. Spacing		
	Copperweld	Copper							d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
E	7x.1576″	12x.1576″	0.788	350 000	32 420	7 409	0.0220	660	0.1658	0.1728	0.1789	0.1812	0.1812	0.1915	0.201	0.204	0.1929	0.386	0.463	0.243	0.1216	0.1014
EK	4x.1470″	15x.1470″	0.735	350 000	23 850	6 536	0.0245	680	0.1658	0.1682	0.1700	0.1705	0.1812	0.1845	0.1873	0.1882	0.1875	0.375	0.450	0.248	0.1241	0.1034
V	3x.1751″	9x.1893″	0.754	350 000	23 480	6 578	0.0226	650	0.1655	0.1725	0.1800	0.1828	0.1809	0.1910	0.202	0.206	0.1915	0.383	0.460	0.246	0.1232	0.1027
E	7x.1459″	12x.1459″	0.729	300 000	27 770	6 351	0.0204	600	0.1934	0.200	0.207	0.209	0.211	0.222	0.232	0.235	0.1969	0.394	0.473	0.249	0.1244	0.1037
EK	4x.1361″	15x.1361″	0.680	300 000	20 960	5 602	0.0227	610	0.1934	0.1958	0.1976	0.1981	0.211	0.215	0.218	0.219	0.1914	0.383	0.460	0.254	0.1269	0.1057
V	3x.1621″	9x.1752″	0.698	300 000	20 730	5 639	0.0209	590	0.1930	0.200	0.208	0.210	0.211	0.222	0.233	0.237	0.1954	0.391	0.469	0.252	0.1259	0.1050
E	7x.1332″	12x.1332″	0.666	250 000	23 920	5 292	0.01859	540	0.232	0.239	0.245	0.248	0.254	0.265	0.275	0.279	0.202	0.403	0.484	0.255	0.1276	0.1064
EK	4x.1242″	15x.1242″	0.621	250 000	17 840	4 669	0.0207	540	0.232	0.235	0.236	0.237	0.254	0.258	0.261	0.261	0.1960	0.392	0.471	0.260	0.1301	0.1084
V	3x.1480″	9x.1600″	0.637	250 000	17 420	4 699	0.01911	530	0.232	0.239	0.246	0.249	0.253	0.264	0.276	0.281	0.200	0.400	0.480	0.258	0.1292	0.1077
E	7x.1225″	12x.1225″	0.613	4/0	20 730	4 479	0.01711	480	0.274	0.281	0.287	0.290	0.300	0.312	0.323	0.326	0.206	0.411	0.493	0.261	0.1306	0.1088
G	2x.1944″	5x.1944″	0.583	4/0	15 640	4 168	0.01409	460	0.273	0.284	0.294	0.298	0.299	0.318	0.336	0.342	0.215	0.431	0.517	0.265	0.1324	0.1103
EK	4x.1143″	15x.1143″	0.571	4/0	15 370	3 951	0.01903	490	0.274	0.277	0.278	0.279	0.300	0.304	0.307	0.308	0.200	0.401	0.481	0.266	0.1331	0.1109
V	3x.1361″	9x.1472″	0.586	4/0	15 000	3 977	0.01758	470	0.274	0.281	0.288	0.291	0.299	0.311	0.323	0.328	0.204	0.409	0.490	0.264	0.1322	0.1101
F	1x.1833″	6x.1833″	0.550	4/0	12 290	3 750	0.01558	470	0.273	0.280	0.285	0.287	0.299	0.309	0.318	0.322	0.210	0.421	0.505	0.269	0.1344	0.1120
E	7x.1091″	12x.1091″	0.545	3/0	16 800	3 552	0.01521	420	0.346	0.353	0.359	0.361	0.378	0.391	0.402	0.407	0.212	0.423	0.508	0.270	0.1348	0.1123
J	3x.1851″	4x.1851″	0.555	3/0	16 170	3 732	0.01156	410	0.344	0.356	0.367	0.372	0.377	0.398	0.419	0.428	0.225	0.451	0.541	0.268	0.1341	0.1118
G	2x.1731″	2x.1731″	0.519	3/0	12 860	3 305	0.01254	400	0.344	0.355	0.365	0.369	0.377	0.397	0.416	0.423	0.221	0.443	0.531	0.273	0.1365	0.1137
EK	4x.1018″	4x.1018″	0.509	3/0	12 370	3 134	0.01697	420	0.346	0.348	0.350	0.351	0.378	0.382	0.386	0.386	0.206	0.412	0.495	0.274	0.1372	0.1143
V	3x.1311″	9x.1311″	0.522	3/0	12 220	3 154	0.01566	410	0.345	0.352	0.360	0.362	0.377	0.390	0.403	0.408	0.210	0.420	0.504	0.273	0.1363	0.1136
F	1x.1632″	6x.1632″	0.490	3/0	9 980	2 974	0.01388	410	0.344	0.351	0.356	0.358	0.377	0.388	0.397	0.401	0.216	0.432	0.519	0.277	0.1385	0.1155
K	4x.1780″	3x.1780″	0.534	2/0	17 600	3 411	0.00912	360	0.434	0.447	0.459	0.466	0.475	0.499	0.524	0.535	0.237	0.475	0.570	0.271	0.1355	0.1129
J	3x.1648″	4x.1648″	0.494	2/0	13 430	2 960	0.01029	350	0.434	0.446	0.457	0.462	0.475	0.498	0.520	0.530	0.231	0.463	0.555	0.277	0.1383	0.1152
G	2x.1542″	5x.1542″	0.463	2/0	10 510	2 622	0.01119	350	0.434	0.445	0.456	0.459	0.475	0.497	0.518	0.525	0.227	0.454	0.545	0.281	0.1406	0.1171
V	3x.1080″	9x.1167″	0.465	2/0	9 846	2 502	0.01395	360	0.435	0.442	0.450	0.452	0.476	0.489	0.504	0.509	0.216	0.432	0.518	0.281	0.1404	0.1170
F	1x.1454″	6x.1454″	0.436	2/0	8 094	2 359	0.01235	350	0.434	0.441	0.446	0.448	0.475	0.487	0.497	0.501	0.222	0.444	0.533	0.285	0.1427	0.1189
K	4x.1585″	3x.1585″	0.475	1/0	14 490	2 703	0.00812	310	0.548	0.560	0.573	0.579	0.599	0.625	0.652	0.664	0.243	0.487	0.584	0.279	0.1397	0.1164
J	3x.1467″	4x.1467″	0.440	1/0	10 970	2 346	0.00917	310	0.548	0.559	0.570	0.576	0.599	0.624	0.648	0.659	0.237	0.474	0.569	0.285	0.1423	0.1186
G	2x.1373″	5x.1373″	0.412	1/0	8 563	2 078	0.00996	310	0.548	0.559	0.568	0.573	0.599	0.623	0.645	0.654	0.233	0.466	0.559	0.289	0.1447	0.1206
F	1x.1294″	6x.1294″	0.388	1/0	6 536	1 870	0.01099	310	0.548	0.554	0.559	0.562	0.599	0.612	0.622	0.627	0.228	0.456	0.547	0.294	0.1469	0.1224
N	5x.1546″	2x.1546″	0.464	1	15 410	2 541	0.00638	280	0.691	0.705	0.719	0.726	0.755	0.787	0.818	0.832	0.256	0.512	0.614	0.281	0.1405	0.1171
K	4x.1412″	3x.1412″	0.423	1	11 900	2 144	0.00723	270	0.691	0.704	0.716	0.722	0.755	0.784	0.813	0.825	0.249	0.498	0.598	0.288	0.1438	0.1198
J	3x.1307″	4x.1307″	0.392	1	9 000	1 861	0.00817	270	0.691	0.703	0.714	0.719	0.755	0.783	0.808	0.820	0.243	0.486	0.583	0.293	0.1465	0.1221
G	2x.1222″	5x.1222″	0.367	1	6 956	1 649	0.00887	260	0.691	0.702	0.712	0.716	0.755	0.781	0.805	0.815	0.239	0.478	0.573	0.298	0.1488	0.1240
F	1x.1153″	6x.1153″	0.346	1	5 266	1 483	0.00980	270	0.691	0.698	0.704	0.705	0.755	0.769	0.781	0.786	0.234	0.468	0.561	0.302	0.1509	0.1258
P	6x.1540″	1x.1540″	0.462	2	16 870	2 487	0.00501	250	0.871	0.886	0.901	0.909	0.952	0.988	1.024	1.040	0.268	0.536	0.643	0.281	0.1406	0.1172
N	5x.1377″	2x.1377″	0.413	2	12 680	2 015	0.00568	240	0.871	0.885	0.899	0.906	0.952	0.986	1.020	1.035	0.261	0.523	0.627	0.289	0.1446	0.1205
K	4x.1257″	3x.1257″	0.377	2	9 730	1 701	0.00644	240	0.871	0.884	0.896	0.902	0.952	0.983	1.014	1.028	0.255	0.510	0.612	0.296	0.1479	0.1232
J	3x.1164″	4x.1164″	0.349	2	7 322	1 476	0.00727	230	0.871	0.883	0.894	0.899	0.952	0.982	1.010	1.022	0.249	0.498	0.598	0.301	0.1506	0.1255
A	1x.1699″	2x.1699″	0.366	2	5 876	1 356	0.00763	240	0.869	0.875	0.880	0.882	0.950	0.962	0.973	0.979	0.247	0.493	0.592	0.298	0.1489	0.1241
G	2x.1089″	5x.1089″	0.327	2	5 626	1 307	0.00790	230	0.871	0.882	0.892	0.896	0.952	0.980	1.006	1.016	0.245	0.489	0.587	0.306	0.1529	0.1275
F	1x.1026″	6x.1026″	0.308	2	4 233	1 176	0.00873	230	0.871	0.878	0.884	0.885	0.952	0.967	0.979	0.985	0.230	0.479	0.575	0.310	0.1551	0.1292
P	6x.1371″	1x.1371″	0.411	3	13 910	1 973	0.00445	220	1.098	1.113	1.127	1.136	1.200	1.239	1.273	1.296	0.274	0.547	0.657	0.290	0.1448	0.1207
N	5x.1226″	2x.1226″	0.368	3	10 390	1 598	0.00506	210	1.098	1.112	1.126	1.133	1.200	1.237	1.273	1.289	0.267	0.534	0.641	0.298	0.1487	1.1239
K	4x.1120″	3x.1120″	0.336	3	7 910	1 349	0.00574	210	1.098	1.111	1.123	1.129	1.200	1.233	1.267	1.281	0.261	0.522	0.626	0.304	0.1520	0.1266
J	3x.1036″	4x.1036″	0.311	3	5 955	1 171	0.00648	200	1.098	1.110	1.121	1.126	1.200	1.232	1.262	1.275	0.255	0.509	0.611	0.309	0.1547	0.1289
A	1x.1513″	2x.1513″	0.326	3	4 810	1 075	0.00679	210	1.096	1.102	1.107	1.109	1.198	1.211	1.225	1.229	0.252	0.505	0.606	0.306	0.1531	0.1275
P	6x.1221″	1x.1221″	0.366	4	11 420	1 564	0.00397	190	1.385	1.400	1.414	1.423	1.514	1.555	1.598	1.616	0.280	0.559	0.671	0.298	0.1489	0.1241
N	5x.1092″	2x.1092″	0.328	4	8 460	1 267	0.00451	180	1.385	1.399	1.413	1.420	1.514	1.554	1.593	1.610	0.273	0.546	0.655	0.306	0.1528	0.1274
D	2x.1615″	1x.1615″	0.348	4	7 340	1 191	0.00566	190	1.382	1.389	1.396	1.399	1.511	1.529	1.544	1.542	0.262	0.523	0.628	0.301	0.1507	0.1256
A	1x.1347″	2x.1347″	0.290	4	3 938	853	0.00604	180	1.382	1.388	1.393	1.395	1.511	1.525	1.540	1.545	0.258	0.517	0.620	0.314	0.1572	0.1310
P	6x.1087″	1x.1087″	0.326	5	9 311	1 240	0.00353	160	1.747	1.762	1.776	1.785	1.909	1.954	2.00	2.02	0.285	0.571	0.685	0.306	0.1531	0.1275
D	2x.1438″	1x.1438″	0.310	5	6 035	944	0.00504	160	1.742	1.749	1.756	1.759	1.905	1.924	1.941	1.939	0.268	0.535	0.642	0.310	0.1548	0.1290
A	1x.1200″	2x.1200″	0.258	5	3 193	676	0.00538	160	1.742	1.748	1.753	1.755	1.905	1.920	1.936	1.941	0.264	0.528	0.634	0.323	0.1614	0.1345
D	2x.1281″	1x.1281″	0.276	6	4 942	749	0.00449	140	2.20	2.21	2.21	2.22	2.40	2.42	2.44	2.44	0.273	0.547	0.656	0.318	0.1590	0.1325
A	1x.1068″	2x.1068″	0.230	6	2 585	536	0.00479	140	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.270	0.540	0.648	0.331	0.1655	0.1379
[illegible]	1x.1046″	2x.1046″	0.225	6	2 143	514	0.00469	130	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.271	0.542	0.651	0.333	0.1663	0.1386
D	2x.1141″	1x.1141″	0.246	7	4 022	594	0.00400	120	2.77	2.78	2.79	2.79	3.03	3.05	3.07	3.07	0.279	0.558	0.670	0.326	0.1631	0.1359
A	1x.1266″	2x.0895″	0.223	7	2 754	495	0.00441	120	2.77	2.78	2.78	2.78	3.03	3.05	3.07	3.07	0.274	0.548	0.658	0.333	0.1666	0.1388
D	2x.1016″	1x.1016″	0.219	8	3 256	471	0.00356	110	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.285	0.570	0.684	0.334	0.1672	0.1393
A	1x.1127″	2x.0797″	0.199	8	2 233	392	0.00394	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.87	0.280	0.560	0.672	0.341	0.1706	0.1422
[illegible]	1x.0808″	2x.0834″	0.179	8	1 362	320	0.00373	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.283	0.565	0.679	0.349	0.1744	0.1453
D	2x.0808″	1x.0808″	0.174	9½	1 743	298	0.00283	85	4.91	4.92	4.92	4.93	5.37	5.39	5.42	5.42	0.297	0.593	0.712	0.351	0.1754	0.1462

*Based on a conductor temperature of 75°C. and an ambient of 25°C., wind 1.4 miles per hour (2 ft/sec.), frequency=60 cycles, average tarnished surface.
**Resistances at 50°C. total temperature, based on an ambient of 25°C. plus 25°C. rise due to heating effect of current. The approximate magnitude of current necessary to produce the 25° C. rise is 75% of the "Approximate Current Carrying Capacity at 60 cycles."

Table 4.B Characteristics of Copperweld Conductors

Nominal Conductor Size	Number and Size of Wires	Outside Diameter Inches	Area of Conductor Circular Mils	Rated Breaking Load Pounds Strength		Weight Pounds per Mile	Geometric Mean Radius at 60 cycles and Average Currents Feet	Approx. Current Carrying Capacity* Amps at 60 Cycles	r_a Resistance Ohms per Conductor per Mile at 25°C. (77°F.) Small Currents				r_a Resistance Ohms per Conductor per Mile at 75°C. (167°F.) Current Approx. 75% of Capacity**				x_a Inductive Reactance Ohms per Conductor per Mile One Ft. Spacing Average Currents			x_a' Capacitive Reactance Megohms per Conductor per Mile One Ft. Spacing		
				High	Extra High				d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
30% Conductivity																						
7/8"	19 No. 5	0.910	628 900	55 570	66 910	9 344	0.00758	620	0.306	0.316	0.326	0.331	0.363	0.419	0.476	0.499	0.261	0.493	0.592	0.233	0.1165	0.0971
13/16"	19 No. 6	0.810	498 800	45 830	55 530	7 410	0.00675	540	0.386	0.396	0.406	0.411	0.458	0.518	0.580	0.605	0.267	0.505	0.606	0.241	0.1206	0.1005
23/32"	19 No. 7	0.721	395 500	37 740	45 850	5 877	0.00601	470	0.486	0.496	0.506	0.511	0.577	0.643	0.710	0.737	0.273	0.517	0.621	0.250	0.1248	0.1040
21/32"	19 No. 8	0.642	313 700	31 040	37 690	4 660	0.00535	410	0.613	0.623	0.633	0.638	0.728	0.799	0.872	0.902	0.279	0.529	0.635	0.258	0.1289	0.1074
9/16"	19 No. 9	0.572	248 800	25 500	30 610	3 696	0.00477	360	0.773	0.783	0.793	0.798	0.917	0.995	1.075	1.106	0.285	0.541	0.649	0.266	0.1330	0.1109
5/8"	7 No. 4	0.613	292 200	24 780	29 430	4 324	0.00511	410	0.656	0.664	0.672	0.676	0.778	0.824	0.870	0.887	0.281	0.533	0.640	0.261	0.1306	0.1088
9/16"	7 No. 5	0.546	231 700	20 470	24 650	3 429	0.00455	360	0.827	0.835	0.843	0.847	0.981	1.030	1.080	1.099	0.287	0.545	0.654	0.269	0.1347	0.1122
1/2"	7 No. 6	0.486	183 800	16 890	20 460	2 719	0.00405	310	1.042	1.050	1.058	1.062	1.237	1.290	1.343	1.364	0.293	0.557	0.668	0.278	0.1388	0.1157
7/16"	7 No. 7	0.433	145 700	13 910	16 890	2 157	0.00361	270	1.315	1.323	1.331	1.335	1.560	1.617	1.675	1.697	0.299	0.569	0.683	0.286	0.1429	0.1191
3/8"	7 No. 8	0.385	115 600	11 440	13 890	1 710	0.00321	230	1.658	1.666	1.674	1.678	1.967	2.03	2.09	2.12	0.305	0.581	0.697	0.294	0.1471	0.1226
11/32"	7 No. 9	0.343	91 650	9 393	11 280	1 356	0.00286	200	2.09	2.10	2.11	2.11	2.48	2.55	2.61	2.64	0.311	0.592	0.711	0.303	0.1512	0.1260
5/16"	7 No. 10	0.306	72 680	7 758	9 196	1 076	0.00255	170	2.64	2.64	2.65	2.66	3.13	3.20	3.27	3.30	0.316	0.604	0.725	0.311	0.1553	0.1294
3 No. 5	3 No. 5	0.392	99 310	9 262	11 860	1 467	0.00457	220	1.926	1.931	1.936	1.938	2.29	2.31	2.34	2.35	0.289	0.545	0.654	0.293	0.1465	0.1221
3 No. 6	3 No. 6	0.349	78 750	7 639	9 754	1 163	0.00407	190	2.43	2.43	2.44	2.44	2.88	2.91	2.94	2.95	0.295	0.556	0.668	0.301	0.1506	0.1255
3 No. 7	3 No. 7	0.311	62 450	6 291	7 922	922.4	0.00363	160	3.06	3.07	3.07	3.07	3.63	3.66	3.70	3.71	0.301	0.568	0.682	0.310	0.1547	0.1289
3 No. 8	3 No. 8	0.277	49 530	5 174	6 282	731.5	0.00323	140	3.86	3.87	3.87	3.87	4.58	4.61	4.65	4.66	0.307	0.580	0.696	0.318	0.1589	0.1324
3 No. 9	3 No. 9	0.247	39 280	4 250	5 129	580.1	0.00288	120	4.87	4.87	4.88	4.88	5.78	5.81	5.85	5.86	0.313	0.591	0.710	0.326	0.1629	0.1358
3 No. 10	3 No. 10	0.220	31 150	3 509	4 160	460.0	0.00257	110	6.14	6.14	6.15	6.15	7.28	7.32	7.36	7.38	0.319	0.603	0.724	0.334	0.1671	0.1392
40% Conductivity																						
7/8"	19 No. 5	0.910	628 900	50 240	......	9 344	0.01175	690	0.229	0.239	0.249	0.254	0.272	0.321	0.371	0.391	0.236	0.449	0.539	0.233	0.1165	0.0971
13/16"	19 No. 6	0.810	498 800	41 600	......	7 410	0.01046	610	0.289	0.299	0.309	0.314	0.343	0.396	0.450	0.472	0.241	0.461	0.553	0.241	0.1206	0.1005
23/32"	19 No. 7	0.721	395 500	34 390	......	5 877	0.00931	530	0.365	0.375	0.385	0.390	0.433	0.490	0.549	0.573	0.247	0.473	0.567	0.250	0.1248	0.1040
21/32"	19 No. 8	0.642	313 700	28 380	......	4 660	0.00829	470	0.460	0.470	0.480	0.485	0.546	0.608	0.672	0.698	0.253	0.485	0.582	0.258	0.1289	0.1074
9/16"	19 No. 9	0.572	248 800	23 390	......	3 696	0.00739	410	0.580	0.590	0.600	0.605	0.688	0.756	0.826	0.853	0.259	0.496	0.595	0.266	0.1330	0.1109
5/8"	7 No. 4	0.613	292 200	22 310	......	4 324	0.00792	470	0.492	0.500	0.508	0.512	0.584	0.624	0.664	0.680	0.255	0.489	0.587	0.261	0.1306	0.1088
9/16"	7 No. 5	0.546	231 700	18 510	......	3 429	0.00705	410	0.620	0.628	0.636	0.640	0.736	0.780	0.843	0.840	0.261	0.501	0.601	0.269	0.1347	0.1122
1/2"	7 No. 6	0.486	183 800	15 330	......	2 719	0.00628	350	0.782	0.790	0.798	0.802	0.928	0.975	1.021	1.040	0.267	0.513	0.615	0.278	0.1388	0.1157
7/16"	7 No. 7	0.433	145 700	12 670	......	2 157	0.00559	310	0.986	0.994	1.002	1.006	1.170	1.220	1.271	1.291	0.273	0.524	0.629	0.286	0.1429	0.1191
3/8"	7 No. 8	0.385	115 600	10 460	......	1 710	0.00497	270	1.244	1.252	1.260	1.264	1.476	1.530	1.584	1.606	0.279	0.536	0.644	0.294	0.1471	0.1226
11/32"	7 No. 9	0.343	91 650	8 616	......	1 356	0.00443	230	1.568	1.576	1.584	1.588	1.861	1.919	1.978	2.00	0.285	0.548	0.658	0.303	0.1512	0.1260
5/16"	7 No. 10	0.306	72 680	7 121	......	1 076	0.00395	200	1.978	1.986	1.994	1.998	2.35	2.41	2.47	2.50	0.291	0.559	0.671	0.311	0.1553	0.1294
3 No. 5	3 No. 5	0.392	99 310	8 373	......	1 467	0.00621	250	1.445	1.450	1.455	1.457	1.714	1.738	1.762	1.772	0.269	0.514	0.617	0.293	0.1465	0.1221
3 No. 6	3 No. 6	0.349	78 750	6 934	......	1 163	0.00553	220	1.821	1.826	1.831	1.833	2.16	2.19	2.21	2.22	0.275	0.526	0.631	0.301	0.1506	0.1255
3 No. 7	3 No. 7	0.311	62 450	5 732	......	922.4	0.00492	190	2.30	2.30	2.31	2.31	2.73	2.75	2.78	2.79	0.281	0.537	0.645	0.310	0.1547	0.1289
3 No. 8	3 No. 8	0.277	49 530	4 730	......	731.5	0.00439	160	2.90	2.90	2.91	2.91	3.44	3.47	3.50	3.51	0.286	0.549	0.659	0.318	0.1589	0.1324
3 No. 9	3 No. 9	0.247	39 280	3 898	......	580.1	0.00391	140	3.65	3.66	3.66	3.66	4.33	4.37	4.40	4.41	0.292	0.561	0.673	0.326	0.1629	0.1358
3 No. 10	3 No. 10	0.220	31 150	3 221	......	460.0	0.00348	120	4.61	4.61	4.62	4.62	5.46	5.50	5.53	5.55	0.297	0.572	0.687	0.334	0.1671	0.1392
3 No. 12	3 No. 12	0.174	19 590	2 236	......	289.3	0.00276	90	7.32	7.33	7.33	7.34	8.69	8.73	8.77	8.78	0.310	0.596	0.715	0.351	0.1754	0.1462

*Based on conductor temperature of 125°C. and an ambient of 25°C.

**Resistance at 75°C. total temperature, based on an ambient of 25°C. plus 50°C. rise due to heating effect of current. The approximate magnitude of current necessary to produce the 50°C. rise is 75% of the "Approximate Current Carrying Capacity at 60 Cycles."

Table 5 Skin Effect Table

X	K	X	K	X	K	X	K
0.0	1.00000	1.0	1.00519	2.0	1.07816	3.0	1.31809
0.1	1.00000	1.1	1.00758	2.1	1.09375	3.1	1.35102
0.2	1.00001	1.2	1.01071	2.2	1.11126	3.2	1.38504
0.3	1.00004	1.3	1.01470	2.3	1.13069	3.3	1.41999
0.4	1.00013	1.4	1.01969	2.4	1.15207	3.4	1.45570
0.5	1.00032	1.5	1.02582	2.5	1.17538	3.5	1.49202
0.6	1.00067	1.6	1.03323	2.6	1.20056	3.6	1.52879
0.7	1.00124	1.7	1.04205	2.7	1.22753	3.7	1.56587
0.8	1.00212	1.8	1.05240	2.8	1.25620	3.8	1.60314
0.9	1.00340	1.9	1.06440	2.9	1.28644	3.9	1.64051

Table 6 Inductive Reactance Spacing Factor (x_d) Ohms per Conductor per Mile

Table 7 (Insert, bottom right) Zero-Sequence Resistance and Inductive Reactance Factors (r_e, x_e)*

25 CYCLES

SEPARATION

Feet	INCHES 0	1	2	3	4	5	6	7	8	9	10	11
0	–	−0.1256	−0.0906	−0.0701	−0.0555	−0.0443	−0.0350	−0.0273	−0.0205	−0.0145	−0.0092	−0.0044
1	0	0.0040	0.0078	0.0113	0.0145	0.0176	0.0205	0.0232	0.0258	0.0283	0.0306	0.0329
2	0.0350	0.0371	0.0391	0.0410	0.0428	0.0446	0.0463	0.0480	0.0496	0.0511	0.0527	0.0541
3	0.0555	0.0569	0.0583	0.0596	0.0609	0.0621	0.0633	0.0645	0.0657	0.0668	0.0679	0.0690
4	0.0701	0.0711	0.0722	0.0732	0.0741	0.0751	0.0760	0.0770	0.0779	0.0788	0.0797	0.0805
5	0.0814	0.0822	0.0830	0.0838	0.0846	0.0854	0.0862	0.0869	0.0877	0.0884	0.0892	0.0899
6	0.0906	0.0913	0.0920	0.0927	0.0933	0.0940	0.0946	0.0953	0.0959	0.0965	0.0972	0.0978
7	0.0984	0.0990	0.0996	0.1002	0.1007	0.1013	0.1019	0.1024	0.1030	0.1035	0.1041	0.1046
8	0.1051											
9	0.1111											
10	0.1164											
11	0.1212											
12	0.1256											
13	0.1297											
14	0.1334											
15	0.1369											
16	0.1402											
17	0.1432											
18	0.1461											
19	0.1489											
20	0.1515											
21	0.1539											
22	0.1563											
23	0.1585											
24	0.1607											
25	0.1627											
26	0.1647											
27	0.1666											
28	0.1685											
29	0.1702											
30	0.1720											
31	0.1736											
32	0.1752											
33	0.1768											
34	0.1783											
35	0.1798											
36	0.1812											
37	0.1826											
38	0.1839											
39	0.1852											
40	0.1865											
41	0.1878											
42	0.1890											
43	0.1902											
44	0.1913											
45	0.1925											
46	0.1936											
47	0.1947											
48	0.1957											
49	0.1968											

x_d at 25 cycles

$x_d=0.1164\log_{10}d$

d=separation, feet.

FUNDAMENTAL EQUATIONS

$$z_1=z_2=r_a+j(x_a+x_d)$$

$$z_o=r_a+r_e+j(x_a+x_e-2x_d)$$

50 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	–	−0.2513	−0.1812	−0.1402	−0.1111	−0.0885	−0.0701	−0.0545	−0.0410	−0.0291	−0.0184	−0.0088
1	0	0.0081	0.0156	0.0226	0.0291	0.0352	0.0410	0.0465	0.0517	0.0566	0.0613	0.0658
2	0.0701	0.0742	0.0782	0.0820	0.0857	0.0892	0.0927	0.0960	0.0992	0.1023	0.1053	0.1082
3	0.1111	0.1139	0.1166	0.1192	0.1217	0.1242	0.1267	0.1291	0.1314	0.1337	0.1359	0.1380
4	0.1402	0.1423	0.1443	0.1463	0.1483	0.1502	0.1521	0.1539	0.1558	0.1576	0.1593	0.1610
5	0.1627	0.1644	0.1661	0.1677	0.1693	0.1708	0.1724	0.1739	0.1754	0.1769	0.1783	0.1798
6	0.1812	0.1826	0.1839	0.1853	0.1866	0.1880	0.1893	0.1906	0.1918	0.1931	0.1943	0.1956
7	0.1968	0.1980	0.1991	0.2003	0.2015	0.2026	0.2037	0.2049	0.2060	0.2071	0.2081	0.2092
8	0.2103											
9	0.2222											
10	0.2328											
11	0.2425											
12	0.2513											
13	0.2594											
14	0.2669											
15	0.2738											
16	0.2804											
17	0.2865											
18	0.2923											
19	0.2977											
20	0.3029											
21	0.3079											
22	0.3126											
23	0.3170											
24	0.3214											
25	0.3255											
26	0.3294											
27	0.3333											
28	0.3369											
29	0.3405											
30	0.3439											
31	0.3472											
32	0.3504											
33	0.3536											
34	0.3566											
35	0.3595											
36	0.3624											
37	0.3651											
38	0.3678											
39	0.3704											
40	0.3730											
41	0.3755											
42	0.3779											
43	0.3803											
44	0.3826											
45	0.3849											
46	0.3871											
47	0.3893											
48	0.3914											
49	0.3935											

x_d at 50 cycles

$x_d=0.2328\log_{10}d$

d=separation, feet.

60 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	–	−0.3015	−0.2174	−0.1682	−0.1333	−0.1062	−0.0841	−0.0654	−0.0492	−0.0349	−0.0221	−0.0106
1	0	0.0097	0.0187	0.0271	0.0349	0.0423	0.0492	0.0558	0.0620	0.0679	0.0735	0.0789
2	0.0841	0.0891	0.0938	0.0984	0.1028	0.1071	0.1112	0.1152	0.1190	0.1227	0.1264	0.1299
3	0.1333	0.1366	0.1399	0.1430	0.1461	0.1491	0.1520	0.1549	0.1577	0.1604	0.1631	0.1657
4	0.1682	0.1707	0.1732	0.1756	0.1779	0.1802	0.1825	0.1847	0.1869	0.1891	0.1912	0.1933
5	0.1953	0.1973	0.1993	0.2012	0.2031	0.2050	0.2069	0.2087	0.2105	0.2123	0.2140	0.2157
6	0.2174	0.2191	0.2207	0.2224	0.2240	0.2256	0.2271	0.2287	0.2302	0.2317	0.2332	0.2347
7	0.2361	0.2376	0.2390	0.2404	0.2418	0.2431	0.2445	0.2458	0.2472	0.2485	0.2498	0.2511
8	0.2523											
9	0.2666											
10	0.2794											
11	0.2910											
12	0.3015											
13	0.3112											
14	0.3202											
15	0.3286											
16	0.3364											
17	0.3438											
18	0.3507											
19	0.3573											
20	0.3635											
21	0.3694											
22	0.3751											
23	0.3805											
24	0.3856											
25	0.3906											
26	0.3953											
27	0.3999											
28	0.4043											
29	0.4086											
30	0.4127											
31	0.4167											
32	0.4205											
33	0.4243											
34	0.4279											
35	0.4314											
36	0.4348											
37	0.4382											
38	0.4414											
39	0.4445											
40	0.4476											
41	0.4506											
42	0.4535											
43	0.4564											
44	0.4592											
45	0.4619											
46	0.4646											
47	0.4672											
48	0.4697											
49	0.4722											

x_d at 60 cycles

$x_d=0.2794\log_{10}d$

d=separation, feet.

	ρ Meter Ohm	FREQUENCY 25 Cycles	50 Cycles	60 Cycles
r_e	All	0.1192	0.2383	0.2860
x_e	1	0.921	1.736	2.050
	5	1.043	1.980	2.343
	10	1.095	2.085	2.469
	50	1.217	2.329	2.762
	100†	1.270	2.434	2.888
	500	1.392	2.679	3.181
	1000	1.444	2.784	3.307
	5000	1.566	3.028	3.600
	10 000	1.619	3.133	3.726

*From Formulas:

$r_e=0.004764f$

$x_e=0.006985f\log_{10}4\,665\,600\frac{\rho}{f}$

where f=frequency

ρ=Resistivity (meter-ohm)

†This is an average value which may be used in the absence of definite information.

Table 8 Shunt Capacitive Reactance Spacing Factor (x_d') Megohms per Conductor per Mile

Table 9 (Insert, bottom right) Zero-Sequence Shunt Capacitive Reactance Factor

25 CYCLES

SEPARATION

Feet	INCHES 0	1	2	3	4	5	6	7	8	9	10	11
0	—	-0.1769	-0.1276	-0.0987	-0.0782	-0.0623	-0.0494	-0.0384	-0.0289	-0.0205	-0.0130	-0.0062
1	0	0.0057	0.0110	0.0159	0.0205	0.0248	0.0289	0.0327	0.0364	0.0398	0.0432	0.0463
2	0.0494	0.0523	0.0551	0.0577	0.0603	0.0628	0.0652	0.0676	0.0698	0.0720	0.0742	0.0762
3	0.0782	0.0802	0.0821	0.0839	0.0857	0.0875	0.0892	0.0909	0.0925	0.0941	0.0957	0.0972
4	0.0987	0.1002	0.1016	0.1030	0.1044	0.1058	0.1071	0.1084	0.1097	0.1109	0.1122	0.1134
5	0.1146	0.1158	0.1169	0.1181	0.1192	0.1203	0.1214	0.1225	0.1235	0.1246	0.1256	0.1266
6	0.1276	0.1286	0.1295	0.1305	0.1314	0.1324	0.1333	0.1342	0.1351	0.1360	0.1368	0.1377
7	0.1386	0.1394	0.1402	0.1411	0.1419	0.1427	0.1435	0.1443	0.1450	0.1458	0.1466	0.1473
8	0.1481											
9	0.1565											
10	0.1640											
11	0.1707											
12	0.1769											
13	0.1826											
14	0.1879											
15	0.1928											
16	0.1974											
17	0.2017											
18	0.2058											
19	0.2097											
20	0.2133											
21	0.2168											
22	0.2201											
23	0.2233											
24	0.2263											
25	0.2292											
26	0.2320											
27	0.2347											
28	0.2373											
29	0.2398											
30	0.2422											
31	0.2445											
32	0.2468											
33	0.2490											
34	0.2511											
35	0.2532											
36	0.2552											
37	0.2571											
38	0.2590											
39	0.2609											
40	0.2627											
41	0.2644											
42	0.2661											
43	0.2678											
44	0.2695											
45	0.2711											
46	0.2726											
47	0.2742											
48	0.2756											
49	0.2771											

x_d' at 25 cycles
$x_d' = .1640 \log_{10} d$
d = separation, feet.

FUNDAMENTAL EQUATIONS

$$x_1' = x_2' = x_a' + x_d'$$
$$x_o' = x_a' + x_e' - 2x_d'$$

50 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	—	-0.0885	-0.0638	-0.0494	-0.0391	-0.0312	-0.0247	-0.0192	-0.0144	-0.0102	-0.0065	-0.0031
1	0	0.0028	0.0055	0.0079	0.0102	0.0124	0.0144	0.0164	0.0182	0.0199	0.0216	0.0232
2	0.0247	0.0261	0.0275	0.0289	0.0302	0.0314	0.0326	0.0338	0.0349	0.0360	0.0371	0.0381
3	0.0391	0.0401	0.0410	0.0420	0.0429	0.0437	0.0446	0.0454	0.0463	0.0471	0.0478	0.0486
4	0.0494	0.0501	0.0508	0.0515	0.0522	0.0529	0.0535	0.0542	0.0548	0.0555	0.0561	0.0567
5	0.0573	0.0579	0.0585	0.0590	0.0596	0.0601	0.0607	0.0612	0.0618	0.0623	0.0628	0.0633
6	0.0638	0.0643	0.0648	0.0652	0.0657	0.0662	0.0666	0.0671	0.0675	0.0680	0.0684	0.0689
7	0.0693	0.0697	0.0701	0.0705	0.0709	0.0713	0.0717	0.0721	0.0725	0.0729	0.0733	0.0737
8	0.0740											
9	0.0782											
10	0.0820											
11	0.0854											
12	0.0885											
13	0.0913											
14	0.0940											
15	0.0964											
16	0.0987											
17	0.1009											
18	0.1029											
19	0.1048											
20	0.1067											
21	0.1084											
22	0.1100											
23	0.1116											
24	0.1131											
25	0.1146											
26	0.1160											
27	0.1173											
28	0.1186											
29	0.1199											
30	0.1211											
31	0.1223											
32	0.1234											
33	0.1245											
34	0.1255											
35	0.1266											
36	0.1276											
37	0.1286											
38	0.1295											
39	0.1304											
40	0.1313											
41	0.1322											
42	0.1331											
43	0.1339											
44	0.1347											
45	0.1355											
46	0.1363											
47	0.1371											
48	0.1378											
49	0.1386											

x_d' at 50 cycles
$x_d' = 0.08198 \log_{10} d$
d = separation, feet.

60 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	—	-0.0737	-0.0532	-0.0411	-0.0326	-0.0260	-0.0206	-0.0160	-0.0120	-0.0085	-0.0054	-0.0026
1	0	0.0024	0.0046	0.0066	0.0085	0.0103	0.0120	0.0136	0.0152	0.0166	0.0180	0.0193
2	0.0206	0.0218	0.0229	0.0241	0.0251	0.0262	0.0272	0.0282	0.0291	0.0300	0.0309	0.0318
3	0.0326	0.0334	0.0342	0.0350	0.0357	0.0365	0.0372	0.0379	0.0385	0.0392	0.0399	0.0405
4	0.0411	0.0417	0.0423	0.0429	0.0435	0.0441	0.0446	0.0452	0.0457	0.0462	0.0467	0.0473
5	0.0478	0.0482	0.0487	0.0492	0.0497	0.0501	0.0506	0.0510	0.0515	0.0519	0.0523	0.0527
6	0.0532	0.0536	0.0540	0.0544	0.0548	0.0552	0.0555	0.0559	0.0563	0.0567	0.0570	0.0574
7	0.0577	0.0581	0.0584	0.0588	0.0591	0.0594	0.0598	0.0601	0.0604	0.0608	0.0611	0.0614
8	0.0617											
9	0.0652											
10	0.0683											
11	0.0711											
12	0.0737											
13	0.0761											
14	0.0783											
15	0.0803											
16	0.0823											
17	0.0841											
18	0.0858											
19	0.0874											
20	0.0889											
21	0.0903											
22	0.0917											
23	0.0930											
24	0.0943											
25	0.0955											
26	0.0967											
27	0.0978											
28	0.0989											
29	0.0999											
30	0.1009											
31	0.1019											
32	0.1028											
33	0.1037											
34	0.1046											
35	0.1055											
36	0.1063											
37	0.1071											
38	0.1079											
39	0.1087											
40	0.1094											
41	0.1102											
42	0.1109											
43	0.1116											
44	0.1123											
45	0.1129											
46	0.1136											
47	0.1142											
48	0.1149											
49	0.1155											

x_d' at 60 cycles
$x_d' = 0.06831 \log_{10} d$
d = separation, feet.

Conductor Height Above Ground Feet	FREQUENCY 25 Cycles	50 Cycles	60 Cycles
10	0.640	0.320	0.267
15	0.727	0.363	0.303
20	0.788	0.394	0.328
25	0.836	0.418	0.348
30	0.875	0.437	0.364
40	0.936	0.468	0.390
50	0.984	0.492	0.410
60	1.023	0.511	0.426
70	1.056	0.528	0.440
80	1.084	0.542	0.452
90	1.109	0.555	0.462
100	1.132	0.566	0.472

$$x_0' = \frac{12\ 30}{f} \log_{10} 2\,h$$

where h = height above ground.
f = frequency.

Table 10 Typical Constants of Three-Phase Synchronous Machines

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	x_d (unsat)	x_q rated current	x_d' rated voltage	x_d'' rated voltage	x_2 rated current	(*) x_0 rated current	x_p	(†) r_2	(‡) r_1	(‡) r_a	T_{d0}'	T_d'	T_d''	T_a	H
2-Pole turbine generators	1.20 0.95–1.45	1.16 0.92–1.42	0.15 0.12–0.21	0.09 0.07–0.14	$=x_d''$	0.03 0.01–0.08	0.10 0.07–0.17	0.025–0.04	0.004–0.011	0.001–0.007	5.0	0.6	0.035 0.02–0.05	0.13 0.04–0.24	
4-Pole turbine generators	1.20 1.00–1.45	1.16 0.92–1.42	0.23 0.20–0.28	0.14 0.12–0.17	$=x_d''$	0.08 0.015–0.14	0.17 0.12–0.24	0.03–0.045	0.003–0.008	0.001–0.005	8.0	1.0	0.035 0.02–0.05	0.20 0.15–0.35	
Salient-pole generators and motors (with dampers)	1.25 0.60–1.50	0.70 0.40–0.80	0.30 0.20–0.50(*)	0.20 0.13–0.32(*)	0.20 0.13–0.32(*)	0.18 0.03–0.23	0.28 0.17–0.40	0.012–0.020	0.005–0.020	0.003–0.015	3.0–5.0 1.5–10	1.5 0.5–3.3	0.035 0.01–0.05	0.15 0.03–0.25	
Salient-pole generators (without dampers)	1.25 0.60–1.50	0.70 0.40–0.80	0.30 0.20–0.50(*)	0.30 0.20–0.50(*)	0.48 0 35–0.65	0.19 0.03–0.24	0.28 0.17–0.40	0.03–0.045	0.005–0.020	0.003–0.015	3.0–5.0 1.5–10	1.5 0.5–3.3	—	0.30 0.10–0.50	
Condensers air cooled	1.85 1.25–2.20	1.15 0.95–1.30	0.40 0.30–0.50	0.27 0.19–0.30	0.26 0.18–0.40	0.12 0.025–0.15	0.25 0.20–0.35	0.025–0.07	0.0065 0.004–0.009	0.0035 0.0025–0.008	9.0 6.0–14.0	2.0 1.2–2.8	0.035 0.02–0.04	0.17 0.1–0.3	Large 2.4 Small 1.0
Condensers hydrogen cooled at ½ psi kva rating	2.20 1.50–2.65	1.35 1.10–1.55	0.48 0.36–0.60	0.32 0.23–0.36	0.31 0.22–0.48	0.14 0.030–0.18	0.27· 0.22–0.37	0.025–0.07	0.0065 0.005–0.009	0.0035 0.0025–0.005	9.0 6.0–14.0	2.0 1.2–2.8	0.035 0.02–0.04	0.20 0.15–0.3	Large 2.0 Small 1.10

(*) High speed units tend to have low reactance and low speed units high reactance.
(*) X_0 varies so critically with armature winding pitch that an average value can hardly be given. Variation is from 0.1 to 0.7 of x_d''. Low limit is for ⅔ pitch windings.

(†) r_2 varies with damper resistance.
(‡) r_1 and r_a vary with machine rating, limiting values given are for about 50,000 kva and 500 kva.

Appendix B

Vectors and Matrices

B.1 DEFINITION OF A VECTOR

A vector $\mathbf{X}$ is defined as an ordered set of elements. The elements $x_1, x_2, \ldots, x_N$ may be real numbers, complex numbers, or functions of some dependent variable. An N-dimensional vector is shown in Eq. B.1.

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} \tag{B.1}$$

B.2 VECTOR ALGEBRA

Equality of Vectors Two vectors $\mathbf{X}$ and $\mathbf{Y}$ are said to be equal if and only if they have the same size and corresponding elements are equal; that is, $\mathbf{X} = \mathbf{Y}$ iff $x_i = y_i$ for all i.

Product of a Vector with a Scalar To multiply a vector $\mathbf{X}$ by a scalar λ, multiply each element x_i by λ, that is,

$$\lambda\mathbf{X} = \mathbf{X}\lambda = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \\ \vdots \\ \lambda x_N \end{bmatrix} \tag{B.2}$$

Addition (Subtraction) of Vectors The sum (difference) of two vectors $\mathbf{X}$ and $\mathbf{Y}$ results in a new vector $\mathbf{Z}$, which is obtained by adding (subtracting) corresponding elements as follows:

$$\mathbf{Z} = \mathbf{X} \pm \mathbf{Y} \begin{bmatrix} x_1 \pm y_1 \\ x_2 \pm y_2 \\ x_3 \pm y_3 \\ \vdots \\ x_N \pm y_N \end{bmatrix} \tag{B.3}$$

Inner (Dot) Product The inner or dot product of two vectors $\mathbf{X}$ and $\mathbf{Y}$ of equal dimension or size is obtained as follows:

$$\mathbf{X} \cdot \mathbf{Y} = \langle \mathbf{X}, \mathbf{Y} \rangle = \sum_{k=1}^{N} x_k y_k = x_1 y_1 + x_2 y_2 + \cdots + x_N y_N \tag{B.4}$$

If the inner or dot product of two nonzero vectors (each having at least one nonzero element) is equal to zero, the vectors are said to be orthogonal.

B.3 DEFINITION OF A MATRIX

An $M \times N$ matrix $\mathbf{A}$ is defined as a rectangular array of MN elements as shown in Eq. B.5. The MN elements a_{ij} may be real numbers, complex numbers, or functions of some independent variable. The double-subscript notation identifies the position of the element in the array; for example, element a_{ij} is found at the ith row and the jth column. The order (size) of the given matrix $\mathbf{A}$ is said to be $M \times N$ because the matrix contains M rows and N columns.

$$\mathbf{A}_{M\times N} \triangleq \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MN} \end{bmatrix} \triangleq [a_{ij}]_{M\times N} \tag{B.5}$$

A *square matrix* $\mathbf{A}$ is a matrix in which the number of rows is equal to the number of columns; that is, $M = N$. For a square matrix, the main (or principal) diagonal consists of the elements of the form a_{ii}.

A *diagonal matrix* is a square matrix in which all the off-diagonal elements are equal to zero; that is, $a_{ij} = 0$ for all $i \neq j$. The *identity* or *unit matrix* is a diagonal matrix whose elements in the principal diagonal are all equal to unity. The *null matrix* has elements that are all equal to zero.

The matrix $\mathbf{A}^{\mathrm{T}}$ is called the *transpose* of $\mathbf{A}$ if the element a_{ij} in $\mathbf{A}$ is equal to element a_{ji} in $\mathbf{A}^{\mathrm{T}}$ for all i and j. In general, $\mathbf{A}^{\mathrm{T}}$ is formed by interchanging the rows and columns of $\mathbf{A}$.

A *symmetric matrix* has the property $a_{ij} = a_{ji}$ for all i and j; that is, it is symmetric about the principle diagonal. Therefore, it is equal to its own transpose matrix; thus, $\mathbf{A} = \mathbf{A}^{\mathrm{T}}$.

B.4 MATRIX ALGEBRA

Matrix operations are defined for addition, subtraction, and multiplication. Division is not defined; however, it is replaced by matrix inversion.

Equality of Matrices Two matrices $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ are said to be equal matrices if and only if (a) they have the same order (size) and (b) each element a_{ij} is equal to the corresponding b_{ij} for all i and j.

Addition (Subtraction) of Matrices Two matrices $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ can be added (subtracted) if they are of the same order. The sum (difference) $\mathbf{C} = \mathbf{A} \pm \mathbf{B}$ is obtained by adding (subtracting) corresponding elements. Thus,

$$\mathbf{C}_{M\times N} = [c_{ij}]_{M\times N} = \mathbf{A}_{M\times N} \pm \mathbf{B}_{M\times N} = [a_{ij} \pm b_{ij}]_{M\times N} \tag{B.6}$$

Product of a Matrix with a Scalar A matrix is multiplied by a scalar λ by multiplying all mn elements by λ; that is,

$$\lambda\mathbf{A}_{M\times N} = \mathbf{A}_{MN}\lambda = \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} & \cdots & \lambda a_{1N} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} & \cdots & \lambda a_{2N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \lambda a_{M1} & \lambda a_{M2} & \lambda a_{M3} & \cdots & \lambda a_{MN} \end{bmatrix} \tag{B.7}$$

Product of Matrices Two matrices $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ can be multiplied in the order $\mathbf{AB}$ if and only if the number of columns of $\mathbf{A}$ is equal to the number of rows of $\mathbf{B}$. That is, if $\mathbf{A}$ is of order $(M \times P)$, then $\mathbf{B}$ has to be of order $(P \times N)$ where M and N are arbitrary. If the product matrix is denoted by $\mathbf{C} = \mathbf{AB}$, then $\mathbf{C}$ is of order $(M \times N)$. Its elements c_{ij} are given by

$$c_{ij} = \Sigma_{k=1}^{P} a_{ik} b_{kj} \quad \text{for } i = 1, 2, \cdots, M,\ j = 1, 2, \cdots, N \tag{B.8}$$

EXAMPLE B.1

For the following two matrices $\mathbf{A}$ and $\mathbf{B}$, find the matrix product $\mathbf{C} = \mathbf{AB}$.

$$\mathbf{A} = \begin{bmatrix} 1\,4 \\ 2\,5 \\ 3\,6 \end{bmatrix}_{3\times 2} \qquad \mathbf{B} = \begin{bmatrix} 7\,8 \\ 9\,0 \end{bmatrix}_{2\times 2}$$

Solution

$$\mathbf{C} = \begin{bmatrix} 1\,4 \\ 2\,5 \\ 3\,6 \end{bmatrix}_{3\times 2} \begin{bmatrix} 7\,8 \\ 9\,0 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} (1\times 7 + 4\times 9) & (1\times 8 + 4\times 0) \\ (2\times 7 + 5\times 9) & (2\times 8 + 5\times 0) \\ (3\times 7 + 6\times 9) & (3\times 8 + 6\times 0) \end{bmatrix}_{3\times 2} = \begin{bmatrix} 43 & 8 \\ 59 & 16 \\ 75 & 24 \end{bmatrix}_{3\times 2}$$

In general, the matrix multiplication is not commutative; that is, $\mathbf{AB} \neq \mathbf{BA}$ even if $\mathbf{AB}$ is defined. Indeed, the product $\mathbf{BA}$ may not even be defined, as in Example B.1.

B.5 INVERSE OF A MATRIX

Consider two N-square matrices $\mathbf{C}$ and $\mathbf{D}$. If $\mathbf{CD} = \mathbf{DC} = \mathbf{I}$, then $\mathbf{C}$ is called the inverse of $\mathbf{D}$. Conversely, $\mathbf{D}$ is called the inverse of $\mathbf{C}$. The inverse matrix is written as $\mathbf{C}^{-1}$ or $\mathbf{D}^{-1}$.

An Application of Matrix Inversion The inverse matrix may be used to solve a system of linear algebraic equations. Consider the following matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{N1} & a_{N2} & a_{N3} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \tag{B.9}$$

where x_i represent the unknowns, a_{ij} are the coefficients, and b_i are constants. These N equations can be written in compact form as

$$\mathbf{AX} = \mathbf{B} \tag{B.10}$$

If the inverse of A exists, then multiplying Eq. B.10 by $\mathbf{A}^{-1}$ yields

$$\mathbf{A}^{-1}\mathbf{A}\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} \quad \text{or} \quad \mathbf{X} = \mathbf{A}^{-1}\mathbf{B} \tag{B.11}$$

Gauss–Jordan Method of Finding the Inverse Matrix Form the augmented matrix $\mathbf{A}^{\text{aug}} = [\mathbf{A} \,|\, \mathbf{I}]$. Apply elementary row operations on $\mathbf{A}^{\text{aug}}$ to transform the given matrix **A** into a unit matrix; simultaneously, the unit matrix **I** is transformed into the inverse matrix $\mathbf{A}^{-1}$.

The elementary row operations used are the following:

1. Multiplication of a row by any nonzero scalar
2. Interchange of any two rows
3. Addition of any multiple of one row to another row

EXAMPLE B.2

Solve the following system of equations.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

Solution To find the solution of the system of equations, the inverse of the coefficient matrix **A** is derived as follows:

Step 1 The augmented matrix is first formed.

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

Step 2 $(-2R_1 + R_2)$; $(-R_1 + R_3)$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -2 & 1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 1 \end{array}\right]$$

Step 3 $-R_2$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 1 \end{array}\right]$$

Step 4 $(-2R_2 + R_1)$; $(R_2 + R_3)$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 \end{array}\right]$$

Step 5 $\frac{1}{2}R_3$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

Step 6 $(-R_3 + R_1)$; $(R_3 + R_2)$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

Thus, $\mathbf{A}^{-1}$ is found to be

$$\mathbf{A}^{-1} = \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Therefore, the solution vector $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ is found as follows:

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

B.6 PARTITIONING OF MATRICES

A large matrix can be subdivided into several submatrices of smaller dimension. Consider an $(M \times P)$ matrix **A** and a $(P \times N)$ matrix **B**. **A** and **B** may be partitioned as follows:

$$\mathbf{A} = \left[\begin{array}{c|c|c} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} \\ \hline \end{array}\right] \tag{B.12}$$

$$\mathbf{B} = \left[\begin{array}{c|c} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \hline \mathbf{B}_{21} & \mathbf{B}_{22} \\ \hline \mathbf{B}_{31} & \mathbf{B}_{32} \end{array}\right] \tag{B.13}$$

Product of Partitioned Matrices If the submatrices $\mathbf{A}_{ij}$ and $\mathbf{B}_{ij}$ are chosen such that the number of columns of $\mathbf{A}_{ij}$ is equal to the number of rows of $\mathbf{B}_{ji}$ and such that the numbers of columns of $\mathbf{A}_{ij}$ and $\mathbf{A}_{(i+1)j}$ are equal, for all i and j, then the matrix product $\mathbf{C} = \mathbf{AB}$ is found as

$$\mathbf{C} = \mathbf{AB} = \left[\begin{array}{c|c} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \hline \mathbf{C}_{21} & \mathbf{C}_{22} \end{array}\right] \tag{B.14}$$

where

$$\mathbf{C}_{11} = \mathbf{A}_{11}\mathbf{B}_{11} + \mathbf{A}_{12}\mathbf{B}_{21} + \mathbf{A}_{13}\mathbf{B}_{31} \qquad \mathbf{C}_{12} = \mathbf{A}_{11}\mathbf{B}_{12} + \mathbf{A}_{12}\mathbf{B}_{22} + \mathbf{A}_{13}\mathbf{B}_{32}$$
$$\mathbf{C}_{21} = \mathbf{A}_{21}\mathbf{B}_{11} + \mathbf{A}_{22}\mathbf{B}_{21} + \mathbf{A}_{23}\mathbf{B}_{31} \qquad \mathbf{C}_{22} = \mathbf{A}_{21}\mathbf{B}_{12} + \mathbf{A}_{22}\mathbf{B}_{22} + \mathbf{A}_{23}\mathbf{B}_{32}$$

EXAMPLE B.3

Solve the following system of linear equations.

$$\left[\begin{array}{cc|cc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \\ \hline x_3 \\ x_4 \end{array}\right] = \left[\begin{array}{c} b_1 \\ b_2 \\ \hline 0 \\ 0 \end{array}\right]$$

Solution The system of equations can be written as follows:

$$\left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{X}_1 \\ \hline \mathbf{X}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{B}_1 \\ \hline \mathbf{0} \end{array}\right]$$

Expanding,

$$\mathbf{A}_{11}\mathbf{X}_1 + \mathbf{A}_{12}\mathbf{X}_2 = \mathbf{B}_1$$
$$\mathbf{A}_{21}\mathbf{X}_1 + \mathbf{A}_{22}\mathbf{X}_2 = \mathbf{0}$$

By using the latter equation, the subvector $\mathbf{X}_2$ can be expressed (provided the inverse of $\mathbf{A}_{22}$ exists) in terms of $\mathbf{X}_1$ as follows:

$$\mathbf{X}_2 = -\mathbf{A}_{22}^{-1}\mathbf{A}_{21}\mathbf{X}_1$$

This last expression is substituted for $\mathbf{X}_2$ in the other equation, and the expression for $\mathbf{X}_1$ is derived as

$$\mathbf{X}_1 = (\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21})^{-1}\mathbf{B}_1$$

provided the inverse of the quantity inside the parentheses exists.

Appendix C

Solutions of Linear Equations

There are several ways to obtain the solution of a system of N simultaneous linear equations in N unknowns. Among these are

1. Matrix inversion
2. Gaussian elimination
3. Triangular factorization

Matrix inversion was discussed in Section B.5. In the following sections, the other two solution techniques are described.

C.1 GAUSSIAN ELIMINATION

Consider the following system of linear algebraic equations.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \tag{C.1}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \tag{C.2}$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \tag{C.3}$$

The Gaussian elimination technique proceeds as follows:

Step 1 Normalize the first of the equations; that is, divide Eq. C.1 by a_{11}. Thus,

$$x_1 + U_{12}x_2 + U_{13}x_3 = z_1 \tag{C.4}$$

where

$$\begin{aligned} U_{12} &= a_{12}/a_{11} \\ U_{13} &= a_{13}/a_{11} \\ z_1 &= b_1/a_{11} \end{aligned}$$

Step 2 Multiply Eq. C.4 by a_{21}, and subtract the result from Eq. C.2 eliminating x_1. This yields

$$a'_{22}x_2 + U'_{23}x_3 = z'_2 \tag{C.5}$$

where

$$\begin{aligned} a'_{22} &= a_{22} - a_{21}U_{12} \\ U'_{23} &= a_{23} - a_{21}U_{13} \\ z'_2 &= b_2 - a_{21}z_1 \end{aligned}$$

Step 3 Multiply Eq. C.4 by a_{31}, and subtract the result from Eq. C.3 eliminating x_1. This yields

$$a'_{32}x_2 + U'_{33}x_3 = z'_3 \tag{C.6}$$

where

$$\begin{aligned} a'_{32} &= a_{32} - a_{31}U_{12} \\ a'_{33} &= a_{33} - a_{31}U_{13} \\ z'_3 &= b_3 - a_{31}z_1 \end{aligned}$$

Step 4 Repeat the general procedure of steps 1–3 on Eqs. C.5 and C.6 to eliminate x_2. Thus, dividing Eq. C.5 by a'_{22} yields

$$x_2 + U_{23}x_3 = z_2 \tag{C.7}$$

where

$$\begin{aligned} U_{23} &= U'_{23}/a'_{22} \\ z_2 &= z'_2/a'_{22} \end{aligned}$$

Multiply Eq. C.7 by a'_{32} and subtract from Eq. C.6; thus,

$$a''_{33}x_3 = z''_3 \tag{C.8}$$

where

$$a''_{33} = U'_{33} - a'_{32}U_{23}$$
$$z''_3 = z'_3 - a'_{32}z_2$$

Dividing Eq. C.8 by a'_{33}, the value of x_3 is found as follows:

$$x_3 = z_3 \tag{C.9}$$

where $z_3 = z''_3/a''_{33}$.

Step 5 Substitute the value of x_3 back into Eq. C.7 to solve for the value of x_2.

$$x_2 = z_2 - U_{23}x_3 \tag{C.10}$$

Next, substitute the values of x_2 and x_3 into Eq. C.4 to obtain the value of x_1.

$$x_1 = z_1 - U_{12}x_2 - U_{13}x_3 \tag{C.11}$$

This final process is referred to as back-substitution.

EXAMPLE C.1

Solve the following system by using the Gaussian elimination technique.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

Solution First, augment the constants to the coefficient matrix. The Gaussian elimination procedure is followed.

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 3 & -1 & 3 \\ 1 & 1 & 2 & -3 \end{array}\right]$$

Step 1 Since the first coefficient of the first row is already unity, continue with step 2.

Step 2 $-2R_1 + R_2$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -1 & 1 & -1 \\ 1 & 1 & 2 & -3 \end{array}\right]$$

Step 3 $-R_1 + R_3$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & -1 & 3 & -5 \end{array}\right]$$

Step 4a $-R_2$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 3 & -5 \end{array}\right]$$

Step 4b $R_2 + R_3$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

Step 4c $\frac{1}{2}R_3$

$$\mathbf{A}^{\text{aug}} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

The value of x_3 is read off as the last element in the last column of the final augmented matrix $\mathbf{A}^{\text{aug}}$. Therefore,

$$x_3 = -2$$

Step 5 Substitute the value of x_3 back into the second of the equations resulting from step 4a to solve for the value of x_2.

$$x_2 = 1 - (-1)(-2) = -1$$

Next, substitute the values of x_2 and x_3 into the first of the equations in step 1 to obtain the value of x_1.

$$x_1 = 2 - (2)(-1) - (-1)(-2) = 2$$

Thus, the back-substitution process yields the solution, which is presented in vector form as

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

C.2 TRIANGULAR FACTORIZATION

Triangular factorization is a modification of the Gaussian elimination technique. It is better adapted to computer use, particularly for a repeat solution of the system of equations with a new vector of right-hand side constants.

Consider the following system of three simultaneous linear algebraic equations in terms of the three unknown variables x_1, x_2, and x_3, where a_{ij} are constants for all i and j.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \tag{C.12}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \tag{C.13}$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \tag{C.14}$$

These equations can also be written as follows:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{C.15}$$

In matrix notation, Eq. C.15 can be expressed as

$$\mathbf{AX} = \mathbf{B} \tag{C.16}$$

Assume that the coefficient matrix **A** can be written as the product of two matrices:

$$\mathbf{A} = \mathbf{LU} \tag{C.17}$$

Let **L** be a lower triangular matrix whose elements above the main or principal diagonal are all equal to zero. Also, let **U** be an upper triangular matrix whose elements on the principal diagonal are all unity and elements below the principal diagonal are all equal to zero. The matrices **L** and **U** are identified as the triangular factors of the coefficient matrix **A**, and their standard forms are shown in Eqs. C.18 and C.19.

$$\mathbf{L} = \begin{bmatrix} L_{11} & 0 & 0 & \cdots & 0 \\ L_{21} & L_{22} & 0 & \cdots & 0 \\ L_{31} & L_{32} & L_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{N1} & L_{N2} & L_{N3} & \cdots & L_{NN} \end{bmatrix} \tag{C.18}$$

$$\mathbf{U} = \begin{bmatrix} 1 & U_{12} & U_{13} & \cdots & U_{1N} \\ 0 & 1 & U_{23} & \cdots & U_{2N} \\ 0 & 0 & 1 & \cdots & U_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{C.19}$$

Equation C.16 can, therefore, be written as follows:

$$\mathbf{AX} = \mathbf{LUX} = \mathbf{B} \tag{C.20}$$

Equation C.20 may also be written as

$$\mathbf{LZ} = \mathbf{B} \tag{C.21}$$

where

$$\mathbf{UX} = \mathbf{Z} \tag{C.22}$$

The vector **Z** is found by solving Eq. C.21; this process is called the forward pass. Then the solution vector **X** is found by solving Eq. C.22 using the previously determined vector **Z**; this is the backward substitution process.

The matrix triangular factors **L** and **U** are found by multiplying Eqs. C.18 and C.19 and equating the product to the coefficient matrix **A**. The following relationships are derived.

$$L_{ij} = a_{ij} - \sum_{k=1}^{j-1} L_{ik}U_{kj} \quad \text{for } i \geq j \tag{C.23}$$

$$U_{ij} = \frac{1}{L_{ii}}\left(a_{ij} - \sum_{k=1}^{i-1} L_{ik}U_{kj}\right) \quad \text{for } i < j \tag{C.24}$$

Equations C.23 and C.24 are used alternately to evaluate the elements of the triangular factors. The first column of **L** is initially determined by using Eq. C.23, followed by the first row of **U** by using Eq. C.24. Next, the second column of **L** is computed, followed by the second row of **U**, and so forth.

Once the triangular factors **L** and **U** are found, they are used for repeat solutions of the system of equations. That is, for any vector of constants **B**, there is no need to recalculate the elements of the vectors **L** and **U**. The solutions of Eqs. C.21 and C.22 may be implemented by using the following relations.

$$z_i = \frac{1}{L_{ii}}\left(b_i - \sum_{k=1}^{i-1} L_{ik}z_k\right) \quad \text{for } i = 1, 2, \ldots, N \tag{C.25}$$

$$x_i = z_i - \sum_{k=i+1}^{N} U_{ik}x_k \quad \text{for } i = N, N-1, \ldots, 2, 1 \tag{C.26}$$

EXAMPLE C.2

Solve Example C.1 by using the triangular factorization technique.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

Solution By using Eqs. C.23 and C.24, the triangular factors are found as follows:

$$L_{11} = a_{11} = 1$$
$$L_{21} = a_{21} = 2$$
$$L_{31} = a_{31} = 1$$
$$U_{12} = \frac{1}{L_{11}}a_{12} = \frac{1}{1}(2) = 2$$
$$U_{13} = \frac{1}{L_{11}}a_{13} = \frac{1}{1}(-1) = -1$$
$$L_{22} = a_{22} - L_{21}U_{12} = 3 - (2)(2) = -1$$
$$L_{32} = a_{32} - L_{31}U_{12} = 1 - (1)(2) = -1$$
$$U_{23} = \frac{1}{L_{22}}(a_{23} - L_{21}U_{13}) = \frac{1}{-1}[-1 - (2)(-1)] = -1$$
$$\begin{aligned} L_{33} &= a_{33} - L_{31}U_{13} - L_{32}U_{23} \\ &= 2 - (1)(-1) - (-1)(-1) = 2 \end{aligned}$$

The forward pass is performed by using Eq. C.25 to solve for the $\mathbb{Z}$ vector as follows:

$$z_1 = \frac{1}{L_{11}}b_1 = \frac{1}{1}(2) = 2$$
$$z_2 = \frac{1}{L_{22}}(b_2 - L_{21}z_1) = \frac{1}{-1}[3 - (2)(2)] = 1$$

$$\begin{aligned} z_3 &= \frac{1}{L_{33}}(b_3 - L_{31}z_1 - L_{32}z_2) \\ &= \tfrac{1}{2}[-3 - (1)(2) - (-1)(1)] = -2 \end{aligned}$$

By using Eq. C.26, the back substitution process yields

$$x_3 = z_3 = -2$$
$$x_2 = z_2 - U_{23}z_3 = 1 - (-1)(-2) = -1$$
$$x_1 = z_1 - U_{12}x_2 - U_{13}x_3 = 2 - (2)(-1) - (-1)(-2) = 2$$

Appendix D

Maxwell's Equations

Maxwell's equations are a collection, and are generalizations, of various phenomena and laws previously described by different scientists on the relationships among currents, charges, electric fields, and magnetic fields. These fields have both magnitudes and directions; thus, they are represented as vectors.

Maxwell's equations can be written in either integral or differential form. They are first presented in integral form in the next section, and the differential form is given in the following section.

D.1 INTEGRAL FORM OF MAXWELL'S EQUATIONS

Maxwell's equations in integral form are used to analyze electromagnetic systems that exhibit symmetry (e.g., rectangular, cylindrical) with respect to one or more dimensions that are usually found in electromechanical conversion devices and systems. These equations describe the relationships among electric fields, magnetic fields, charge densities, and current densities over a specified area or volume in space. The following notation is used.

$\mathbf{E}$ = electric field intensity (volts/meter)
$\mathbf{H}$ = magnetic field intensity (amperes/meter)
$\mathbf{D}$ = electric flux density (coulombs/square meter)
$\mathbf{B}$ = magnetic flux density (webers/square meter)
$\mathbf{M}_\mathrm{i}$ = applied magnetic current density (volts/square meter)
$\mathbf{J}_\mathrm{i}$ = applied electric current density (amperes/square meter)
ρ_e = electric charge density (coulombs/cubic meter)
ρ_m = magnetic charge density (webers/cubic meter)

The first of Maxwell's equations in integral form can be written as

$$\oint_C \mathbf{E}\cdot\mathbf{dl} = -\iint_S \mathbf{M}_\mathrm{i}\cdot\mathbf{dS} - \frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot\mathbf{dS} \qquad \text{(D.1)}$$

where $\mathbf{dS}$ is the normal vector to the surface $\mathbf{S}$ which is bounded by the contour C. When there is no applied magnetic current density ($\mathbf{M}_\mathrm{i} = 0$), Eq. D.1 reduces to Faraday's law, which states that the electromagnetic force (emf) induced across the open-circuited terminals of a coil is equal to the time rate of change of magnetic flux linking the coil.

The second of Maxwell's equations in integral form can be written as

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = -\iint_S \mathbf{J}_\mathrm{i}\cdot\mathbf{dS} - \frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot\mathbf{dS} \qquad \text{(D.2)}$$

In the absence of the applied electric current density ($\mathbf{J}_\mathrm{i} = 0$), Eq. D.2 reduces to Ampère's law, which states that the line integral of the electric field about any closed path is equal to the current enclosed by that path.

The next two Maxwell's equations in integral form can be written as follows:

$$\oint\oint_S \mathbf{D}\cdot\mathbf{dS} = \iiint_V \rho_\mathrm{e}\,dv = Q_\mathrm{e} \qquad \text{(D.3)}$$

$$\oint\oint_S \mathbf{B}\cdot\mathbf{dS} = \iiint_V \rho_\mathrm{m}\,dv = 0 \qquad \text{(D.4)}$$

Equations D.3 and D.4 are Gauss's law for electric fields and magnetic fields, respectively. Gauss's law for electric fields states that the total electric flux passing through a closed surface is equal to the total charge enclosed by that surface. Since no magnetic source for magnetic flux lines has ever been discovered, the right-hand side of Gauss's law for magnetic fields is identically equal to zero.

D.2 DIFFERENTIAL FORM OF MAXWELL'S EQUATIONS

Maxwell's equations in differential form are used to describe the relationships among electric fields, magnetic fields, current densities, and charge densities at any point in space at any time. It is assumed that the field vectors are analytic functions of both position and time with continuous derivatives. Electromagnetic fields exhibit these characteristics except where there are abrupt changes in charge and current densities, which usually occur when there are changes in the properties of the medium or flux path. At these discontinuities, the associated boundary conditions need to be specified.

Maxwell's equations in differential form can be written as

$$\nabla \times \mathbf{E} = -\mathbf{M}_i - \frac{\partial \mathbf{B}}{\partial t} \tag{D.5}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_i + \frac{\partial \mathbf{D}}{\partial t} \tag{D.6}$$

$$\nabla \cdot \mathbf{D} = \rho_e \tag{D.7}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{D.8}$$

Appendix E

Constants and Conversion Factors

E.1 CONSTANTS

Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$ F/m
Resistivity of annealed copper	$\rho = 1.72 \times 10^{-8}$ Ω-m
Acceleration due to gravity	$g = 9.807$ m/s^2

E.2 CONVERSION FACTORS

Length	1 meter (m)	= 3.281 feet (ft) = 39.36 inches (in)
	1 mile (mi)	= 1.609 kilometers (km)
Mass	1 kilogram (kg)	= 0.0685 slug = 2.205 pounds (lb)
Time	1 second (s)	= 1/60 minute (min) = 1/3600 hour (h)
Force	1 Newton (N)	= 0.02248 lb force (lbf) = 0.102 kg force
Torque	1 N-m	= 0.7376 lbf-ft
Moment of inertia	1 kg-m^2	= 23.7 lb-ft^2

Power	1 watt (W)	= 0.7376 ft-lbf/s
	1 horsepower (hp)	= 746 W
Energy	1 joule (J)	= 1 W-s = 0.7376 ft-lbf = 2.778×10^{-7} kWh
Current	1 ampere (A)	= 1 coulomb/s
Voltage	1 volt (V)	= 1 watt/ampere
Magnetic flux	1 weber (Wb)	= 10^8 maxwells or lines
Magnetic flux density	1 tesla (T)	= 1 Wb/m^2 = 64,500 lines/in^2
Magnetic field intensity	1 A-turn/m	= 0.0254 A-turn/in
Flux linkage	1 Wb-turn	
Resistance	1 ohm (Ω)	
Inductance	1 henry (H)	
Capacitance	1 farad (F)	

Appendix F

Trigonometric Identities

a. $\sin^2\theta + \cos^2\theta = 1$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

c. $\sin 2\theta = 2\sin\theta\cos\theta$

d. $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

e. $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

f. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

g. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

h. $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

i. $\cos\theta + \cos(\theta - 120°) + \cos(\theta - 240°) = 0$

j. $\sin\theta + \sin(\theta - 120°) + \sin(\theta - 240°) = 0$

k. $\cos^2\theta + \cos^2(\theta - 120°) + \cos^2(\theta - 240°) = \frac{3}{2}$

l. $\sin^2\theta + \sin^2(\theta - 120°) + \sin^2(\theta - 240°) = \frac{3}{2}$

m. $\sin\theta\cos\theta + \sin(\theta - 120°)\cos(\theta - 120°) + \sin(\theta - 240°)\cos(\theta - 240°) = 0$

n. $\tan\theta = \sin\theta/\cos\theta$

o. $\cot\theta = \cos\theta/\sin\theta$

p. $\sec\theta = 1/\cos\theta$

q. $\csc\theta = 1/\sin\theta$

r. $d(\sin\theta) = \cos\theta \, d\theta$

s. $d(\cos\theta) = -\sin\theta \, d\theta$

t. $d(\tan\theta) = \sec^2\theta \, d\theta$

u. $d(\cot\theta) = -\csc^2\theta \, d\theta$

v. $d(\sec\theta) = \sec\theta \tan\theta \, d\theta$

w. $d(\csc\theta) = -\csc\theta \cot\theta \, d\theta$

Appendix G

Glossary

The key terms as well as some of the most commonly used terms that have been highlighted in this book are defined in the following pages. Most of the definitions given in this glossary are based on the *IEEE Standard Dictionary of Electrical and Electronics Terms,* 4th ed, IEEE, New York, 1988.

Air-gap power The power transferred across the air gap from the stator to the rotor.

Aluminum cable steel-reinforced (ACSR) A composite conductor made up of a combination of aluminum wires surrounding the steel.

Ampère's law The magnetic field strength, at any point in the neighborhood of a circuit in which there is a current i, is equal to the vector sum of the contributions from all the differential elements of the circuit. The contribution, $d\mathbf{H}$, caused by a current i in an element $\mathbf{ds}$ at a distance $\mathbf{r}$ from a point P is given by

$$d\mathbf{H} = \frac{i[\mathbf{r} \times \mathbf{ds}]}{r^2}$$

An-bn-cn or abc sequence The order in which the successive members of the set reach their positive maximum values. Phase a is followed by phase b and then by phase c.

Apparent power The product of the root-mean-square voltage and the root-mean-square current.

Armature winding The winding in which alternating voltage is generated by virtue of relative motion with respect to a magnetic flux field.

Autotransformer A transformer in which at least two windings have a common section.

Average power The time average of the instantaneous power, the average being taken over one period.

Back emf. *See* Counter emf.

Balanced set A set of phasor currents (or phasor voltages) that have equal magnitudes and are separated from each other by equal phase angles.

Blocked-rotor test A test applied to an induction motor, in which the rotor is blocked so it cannot rotate.

Bundled conductors An assembly of two or more conductors used as a single conductor and employing spacers to maintain a predetermined configuration. The individual conductors of this assembly are called subconductors.

Bus A conductor, or group of conductors, that serve as a common connection for two or more circuits.

Bus admittance matrix A matrix whose elements have the dimension of admittance and, when multiplied into the vector of bus voltages, gives the vector of bus currents.

Capacitive reactance at 1-ft spacing, x'_a Capacitive reactance of one conductor to neutral of a circuit consisting of two conductors 1 foot apart.

Capacitive reactance spacing factor, x'_d Capacitive reactance of one conductor of a circuit consisting of two conductors separated by a distance D expressed in feet.

Circuit breaker A switching device capable of making, carrying, and breaking currents under normal circuit conditions and also making, carrying for a specified time, and breaking currents under specified abnormal conditions such as those of a short circuit.

Circular mil A unit of area equal to $\pi/4$ of a square mil (0.7854 square mil). The cross-sectional area of a circle in circular mils is therefore equal to the square of its diameter in mils.

Coenergy A function associated with the field energy function such that the sum of the energy and coenergy functions is equal to the sum of the products of the flux linkage of a coil multiplied by the corresponding current flowing through it.

Complex power The product of the phasor voltage multiplied by the complex conjugate of the phasor current.

Core losses The power dissipated in a magnetic core subjected to a time-varying magnetizing force.

Counter emf The effective electromotive force within the system that opposes the passage of current in a specified direction.

Critical resistance The value of field circuit resistance of a DC shunt generator above which the generator fails to build up.

DC machine An electromechanical conversion device whose armature terminal voltage and current, as well as field voltage and excitation current, are all DC.

Developed power The power converted from electrical to mechanical or from mechanical to electrical.

Diagonal matrix A square matrix in which all the elements not in the principal diagonal are equal to zero; that is, $a_{ij} = 0$ for all $i \neq j$.

Direct-axis synchronous reactance, X_d The ratio of the sustained fundamental component of armature voltage that is produced by the total direct-axis flux due to direct-axis armature current to the fundamental component of this current, the machine running at rated speed.

Economic dispatch The distribution of total generation requirements among alternative sources for optimum system economy, with due consideration of both incremental generating costs and incremental transmission losses.

Effective value. *See* Root-mean-square value.

Efficiency The ratio of the useful power output to the total power input.

Electrical load Electric power used by devices connected to an electrical generating system.

Electromotive force (emf) A voltage produced in a closed path or circuit by the relative motion of the circuit or its parts with respect to magnetic flux.

Equivalent radius The conductor radius r multiplied by $e^{-1/4}$ (i.e., $re^{-1/4}$) where e is the base of the natural logarithm.

Faraday's law The electromotive force induced is proportional to the time rate of change of magnetic flux linked with the circuit.

Fast-decoupled method An iterative technique for solving the nonlinear power flow equations by separately and alternately solving for the voltage angles and voltage magnitudes.

Fault A physical condition that causes a device, a component, or an element to fail to perform in a required manner, for example, a short circuit, a broken wire, or an intermittent connection.

Field energy The energy stored in the magnetic field of an electromagnetic system.

Field winding A winding on either the stationary or the rotating part of a machine whose sole purpose is the production of the main electromagnetic field of the machine.

Flux linkages The sum of the fluxes linking the turns forming the coil.

Gauss–Seidel method An iterative technique for solving a set of nonlinear equations (e.g., the power flow equations), which uses the latest values of the variables in seeking improved values of the other variables.

Gauss's law The integral over any closed surface of the normal component of the electric flux density is equal in a rationalized system to the electric charge Q_0 within the surface.

Geometric mean distance The mean of (n) distances produced by taking the nth root of their product.

Geometric mean radius The equivalent radius of a multistrand conductor.

Ground wire A conductor having grounding connections at intervals, which is suspended usually above but not necessarily over the line conductor to provide a degree of protection against lightning discharges.

Ideal transformer A transformer characterized by no winding resistance, no leakage flux, and a lossless and infinitely permeable magnetic core.

Identity, or unit, matrix A diagonal matrix whose elements in the principal diagonal are all equal to unity and all elements not in the principal diagonal are equal to zero.

Induction machine An asynchronous AC machine that comprises a magnetic circuit interlinked with two electric circuits, rotating with respect to each other, and in which power is transferred from one circuit to another by electromagnetic induction.

Inductive reactance at 1-ft spacing, x_a The inductive reactance of one conductor of a circuit consisting of two conductors 1 foot apart.

Inductive reactance spacing factor, x_d The inductive reactance of one conductor of a circuit consisting of two conductors separated by a distance D expressed in feet.

Iron losses. *See* Core losses.

Lagging power factor An operating power factor condition such that the phasor current lags the phasor voltage.

Leading power factor An operating power factor condition such that the phasor current leads the phasor voltage.

Linear electromagnetic system An electromagnetic system whose flux linkages are expressed as linear combinations of the currents in terms of the self-inductance of each winding and mutual inductances between the windings.

Load characteristic. *See* Terminal characteristic.

Load curve A curve of power versus time showing the value of a specific load for each unit of the period covered.

Magnetic circuit The region containing essentially all the flux, such as the core of a transformer.

Magnetic flux The surface integral of the normal component of the magnetic induction over the area.

Magnetic flux density Flux per unit area through an element normal to the direction of flux.

Magnetization curve. *See* Saturation curve.

Magnetomotive force (mmf) The line integral of the magnetizing force around the path.

Maximum power The maximum output that an electric machine is capable of developing at rated voltage and speed.

Mutual inductance The common property of two electric circuits whereby an electromotive force is induced in one circuit by a change of current in the other circuit.

Negative-sequence impedance The quotient of that component of negative-sequence sinusoidal voltage that is due to the negative-sequence component of the current, divided by the negative-sequence component of the current at the same frequency.

Negative-sequence network The equivalent representation of a power system constructed by using only the negative-sequence impedances of the various components.

Newton–Raphson method An iterative technique for solving a set of nonlinear equations (e.g., the power flow equations) by solving a succession of linearized equations to derive improvements to the latest estimate of the solution.

Node. *See* Bus.

No-load characteristic The saturation curve of a machine at no load.

No-load test A test applied to a machine on no load at rated voltage and frequency.

Nominal π circuit A network composed of three branches connected in series with each other to form a mesh, the three junction points forming an input terminal, an output terminal, and a common input and output terminal, respectively.

Nonideal or actual transformer A transformer having winding resistance and leakage flux, and a magnetic core having finite permeability and core losses.

Nonsalient, round, or cylindrical The part of a core, usually circular, that by virtue of DC excitation of a winding embedded in slots and distributed over the interpolar space acts as a pole.

Normally excited The operating condition of a synchronous machine at unity power factor.

Null matrix A matrix whose elements are all equal to zero.

One-line, or single-line, diagram A diagram that shows, by means of single lines and graphic symbols, the course of an electric circuit or system of circuits and the component devices or parts used therein.

Open-circuit characteristic (OCC) The saturation curve of a machine with an open-circuited armature winding.

Open-circuit test A test in which the machine is run as a generator with its terminals open-circuited.

Overexcited The operating condition of a synchronous machine delivering reactive power.

Pickup The action of a relay as it makes designated responses to a progressive increase of input.

Pickup value The minimum input that will cause a device to complete contact operation or similar designated action.

Positive-sequence impedance The quotient of that component of positive-sequence sinusoidal voltage that is due to the positive-sequence component of current, divided by the positive-sequence component of the current at the same frequency.

Positive-sequence network The equivalent representation of a power system constructed by using only the positive-sequence impedances of the various components.

Power angle The phase angle between the generated voltage phasor and the terminal voltage phasor.

Power-angle characteristic The expression for the real power developed by a synchronous machine in terms of its generated voltage, terminal voltage, synchronous reactance, and power angle.

Power-angle curve The plot of the power-angle characteristic of a synchronous machine.

Power factor The ratio of the average power in watts to the root-mean-square (RMS) volt-amperes.

Power factor angle The angle whose cosine is the power factor.

Power flow equations The system of nonlinear algebraic equations relating the phasor bus voltages to the complex power injections into the buses of the power system.

Protective relay A device whose function is to detect defective lines or apparatus or other power system conditions of an abnormal or dangerous nature and to initiate appropriate control action.

Pull-out torque The maximum sustained torque that the synchronous machine will develop at synchronous speed with rated voltage applied at rated

Quadrature-axis synchronous reactance, X_q The ratio of the fundamental component of reactive armature voltage, due to the fundamental quadrature-axis component of armature current, to this component of current under steady-state conditions and at rated frequency.

Reactive power The product of voltage and out-of-phase components of alternating current.

Real power The average power, or active power, or the real part of the complex power.

Reluctance The ratio of the magnetomotive force to the magnetic flux through any cross section of the magnetic circuit.

Reluctance power The component of the power delivered by a synchronous generator representing the effects of generator saliency.

Reset The action of a relay as it makes designated responses to decreases in input.

Reset value The maximum value of an input quantity reached by progressive decreases that will permit the relay to reach the state of complete reset from pickup.

Residual voltage The generated voltage due to the residual flux in the field poles even when the field circuit remains unexcited.

Root-mean-square value The square root of the average of the square of the value of the function taken throughout one period.

Rotor The rotating member of a machine, with shaft.

Salient, or projecting, pole A field pole that projects from the yoke or hub toward the primary winding core.

Saturation curve A characteristic curve that expresses the degree of magnetic saturation as a function of some property of the magnetic excitation.

Self-inductance The property of an electric circuit whereby an electromotive force is induced in that circuit by a change of current in the circuit.

Short-circuit characteristic (SCC) The relationship between the current in the short-circuited armature winding and the field current.

Short-circuit test A test applied to a transformer with one winding short-circuited and reduced voltage applied to the other winding such that rated current flows in the windings.

Single-phase transformer A device consisting of two or more windings coupled by a magnetic core that is used to transform a single-phase voltage.

Skin effect The tendency of an alternating current to concentrate in the areas of lowest impedance.

Slip rpm The difference between the synchronous speed and the actual speed of the rotor, expressed in revolutions per minute.

Slip s The quotient of the difference between the synchronous speed and the actual speed of a rotor, to the synchronous speed, expressed as a ratio or as a percentage.

Speed regulation The relationship between the speed and the load of a motor under specified conditions.

Square matrix A matrix in which the number of rows is equal to the number of columns.

Squirrel-cage rotor A rotor core assembly consisting of a number of conducting bars having their extremities connected by metal rings or plates at each end, like a squirrel's cage.

Stator The portion of the rotating machine that includes and supports the stationary active parts.

Stranded conductor A conductor composed of a group of wires or of any combination of groups of wires.

Swing equation The differential equation used to describe the dynamic motion of a synchronous machine.

Symmetric matrix A matrix whose elements are symmetric about the principal diagonal, that is, $a_{ij} = a_{ji}$, for all i and j. Therefore, it is equal to its own transpose matrix; thus, $\mathbf{A} = \mathbf{A}^{\mathrm{T}}$.

Synchronization The process whereby a synchronous machine, with its voltage and phase suitably adjusted, is paralleled with another synchronous machine or system.

Synchronous machine A machine in which the average speed of normal operation is exactly proportional to the frequency of the system to which it is connected.

Synchronous reactance The steady-state reactance of a generator during fault conditions used to calculate the steady-state fault current.

Synchronous speed The speed of rotation of the magnetic flux, produced by or linking the primary winding.

Terminal characteristic A plot of the terminal voltage versus load current.

Three-phase transformer A transformer consisting of three pairs of windings used to transform a balanced set of three-phase voltages from one voltage level to another.

Transpose The matrix $\mathbf{A}^{\mathrm{T}}$ whose element a_{ji} is equal to the element a_{ij} of matrix $\mathbf{A}$ for all i and j. In general, $\mathbf{A}^{\mathrm{T}}$ is formed by interchanging the rows and columns of $\mathbf{A}$.

Turns ratio The ratio of the primary winding turns to the secondary winding turns.

Underexcited The operating condition of a synchronous machine absorbing

V-curve The characteristic of a synchronous motor showing the variation of the stator current versus the field current.

Voltage regulation (1) In a transformer, the change in output (secondary) voltage that occurs when the load (at a specified power factor) is reduced from rated value to zero, with the primary impressed terminal voltage maintained constant. (2) In a DC generator, the final change in voltage with constant field-rheostat setting when the specified load is reduced gradually to zero, expressed as a percent of rated-load voltage, the speed being kept constant. (3) In a synchronous generator, the rise in voltage with constant field current, when, with the synchronous generator operated at rated voltage and rated speed, the specified load at the specified power factor is reduced to zero, expressed as a percent of rated voltage.

Winding factor The product of the distribution factor and the pitch factor.

Wound rotor A rotor core assembly having a winding made up of individually insulated wires.

Zero-sequence impedance The quotient of the zero-sequence component of the voltage, assumed to be sinusoidal, supplied to a synchronous machine, and the zero-sequence component of the current at the same frequency.

Zero-sequence network The equivalent representation of a power system constructed by using only the zero-sequence impedances of the various components.

Index