The full solution of the great circle problem is to solve for initial course, final course and the great circle distance. Then in order to plot a great circle route on a Mercator chart a series of positions along the track has to be calculated with the course at each of these positions.

Definitions

Great Circle. This is a circle on the surface of a sphere whose plane passes through the centre of the sphere. It will divide the sphere into two equal hemispheres. There is only one great circle that can be drawn through any two positions unless the two positions are diametrically opposed on the sphere, in which case there is an infinite number.

Vertex of a great circle is that point on the great circle which is closest to a pole. Thus any great circle other than the equator will have two vertices, a northerly and a southerly.

Spherical triangle. This is formed on the surface of a sphere by the intersection of three great circles.

A right angled spherical triangle may have one, two or three 90 degree angles.

A right sided spherical triangle may have one, two or three right or 90 degree sides.

A great circle may be unambiguously defined by stating its inclination to the equator and the longitude at which it crosses the equator. The great circle will cross the equator at two points diametrically opposed and the two vertices will be 90° removed in

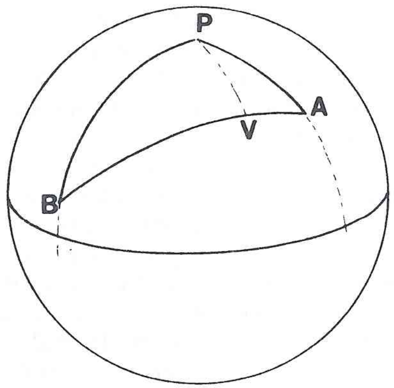


FIG. 1.6.5

longitude from these two points. The latitude of the vertices will be equal to the inclination to the equator.

Great Circles on a Mercator Chart

A great circle other than the equator or a meridian will appear on a Mercator chart as a curve. The curvature will be more pronounced in higher latitudes than near the equator and the curve will always appear concave towards the equator. This means that the Mercator chart is unsuitable for laying off great circle tracks and the great circle sailing problem must be solved mathematically to find a series of points through which the track passes, and the course at those points. These can then be put onto a Mercator chart. Alternatively this task can be done with the aid of a gnomonic chart as described later.

It should be noted that lines of sight projected down onto the earth's surface will result in a great circle. Thus an observation of bearing by radar or visually by compass gives the great circle bearing of the point observed. Over the distances concerned with visual or radar bearings the divergence from the rhumb line is usually insignificant and is ignored. This is not the case for radio bearings which can be taken over considerable distances. In the past these were used extensively in marine navigation but this is no longer the case. The methods used for correcting observed great circle bearings in order to lay off a rhumb line bearing on a Mercator chart are however included here.

Convergency of the Meridians

All meridians converge towards the poles. The convergency between two points on the earth's surface is defined as the change of direction of the great circle which passes through the points, between them.

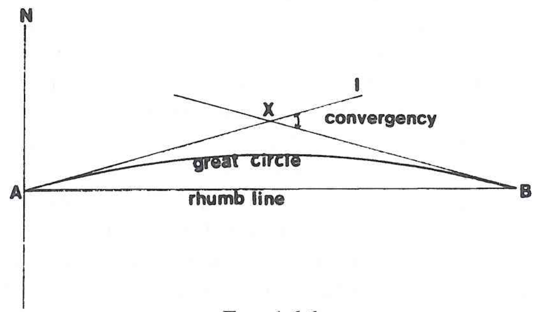


FIG. 1.6.6

Figure 1.6.6 shows two positions in the same latitude with the great circle between them shown as it would appear on a Mercator chart. The rhumb line is shown also and in this case it lies along the parallel of latitude of the two points.

In Figure 1.6.6 the direction AI is the direction of the great circle at A. The direction XB is the direction of the great circle at B. The angle IXB is therefore the change in direction of the great circle between the two positions, that is by definition the convergency.

Angle IXB = angle XAB + angle XBA (exterior angle of a triangle is equal to the sum of the two interior and opposite angles). As the two interior angles XAB and XBA are equal then each is equal to half the convergency.

Thus at each position the difference between the direction of the great circle and the direction of the rhumb line is half the convergency of the meridians.

The half convergency is given, as long as the distance is not too great by the approximate formula:

$$\tfrac{1}{2}\text{ conv.} = \frac{\text{d'long sine mean latitude}}{2}$$

Alternatively it may be taken from a tabulation in nautical tables.

In practice the great circle bearing measured is corrected by the half convergency and the rhumb line is laid off on the Mercator chart and assumed to be the position line. This in itself is a false assumption but in practice the errors caused are not significant compared to other sources of error when observing bearings over long distances and the rhumb line as invariably used. In fact the true position line is the curve of constant bearing whose direction at the ship differs from that of the great circle by the full convergency, but can only be assumed to be a straight line over a short distance near the ship's position. The problem then is to find a position through which this curve passes in order that it can be drawn on the chart. This involves further calculations which although not difficult are time consuming and the approximate solution of using the rhumb line is acceptable.

The great circle between two points always lies on the polar side of the rhumb line (in certain circumstances they are coincident). This is illustrated if Figure 1.6.7 which shows the appearance of the great circle and the rhumb line on a Mercator chart in the four cases:

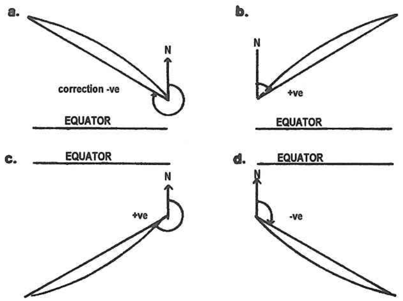


FIG. 1.6.7

(a) westerly bearing in north latitude,
(correction to great circle bearing is –ve)
(b) easterly bearing in north latitude,
(correction to great circle bearing is +ve)
(c) westerly bearing in south latitude,
(correction to great circle bearing is +ve)
(d) easterly bearing in south latitude,
(correction to great circle bearing is –ve)

Example

From a ship in DR position 44° 10′ S 144° 50′ E the bearing of a radio station was observed by direction finder to be 055°. If the position of the station was 42° 53′ S 147° 14′ E find the direction of the rhumb line.

	44° 10′ S	144° 50′ E	
	42° 53′ S	147° 14′ E	
	1° 17′ N	2° 24′ E	Mean latitude = 43° 31.5′ S
=	77′ N	144′ E	

convergency = d'long × sine mean latitude
= 144 × sine 43° 31′
= 99.2

half convergency = 49.6′

great circle bearing = 055°
half convergency = 49.6′
rhumb line bearing = 054° 10.4′ or $054\tfrac{1}{4}$°

The Great Circle Sailing Problem

Over long ocean passages it is normal practice to follow great circle tracks. The saving in distance over the rhumb line track can be considerable. In general the greater the difference of longitude to cover and the higher the latitude then the greater will be the saving in distance. The great circle track however may take the vessel into much higher latitudes than the rhumb line track and the advisability of this must be considered.

In order to plot a great circle track on a Mercator chart then it is necessary to calculate a series of waypoints through which the great circle passes. These can then be joined with a series of short rhumb lines to approximate to the great circle. The course at each of the waypoints must also be calculated.

Figure 1.6.5 shows two positions in north latitude with the great circle between them. A spherical triangle is formed by this great circle and the two meridians which pass through the positions. Thus the three points of the triangle are the two positions and one of the earth's poles. The three sides of the triangle are the great circle distance, the complement of latitude A and the complement of latitude B.

To Find the Great Circle Distance

The side AB in Figure 1.6.5 expressed in minutes of arc will give the great circle distance in nautical miles.

The spherical cosine formula states that in triangle PAB:

$$\cos AB = \cos AP \cos BP + \text{sine } AP \text{ sine } BP \cos P$$

To Find the Initial Course

Angle A will provide the initial course from the departure point. If the vertex of the great circle lies between the two positions angle A will be the course in quadrantal notation. If the vertex lies outside the two positions then angle A will be the supplement of the course.

The spherical cosine formula states that in triangle PAB:

$$\cos A = \frac{\cos BP - \cos AP \cos AB}{\sin AP \sin AB}$$

Note that the side AB must be found first and used in this formula.

To calculate waypoints along the track

This is best done by first finding the position of the vertex. The longitude of the required waypoints can then be assumed at convenient intervals of longitude, and the latitude corresponding to each calculated. By using the vertex then all calculations can be done using Napier's rules for solving right angled or right sided spherical triangles.

To Find the Position of the Vertex

In Figure 1.6.8 V is the vertex of the great circle through A and B. The triangles PVA and PVB are right angled at V.

The side PV in triangle PVA is the co-latitude of the vertex. Angle P is the d'long between A and the vertex.

Napier's rules state that in triangle PVA:

$$\text{sine } PV = \sin PA \sin A$$

and

$$\cot P = \cos PA \tan A$$

Thus the latitude of the vertex is given by (90 – PV) and the longitude of the vertex can be found by applying the angle P (d'long) to the longitude of A.

To find a series of waypoints along the track

This may be done by assuming a series of longitudes at equal intervals and calculating the latitude which corresponds to each on the track. The course at each point can also be calculated. Figure 1.6.8 shows one such waypoint denoted as X.

If a longitude is assumed for X then the corresponding latitude may be found by solving the right angled triangle PVX for PX which is the co-latitude of the waypoint. The direction of the great circle or course at X is found by solving for the angle X.

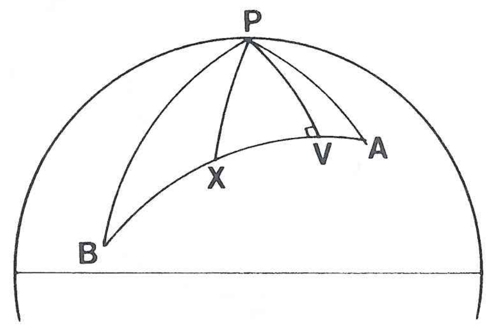


Fig. 1.6.8

In the right angled triangle PVX:

P = d'long between the vertex and the assumed longitude of X
PV = co-latitude of the vertex

By Napier's rules:

$$\cot PX = \sin PX \cot PV$$

and

$$\cos X = \cos PV \sin P$$

If a large number of waypoints are to be found then the repetitive nature of the calculations is best done by a system of tabulation such as that adopted in the examples to follow.

Example 1
Find the great circle distance and the initial course from a position off Cape Palliser (41° 40′ S 175° 25′ E) to a position off Panama (7° 00′ N 80° 50′ W). Also find the positions where the track crosses the meridians of 180°, 160°, 140°, 120°, and 100°, and the course at these positions.

	41° 40.0′ S		175° 25.0′ E
	7° 00.0′ N		80° 50.0′ W
d'lat	48° 40.0′ N	d'long	256° 15.0′ W
			360°
			103° 45.0′ E

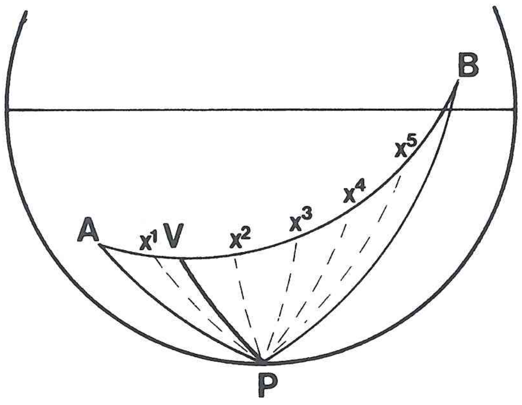


FIG. 1.6.9

By the spherical cosine formula:

$$\cos AB = \cos AP \cos BP + \text{sine } AP \text{ sine } BP \cos P$$

$$\begin{aligned}
\cos \text{dist} &= \cos 48° \, 20' \cos 97° \, 00' + \sin 48° \, 20' \sin 97° \cos 103° \, 45' \\
&= -0.08102 - 0.17623 \\
&= -0.25725 \\
\text{dist} &= 104° \, 54.4' \\
&= 6294.4 \text{ miles}
\end{aligned}$$

Note the negative signs which must be applied to the side BP and the angle P both of which are in the second quadrant.

and

$$\begin{aligned}
\cos A &= \frac{\cos BP - \cos AP \cos AB}{\sin AP \sin AB} \\
&= \frac{\cos 97° - \cos 48° \, 20' \cos 104° \, 54.4'}{\sin 48° \, 20' \sin 104° \, 54.4'} \\
&= \frac{-0.12187 + 0.17102}{0.72188} \\
&= \frac{0.04915}{0.72188} \\
&= 0.06809 \\
&= 86° \, 05.8
\end{aligned}$$

initial course = S 86°E′

To find the position of the vertex
In triangle APV by Napier's rules:

$$\begin{aligned}
\sin PV &= \sin A \sin PA \\
&= \sin 86° \, 05.8' \sin 48° \, 20' \\
PV &= 48° \, 11'
\end{aligned}$$

latitude of vertex = 41° 49′ S

and

$$\cot P = \cos PA \tan A$$
$$= \cos 48^\circ\ 20' \tan 86^\circ\ 05.8'$$
$$P = 5^\circ\ 51.8'$$

longitude of A = 175° 25.0′ E
P = 5° 51.8′

longitude of vertex = 181° 16.8′ E
= 178° 43.2′ W

Position of the vertex = 41° 48.9′ S 178° 43.2′ W

To find the latitudes at which the track cuts the given longitudes and the course

In triangle PVX by Napier's rules:

$$\cot PX = \cos P \cot PV$$

and

$$\cos X = \sin P \cos PV$$

Longitude	180°	160° W	140° W	120° W	100° W
angle P	1° 16.8′	18° 43.2′	38° 43.2′	58° 43.2′	78° 43.2′
cos P	0.99975	0.94710	0.78021	0.51922	0.19560
cot PV x	0.89463	0.89463	0.89463	0.89463	0.89463
cot PX	0.89440	0.84730	0.69800	0.46451	0.17499
PX	48° 11.4′	49° 43.5′	55° 05.1′	65° 05.1′	80° 04.5′
Latitude	41° 48.6′ S	40° 16.5′ S	34° 54.9′ S	24° 54.9′ S	9° 55.5′ S

sin P	0.02234	0.32094	0.62552	0.85464	0.98068
cos PV x	0.66675	0.66675	0.66675	0.66675	0.66675
cos X	0.01490	0.21399	0.41707	0.56983	0.65387
angle X	89° 08.8′	77° 38.6′	65° 21.0′	55° 15.7′	49° 10.0′
course	S 89.1° E	N 77.6° E	N 65.3° E	N 55.2° E	N 49.2° E

Solution

Latitude	Longitude	Course
41° 48.6′ S	180° 00.0′	S 89° E
41° 49′ S	178° 43.2′ W	east
40° 16.5′ S	160° 00.0′ W	N 78° E
34° 54.9′ S	140° 00.0′ W	N 65° E
24° 54.9′ S	120° 00.0′ W	N 55° E
9° 55.5′ S	100° 00.0′ W	N 49° E

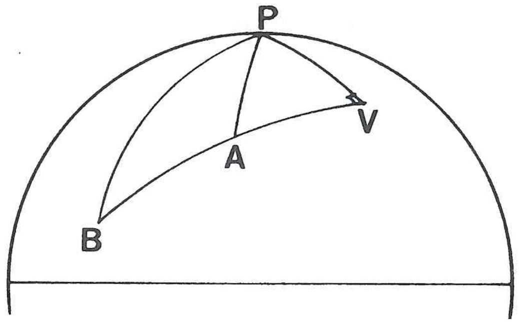


Fig. 1.6.10

Example 2

Find the great circle distance and the initial course on a voyage from Vancouver to Hawaii between the positions 48° 20′ N 125° 00′ W and 21° 15′ N 157° 25′ W. Find also the position of the vertex.

Note that in this case the position of the vertex lies outside the triangle formed by the two positions and the pole. This will only be indicated when the initial course is calculated to be over 90° T and must be subtracted from 180° to give the course angle.

	48° 20.0′ N		125° 00.0′ W
	21° 15.0′ N		157° 25.0′ W
d'lat	27° 05.0′ N	d'long	32° 25.0′ W

By the spherical cosine formula:

$$\cos AB = \cos AP \cos BP + \text{sine } AP \text{ sine } BP \cos P$$

$$\cos \text{dist} = \cos 41^\circ 40' \cos 68^\circ 45' + \sin 41^\circ 40' \sin 68^\circ 45' \cos 32^\circ 25'$$
$$= 0.27075 + 0.52304$$
$$= 0.79379$$
$$\text{dist} = 37^\circ\ 27.5'$$
$$= 2247.5 \text{ miles}$$

and

$$\cos A = \frac{\cos BP - \cos AP \cos AB}{\sin AP \sin AB}$$

$$= \frac{\cos 68° 45' - \cos 41° 40' . \cos 37° 27.5'}{\sin 41° 40' . \sin 37° 27.5'}$$

$$= \frac{0.36244 - 0.59299}{0.40432}$$

$$= -0.57022$$

$$= 124° 45.9'$$

$$\text{course} = S 55° 14.1'W \text{ or } 235°$$

The vertex must lie outside the two positions as angle A was calculated to be more than 90°.

To find the position of the vertex
In triangle APV by Napier's rules:

$$\sin PV = \sin A \sin PA$$
$$= \sin 124° 45.9' \sin 41° 40'$$
$$= 0.54613$$
$$PV = 33° 06.1'$$

latitude of vertex = 56° 53.9′ N

and

$$\cot P = \cos PA \tan A$$
$$= \cos 41° 40' \tan 124° 45.9'$$
$$= 1.07623$$
$$P = 42° 53.8'$$

longitude of A = 125° 00.0′ W
angle P = 42° 53.8′
longitude of vertex = 82° 06.2′ W

Position of vertex 56° 53.9′ N 82° 06.2′ W

EXERCISE 1.6.5

1. Find the great circle distance, and the initial course of the great circle between the following positions.

41° 00′ S 175° 00′ E
33° 00′ S 71° 30′ W

Find also the latitudes where the track cuts the longitudes of 90° W, 110° W, 130° W, 150° W, and 170° W, and the course at these points.

2. Find the great circle distance, and the initial course of the great circle between the following positions.

48° 30′ N 5° 10′ W
22° 00′ S 40° 40′ W

Find also the position of the vertex.

3. Find the great circle distance, and the initial course of the great circle between the following positions.

55° 25′ N 7° 12′ W
51° 12′ N 56° 10′ W

4. Find the great circle distance, and the initial course of the great circle between the following positions.

34° 55′ S 56° 10′ W
33° 55′ S 18° 25′ E

5. Find the saving in distance by steaming the great circle track as opposed to the rhumb line between the following positions.

43° 36′ S 146° 02′ E
26° 12′ S 34° 00′ E

6. Find the great circle distance, and the initial course of the great circle between the following positions.

48° 24′ N 124° 44′ W
34° 50′ N 139° 50′ E

Find also the latitudes where the track cuts the longitudes of 140° W, 160° W, 180° W, 160° E, and the course at these points.

Composite Great Circle Sailing

If the vertex of the great circle lies between the two positions involved in a great circle sailing problem then the great circle track will take the vessel into higher latitudes than either of the two positions. The further the two positions are apart then the higher will be the latitude of the vertex, and in the limit if the two positions are 180° apart in longitude then the great circle will pass through the poles.

Obviously there must be an arbitrary limit placed by the navigator on the maximum latitude during a passage. The composite great circle track minimises the distance while maintaining the latitude within chosen limits. An example of a passage when such a track might be used would be from Cape of Good Hope to Australia.

The track now becomes a great circle down to the chosen maximum parallel meeting the parallel at the vertex, a parallel sailing along the parallel, and then a second great circle track from the parallel to the destination leaving the parallel at the vertex of the second great circle.

Figure 1.6.11 represents such a track in the southern hemisphere. The broken line shows the true great circle between the two positions.

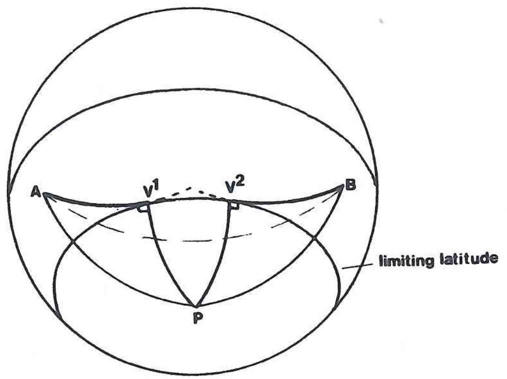


FIG. 1.6.11

To find the total distance and the initial course

The two right angled triangles PAV_1 and PAV_2 can be solved for any part using Napier's rules.

The sides PA and PB are the co-latitudes of the two positions.

The sides PV_1 and PV_2 are equal and are the co-latitude of the chosen maximum latitude.

The angle A will give the initial course.

The sides AV_1 and BV_2 will give the distances along the two great circles.

By Napier's rules:

the initial course is given by:

$$\sin A = \sin PV_1 \operatorname{cosec} PA$$

the distance AV_1 and BV_2 are given by:

$$\cos AV_1 = \cos PA \sec PV_1$$

and

$$\cos BV_2 = \cos PB \sec PV_2$$

The parallel sailing section of the track can be solved for distance by the parallel sailing formula:

$$\frac{\text{departure}}{\text{d'long}} = \cos \text{latitude}$$

The problem of finding waypoints along the two great circle tracks is the same as for great circle sailing.

Example 1

Find the total distance and the initial course along a composite great circle track between a position off Cape of Good Hope, 34° 35′ S 18° 30′ E and a position off Tasmania, 43° 40′ S 146° 50′ E. The maximum latitude is 50° S.

Refer to Figure 1.6.11

34° 35.0′ S	18° 30.0′ E
43° 40.0′ S	146° 50.0′ E
	128° 20.0′ E = total d'long

In triangle APV_1

$$\begin{aligned}\cos AV_1 &= \cos AP \sec PV_1\\ &= \cos 55° 25' \sec 40°\\ &= 0.74095\\ &= 42° 11.2'\\ &= 2531.2 \text{ miles}\end{aligned}$$

and triangle BPV_2

$$\begin{aligned}\cos BV_2 &= \cos BP \sec PV_2\\ &= \cos 46° 20' \sec 40°\\ &= 0.90133\\ &= 25° 40'\\ &= 1540.0 \text{ miles}\end{aligned}$$

$\sin A = \sin PV_1 \operatorname{cosec} PA$
$= \sin 40 \operatorname{cosec} 55° 25'$
$= 0.78074$
$= 51° 19.7'$

$\cos P = \tan PV_1 \cot PA$
$= \tan 40° \cot 55° 25'$
$= 0.57850$
$= 54° 39.3'$

$\cos P = \tan PV_2 \cot PB$
$= \tan 40° \cot 46° 20'$
$= 0.80093$
$= 36° 46.9'$

To find distance V_1V_2
In triangle PV_1V_2

angle P = total d'long $- 54° 39.3' - 36° 46.9'$
$= 128° 20' - 91° 26.2'$
$= 36° 53.8'$
$= 2213.8$

distance $V_1V_2 = 2213.8 \cos 50°$
$= 1423.0$ miles

Thus total distance $= 2531.8 + 1423.0 + 1540.0$
$= 5494.2$ miles

Initial course $=$ S 51° 19.7′ E

Example 2
Find the initial course to steer and the shortest distance between the following positions if the vessel is not to go south of the parallel of 45° S.

10° 18.0′ S	20° 10.0′ E
45° 00.0′ S	160° 10.0′ E
	140° 00.0′ E = total d'long

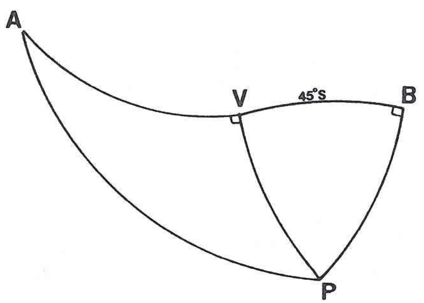


Fig. 1.6.12

In Figure 1.6.12
By Napier's rules:

$\cos AV_1 = \cos AP \sec PV_1$
$= \cos 79° 42' \sec 45°$
$= 0.25286$
$= 75° 21.2'$
$= 4521.2$ miles

$\sin A = \sin PV_1 \operatorname{cosec} PA$
$= \sin 45° \operatorname{cosec} 79° 42'$
$= 0.71869$
$= 45° 56.8'$

$\cos P = \tan PV_1 \cot PA$
$= \tan 45° \cot 79° 42'$
$= 0.18173$
$= 79° 31.8'$

To find distance V_1V_2
In triangle PV_1V_2:

angle P = total d'long $- 79° 31.8'$
$= 60° 28.2'$
$= 3628.2$

distance $V_1V_2 = 3628.2 \cos 45°$
$= 2565.5$

total distance $= 4521.2 + 2565.5$
$= 7086.7$

initial course $=$ S 45° 56.8′ E

EXERCISE 1.6.6

1. Find the initial course and the total distance along a composite great circle track from a position 35° 00′ S 20° 00′ E to a position 43° 40′ S 146° 50′ E. The maximum latitude is to be 48° S.

2. A composite great circle track between Montevideo (34° 55′ S 56° 10′ W) and Cape Town (33° 55′ S 18° 25′ E) is required with limiting latitude 38° S. Find the initial course and the total distance.

3. Find the total distance, the initial course and the longitudes in which the track reaches and leaves the limiting latitude of 45° S on a composite great circle track between a position in 26° 12′ S 34° 00′ E and a position 43° 36′ S 146° 02′ E.

4. Find the total distance and the initial course on a composite great circle track from a position 45° 30′ S 71° 37′ W to a position 46° 40′ S 168° 20′ E if the limiting latitude is to be 49° S.

5. Calculate the shortest distance to steam from a position 4° 00′ N 31° 00′ E to a position 42° 00′ S 145° 00′ E, given that the vessel is not to go south of the parallel of 42° S. State also the direction of the track as it crosses the equator.

The Gnomonic Chart

A quick but less accurate solution of the great circle problem may be obtained with the aid of a gnomonic chart. As the point of projection of such a chart is the centre of the earth, then all great circles will appear as straight lines. Gnomonic charts are produced which cover the oceans of the world for this purpose and it is also used to chart polar regions where the Mercator projection cannot be used.

In order to solve the great circle problem a straight line can be drawn between the departure and arrival points on a gnomonic chart to represent the great circle track. Points along the track can then be lifted off the chart and transferred to a Mercator chart. These points are then joined by a series of short rhumb lines which will approximate to the great circle. The courses can be measured from the Mercator chart or preferably found by using the ABC tables (see Module 2.3). Note that course cannot be measured from the gnomonic chart as it is not an orthomorphic projection.

A composite great circle track can be drawn on a gnomonic chart by drawing the initial great circle as a tangent to the curve which represents the limiting parallel of latitude. Similarly a tangent to this parallel to the arrival position will represent the final great circle. Points along the track can then be lifted off in the same way as for the great circle track.

In practice a great circle passage may be executed by plotting the great circle on the chart and maintaining this great circle. It may also be done by treating each fix obtained as the initial position of a new great circle track. It may be noted that once the vessel has departed from the original great circle there is no practical advantage of regaining the original track. A new initial course may be worked and the vessel put onto this course with

appropriate set to try and maintain the new great circle. Obviously it is necessary to ensure that the new track does not take the vessel close to any navigational hazard. With modern electronic fixing aids such as satellite navigators there is very often a provision for the rapid calculation of great circle tracks. Also an initial course can quickly be obtained by the use of the ABC tables found in nautical tables. The latitude of the initial position is used as the argument latitude in Table A. The latitude of the destination is used as the argument declination in Table B. The d'long is used as the argument hour angle. The initial course can then be extracted from Table C as is the azimuth. This method will become clear when the use of the ABC tables is studied in Module 2.3.

EXERCISE 1.6.7

1. Define 'vertex of a great circle'. What is the course along a great circle at the vertices? State what the relationship between the direction of a great circle at the equator and the latitude of the vertices.

2. Given that a great circle is inclined to the equator at 42°, and crosses the equator in longitude 50° W, the direction here being SW/NE, give the positions of the two vertices. What is the convergency between the vertex and a point where the great circle crosses the equator?

3. Define 'convergency of the meridians'. If a vessel steams directly towards a point which is in sight does she follow a great circle or a rhumb line track? Explain why a D/F bearing must be corrected before laying off on a Mercator chart. Why is this not done for a visual bearing?

4. Two places on a parallel of 50° N have a d'long of 100° between them. Find the difference in steaming distance between the great circle track and the rhumb line track.

5. Find by use of the half convergency formula the great circle bearing of a position in 50° N 176° 14′ E from a position in 50° N 170° 21′ W.

6. Why is the gnomonic projection unsuitable for use in general navigation?

MODULE 1.7

Tides, Tide Tables and Other Tidal Information

Tides

Tides and tidal streams are the result of gravitational attractions between astronomical bodies, mainly the earth and sun, and the earth and moon. The gravitational force between two bodies, varies as the product of the masses and inversely as the square of the distance between the bodies. If the earth moon system is considered, there is therefore a difference between the gravitational force acting between the solid earth and the moon, and that acting between the moon and the water on the earth directly under it. This will cause an excess of water directly beneath the moon, which is referred to as a tidal wave. Similarly the gravitational attraction between the solid earth and the moon is greater than the force between the moon and the water on the opposite side of the earth and a tidal wave is caused on the side of the earth away from the moon. Variation in the height of water at any place on the earth will occur as the earth rotates with respect to these tidal waves, giving two high waters and two low waters with each rotation. The period of rotation for the solar tidal wave is once per day and for the lunar tidal wave once in 24h 50m, which is the period of rotation of the earth with respect to the moon.

The highest high waters and lowest low waters will occur when the lunar and the solar tidal wave coincide that is when the moon and sun are in line or almost so near to new and full moon. At these times the two tides reinforce giving spring tides. At first and third quarter when the two tidal waves are in opposition they partially cancel giving lowest high waters and highest low waters, which are termed neap tides.

This idealistic description is modified dramatically by the effect of the land masses which prevent the flow of such tidal waves. In fact the magnitude of the tidal effects would be relatively small if it were not for the modifying effects of the land masses, and the magnifying effect of resonance within the ocean basins. This occurs particularly in the North Atlantic basin along whose boundary tidal heights are large. They become extremely large

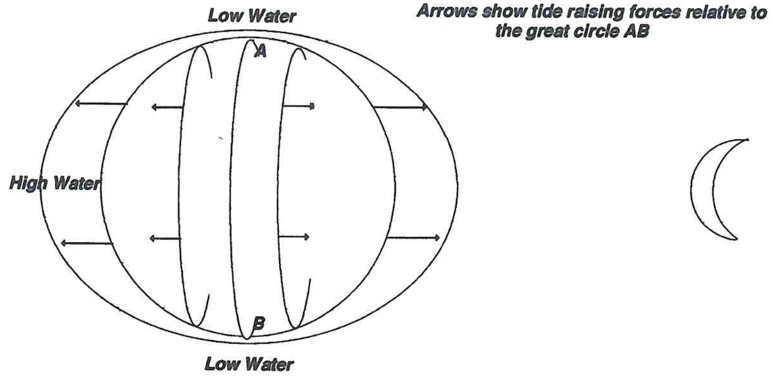


FIG. 1.7.1

when the land gives a funnel and shoaling effect raising the height of the tidal wave. On the other hand in bodies of water whose natural resonant period is very different from the period of the tide raising forces, then tidal effects are small. This is the case in seas such as the Baltic and Mediterranean.

Tidal predictions are made after analysing the results of observations over many years to find the magnitude of constituent astronomical effects. The long period of observation filters out the effects of climate and weather, and then tidal heights can be predicted into the future based upon astronomical predictions. What cannot be done is to predict the effects of weather, so it must be realised that tidal conditions experienced can be very different from those predicted in extreme weather conditions.

Tidal heights are affected by prolonged winds and by atmospheric pressure variations from the mean, while winds can affect the times of tides. A wind will raise sea level towards the direction in which it is blowing and prolonged and strong winds will cause wave motions of long wavelength which themselves will modify the heights of tidal waves lowering the height when the trough of the wave passes and raising it when the crest passes. These long waves are known as storm surges and they themselves may be modified by the effect of the land shape. This happens frequently in the North Sea which is affected by wind condition in the North Atlantic, raising sea levels in the North Sea by as much as a metre with occasional surges of up to 3 metres in the southern North Sea. Similarly tide levels can be lowered by similar amounts which could be critical to large vessels in this area.

Tidal Streams

Tidal streams are the horizontal movements of water relative to the earth's surface due to tide raising forces. Again although their cause is astronomical, the results will be greatly affected by the land masses and the sea bed. A tidal stream may be predominantly rectilinear having only two directions which are usually referred to as ebb and flood streams or denoted by their compass directions. Other tidal streams flow in a rotatory manner within a sea basin such as the southern North Sea.

Tidal stream predictions are made available to the navigator by:

(a) Tidal information on Admiralty charts.
(b) Tidal stream atlases.

Currents

Currents are horizontal movements of water caused by meteorological conditions, or by flow of water from river estuaries. They are not periodic as are tidal streams and those currents which are due to meteorological effects are not included in the tidal predictions. Consistent strong winds may therefore modify the streams to a marked extent. The largely permanent effect of flow from river estuaries is included in the predictions.

Tidal Information on Admiralty Charts

Selected positions on Admiralty charts are chosen for which to give tidal stream information. These positions are marked by a magenta diamond with an identifying letter inside. At some convenient place on the chart a table is given for each tidal diamond, each table being headed by the appropriate letter and the position of the diamond. The table gives the direction, and the spring and neap rates for each hour of the tidal cycle. The hours of the cycle are referred to the time of high water at some standard port, whose tide times are included in the Admiralty Tide Tables. This port may or may not be on the chart, and many charts around the UK are referred to the time of high water at Dover. Information is given from 6 hours before to 6 hours after high water at hourly intervals. In order to relate the information to ship's time, then the time of the nearest high water at the standard port must be extracted from the Admiralty Tide Tables and converted to ship's time.

To find the tidal direction and rate at any position between the tidal diamonds then some interpolation must be done between the

Tidal Levels referred to Datum of Soundings

Place	Lat N	Long W	Heights in metres above datum MHWS	MHWN	MLWN	MLWS	Datum and remarks
Sennen Cove	50° 04′	5° 42′	6·1	4·8	–	–	
Penzance	50 06	5 33	5·6	4·4	2·0	0·8	3·05m below Ordnance Datum (Newlyn)
Porthleven	50 05	5 19	5·5	4·3	2·0	0·8	2·99m below Ordnance Datum (Newlyn)
Lizard Point	49 57	5 12	5·3	4·2	1·9	0·6	2·90m below Ordnance Datum (Newlyn)
Coverack	50 01	5 05	5·3	4·2	1·9	0·6	2·90m below Ordnance Datum (Newlyn)
Helford R Entrance	50 05	5 05	5·3	4·2	1·9	0·6	2·90m below Ordnance Datum (Newlyn)
Falmouth	50 09	5 03	5·3	4·2	1·9	0·6	2·91m below Ordnance Datum (Newlyn)

Tidal Streams referred to HW at DEVONPORT

Hours	Ⓐ 50°07'·2N 5 49·5W Dir	Rate (kn) Sp	Np	Ⓑ 49°58'·5N 5 48·5W Dir	Rate (kn) Sp	Np	Ⓒ 50°01'·5N 5 27·6W Dir	Rate (kn) Sp	Np	Ⓓ 49°52'·2N 5 10·9W Dir	Rate (kn) Sp	Np	Ⓔ 50°02'·4N 5 02·3W Dir	Rate (kn) Sp	Np
Before HW 6	332	0·4	0·2	311	1·7	0·8	280	0·9	0·5	256	1·8	0·9	201	1·0	0·5
5	002	1·5	0·7	327	2·2	1·1	290	0·6	0·3	254	1·2	0·6	309	0·1	0·0
4	009	2·4	1·2	336	2·0	1·0	329	0·3	0·2	234	0·4	0·2	006	1·0	0·5
3	010	2·5	1·2	352	1·6	0·8	049	0·4	0·2	045	0·4	0·2	011	1·4	0·7
2	009	1·6	0·8	023	1·1	0·5	065	0·6	0·3	054	1·0	0·5	015	1·5	0·8
1	031	1·1	0·5	088	1·0	0·5	082	0·8	0·4	059	1·8	0·9	022	1·5	0·7
HW	123	0·6	0·3	113	1·4	0·7	097	0·8	0·4	067	2·3	1·1	028	1·2	0·6
After HW 1	168	1·7	0·8	124	1·8	0·9	107	0·7	0·3	075	1·8	0·9	030	0·5	0·2
2	181	2·5	1·2	137	1·9	0·9	124	0·4	0·2	082	0·8	0·4	202	0·4	0·2
3	194	2·5	1·3	166	1·8	0·9	201	0·3	0·2	203	0·4	0·2	196	1·2	0·6
4	210	2·1	1·0	195	2·0	1·0	234	0·6	0·3	233	1·4	0·7	195	1·7	0·9
5	223	1·1	0·6	232	1·6	0·8	263	0·8	0·4	247	2·3	1·1	197	1·6	0·8
6	295	0·4	0·2	286	1·4	0·7	278	0·9	0·5	257	1·9	0·9	202	1·2	0·6

Ⓕ 50°02'·5N 4 58·7W Dir	Rate (kn) Sp	Np	Ⓖ 50°02'·7N 4 54·8W Dir	Rate (kn) Sp	Np	Ⓗ 50°08'·0N 4 52·3W Dir	Rate (kn) Sp	Np	Ⓙ 50°08'·5N 5 01·5W Dir	Rate (kn) Sp	Np	Hours
215	1·0	0·5	217	0·7	0·4	222	0·4	0·2	339	0·2	0·1	6 Before HW
220	0·5	0·2	232	0·5	0·2	249	0·2	0·1	005	0·6	0·3	5
293	0·1	0·1	277	0·3	0·1	Slack			022	0·9	0·4	4
017	0·5	0·2	349	0·4	0·2	077	0·2	0·1	023	0·6	0·3	3
029	0·9	0·5	014	0·7	0·3	037	0·4	0·2	022	0·4	0·2	2
043	1·2	0·6	051	0·9	0·4	042	0·5	0·3	036	0·2	0·1	1
043	1·2	0·6	060	1·1	0·5	040	0·7	0·3	Slack			HW
040	0·7	0·4	069	0·8	0·4	036	0·5	0·2	217	0·3	0·1	1 After HW
Slack			107	0·3	0·1	Slack			213	0·5	0·2	2
214	0·5	0·3	212	0·4	0·2	210	0·2	0·1	207	0·7	0·3	3
210	0·9	0·5	224	0·7	0·4	219	0·6	0·3	190	0·8	0·4	4
213	1·3	0·6	220	0·8	0·4	211	0·7	0·4	180	0·5	0·2	5
216	1·2	0·6	217	0·8	0·4	216	0·5	0·3	276	0·1	0·0	6

FIG. 1.7.2

Reproduced from British Admiralty Tide Tables with the sanction of the Controller, H.M. Stationery Office and of the Hydrographer of the Navy.

adjacent diamonds. This must be done with some amount of personal judgement as to the likely effect of the coastline shape and underwater banks on the direction of the stream. In this respect it must be remembered that tidal streams tend to flow parallel to coastlines and sand banks and into and out of estuaries, although this may not be the case at the turn of the tide. To help in this respect there are also shown on charts approximate direction of the flood (arrows with feathers), and the ebb (arrows without feathers), or currents, (a wavy arrow). A sample block of tidal data is shown in Figure 1.7.2.

Tidal Stream Atlases

These are published by the Hydrographic Office in a series of booklets to cover the coastal waters of the British Isles. Each booklet, for its area of coverage, contains a chartlet for each hour of the tidal cycle from 6 hours before to 6 hours after high water at a Standard Port such as Dover. The time of high water at the Standard Port must be obtained from the Admiralty Tide Tables for the day in question. On each chartlet the mean direction of the tidal stream for that hour of the cycle is shown by arrows, the length and thickness of which indicate approximately the strength of the stream. Figures are given against some arrows to show the mean neap and spring rates. These are shown thus:

11,24

meaning that the mean neap rate is 1.1 knots and the mean spring rate is 2.4 knots. Interpolation or extrapolation between these figures may be done by taking the range at Dover for that day and comparing it with the mean neap and spring ranges. Interpolation diagrams with full instructions are included to facilitate this. Tidal stream atlases are particularly useful to follow the pattern of tides that a vessel will experience during a passage.

Tidal heights

Definitions

Chart datum. The arbitrary level from which charted soundings and tidal height predictions are expressed. This level is chosen so that the sea level will rarely fall below chart datum. In the United Kingdom chart datum is approximately the level of Lowest Astronomical Tide.

Height of Tide. This is the height of the water surface at any time above the level of chart datum.

Mean High Water Springs. This is the height above chart datum which is the average of the heights of all the two successive high waters when the range of the tide is greatest, throughout a year when the average maximum declination of the moon is 23.5 degrees. The value is adjusted to take an average over an 18.6 year cycle over which the value of MHWS varies.

Mean Low Water Springs. This is the height above chart datum which is the average of the heights of all the two successive low waters when the range of the tide is greatest, throughout a year when the average maximum declination of the moon is 23.5 degrees. The value is adjusted to take average over an 18.6 year cycle over which the value of MLWS varies.

Mean High Water Neaps. This is the height above chart datum which is the average of the heights of all the two successive high waters when the range of the tide is least, throughout a year when the average maximum declination of the moon is 23.5 degrees. The value is adjusted to take average over an 18.6 year cycle over which the value of MHWN varies.

Mean Low Water Neaps. This is the height above chart datum which is the average of the heights of all the two successive low

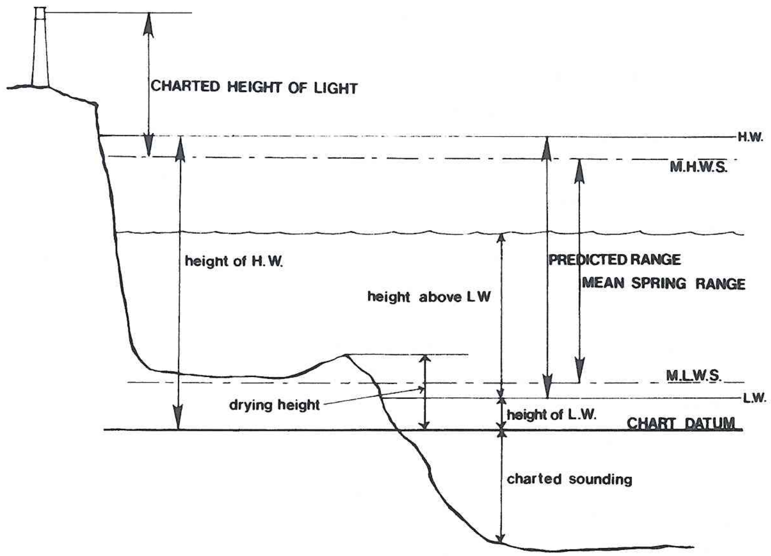


FIG. 1.7.3 shows the relationship between these defined levels.

waters when the range of the tide is least, throughout a year when the average maximum declination of the moon is 23.5 degrees. The value is adjusted to take average over an 18.6 year cycle over which the value of MLWN varies.

Lowest Astronomical Tide. The lowest level that can be predicted under normal meteorological conditions under any combination of astronomical conditions. This will not take into account meteorological effects and these can cause sea level to fall considerably lower than LA T.

Admiralty Tide Tables

Admiralty Tide Tables are published in four volumes.

Volume 1: United Kingdom and Ireland with European Channel Ports.
Volume 2: Europe, Mediterranean Sea and Atlantic Ocean.
Volume 3: Indian Ocean and South China Sea
Volume 4: Pacific Ocean.

Volume 1 contains:

An introduction which gives an explanation of the use of the tide tables together with a number of tables and diagrams which may be used in conjunction with the tide tables.

Part I which gives the predicted times and heights of high and low waters throughout the year, for a selected number of Standard Ports. For each Standard Port a tidal diagram is given to enable prediction of heights between high and low waters.

Part Ia gives for Plymouth, Poole, Southampton, Portsmouth, Rosyth, Liverpool, Avonmouth and St. Helier predictions of the height of tide for each hour throughout the year.

Part II gives tidal data for a large number of secondary ports in the form of time and height differences to be applied to one of the Standard Ports in Part 1.

Part III gives Harmonic Constants for a selection of ports which give an alternative method of tidal prediction for those ports. This method is not in general use in the Merchant Navy and will not be covered here.

Chart Datum

Soundings on Admiralty charts are expressed below chart datum. In normal circumstances because of the choice of chart datum, there will rarely be less water than shown on the chart. The levels of chart datum differs between different ports but are largely standardised to approximate to lowest astronomical tide. The relationships between chart datums at various places and LAT and other levels are shown in Table V in the front of the Admiralty Tide Tables Volume 1. For comparison of datums between charts of different areas Table III should be consulted. This gives the height of chart datum at various places relative to the ordnance datum (Newlyn) which is the datum for the land levelling system of England, Wales and Scotland. For chart datums at places other than these countries the datum used is that established in that country.

In all cases the tidal predictions for ports will be referred to the same datum which is established at that port and which is used on the largest scale chart of that port. The total predicted depth of water at any place therefore will always be the charted sounding on the largest scale chart of the area, plus the predicted height of tide. This means that in practice the navigator need not concern himself unduly with the relationship between the various datums, but he should note that soundings will not agree between charts which are based on differing datum levels.

Tidal Predictions

The tides of UK waters are of a predominantly semi diurnal nature, that means that there are two high and two low waters each lunar day. Part I of ATT Vol 1, gives the predictions of the times and the heights above chart datum of these high and low waters for a number of selected ports throughout the coverage area which includes UK, Ireland and European Channel ports. These ports are called Standard Ports. For each Standard Port there is also a tidal curve plotting the relative tidal height between a low water and the next low water, against the interval of time from the nearest high water. This tidal curve also contains a graphical means of finding heights of tide between the times of high and low waters. Part II of the tide tables gives tidal predictions for a large number of secondary ports. The information is such that the tidal predictions can be made for these ports by application of corrections to a Standard Port which has tides of a similar nature. The Standard Ports are tabulated in geographical order starting in the UK at Falmouth and following the coast anticlockwise followed by Irish ports starting at Dublin, and then the European channel ports from Hook of Holland to Brest including the Channel Islands. This is usually the easiest way of locating a particular port but they are also indexed at the back of the volume, and inside the front cover.

To find the times and heights of high and low water at a Standard Port

These may be extracted directly from Part I of ATT Vol 1 for the required Standard Port, and for the required date. (see extracts from ATT Vol 1). The times will be in the official standard time kept at the place which for the United Kingdom is Universal Time (GMT). For ports which do not keep UT the difference between the zone time and UT is shown at the top of each page of predictions. The sign attached to this correction is appropriate to correct the tabulated times to (UT). The user must apply any corrections to obtain any summer time that may be in force.

Example (refer to extracts from ATT Vol 1)

Find the times of high and low water at Avonmouth on the morning of 29th January and the depth of water at these times at a place off Avonmouth where the charted sounding is 4.2 metres. Find also the predicted range of the tide.

from ATT Vol 1	HW 0723	Height 13.4 metres
	LW 0147	Height 1.4 metres.

Depth of water = charted depth + height of tide

depth at high water	= 13.4 + 4.2 = 17.6 metres
depth at low water	= 1.4 + 4.2 = 5.6 metres
predicted range	12.0 metres

To find the height of tide at times between high and low water at a Standard Port

This is done with the aid of the tidal curve given with the Standard Port. There is one curve for spring tides given in full line, and one for the neap tides given in dotted line where it diverges from the spring curve. For times between springs and neaps then an interpolation between the two curves should be made if necessary. To establish which curve to use the range for the tide in question should be compared to the mean spring and neap ranges given in a small box above the curves. Thus for the example given above the predicted range of 12.0 metres is very close to the mean spring range of 12.2 metres. The mean neap range is 6.0 metres. This tide on 29th January is therefore slightly below a mean spring tide, and the spring curve is used.

Procedure

1. Extract from ATT Vol 1 Part 1 the times and heights of the high and low waters which bracket the time for which the prediction is required. This time should be expressed in the same time zone as the tidal predictions for the port.
2. Subtract the height of low water from the height of high water to give the predicted range.
3. On the bottom scale to the left of the tidal curve, marked L.W.Hts.m. mark the low water obtained for this tide.
4. On the top scale to the left of the curves marked H.W.Hts.m. mark the height of high water obtained for this tide. Join this point with the low water marked in 3, with a straight line.
5. Take the difference between the time for which prediction is required and the time of high water. This is called the interval from high water. Note whether it is negative (rising tide) or positive (falling tide).
6. Compare the predicted range with the mean spring and neap ranges given above the tidal curves. This will determine whether the spring curve or the neap curve should be used and if interpolation is necessary.
7. Enter the tidal curves on the bottom scale directly beneath the curves at the point of the established interval from high water. Draw a vertical to meet the curve to be used (springs or neaps) or to a point between the two if interpolation is to be done. From this point draw a line horizontally to meet the straight line drawn in 4. From this point go up or down to the top or bottom scales and read off the height. This is the height of tide above chart datum at the time selected.

Example (refer to Figure 1.7.4)

Find the height of tide at Avonmouth at 1530 GMT on 9th April and hence the depth of water at a place where the charted sounding is 2.0 metres.

From ATT	HW 1730	Ht 11.3 metres
	LW 1136	Ht 2.5 metres 1
		8.8 metres = predicted range. 2
	time HW	1730
	time required	1530
	int from HW	−0200 (rising tide) 3

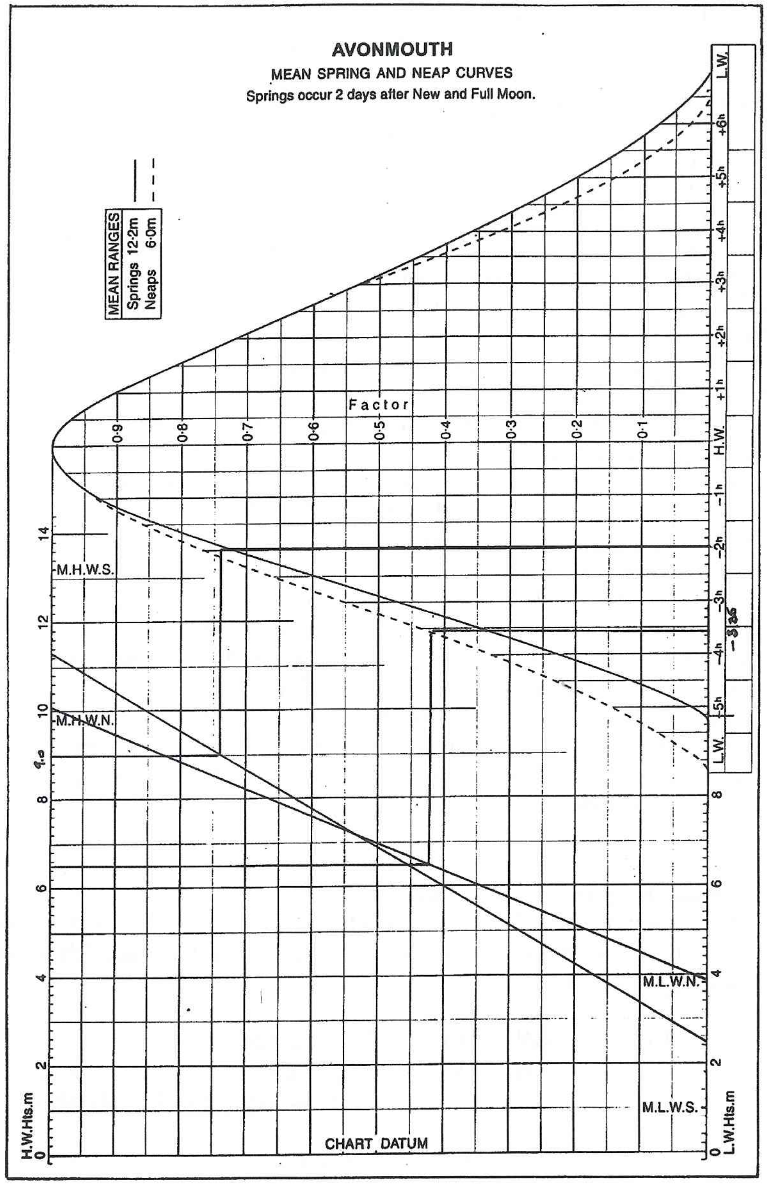


Fig. 1.7.4

From tidal curve	spring range	= 12.2
	neap range	= 6.0
	predicted range	= 8.8 (assume half way between springs and neaps)........................ 4

From Figure 1.7.4	ht of tide	9.0 metres
(see constructed lines)	charted sounding	2.0
	depth of water	11.0

To find the time at which there will be a given depth of water between high and low water at a Standard Port, on a given tide

It will be necessary to determine the height of tide which will produce the required depth by subtracting the charted sounding. The procedure is then the reverse of that of the previous example.

Procedure

1. Extract the times of high and low water for the tide in question, with the heights.
2. Subtract the height of low water from the height of high water to find the predicted range. Compare this with the mean spring and neap range and determine which tidal curve to use or whether interpolation is required.
4. Subtract the charted sounding from the depth of water given to find the height of tide.
5. Plot the heights of low water and high water on their respective scales at the top and bottom of the graph at the left hand side of the tidal curves as in the previous example. Join these points with a straight line.
6. From the top or the bottom scale locate the height of tide found in 4 and draw a line vertically down or up to meet the straight line drawn in 5. Draw a line horizontally from this point to the appropriate tidal curve. From where this line meets the tidal curve, draw a line vertically down to the scale directly beneath the tidal curves. Read off the interval from high water and apply it to the time of high water.

Example (refer to Figure 1.7.4)

Find the time when a vessel of draft 6.5 metres will have a clearance of 1 metre over a shoal of charted depth 1.0 metre off Avonmouth, on the rising tide of the morning of the 23rd February.

draft	6.5 metres
clearance	1.0
depth requ.	7.5
sounding	1.0
Ht of tide	6.5 metres

LW 23rd.	0920	3.9 metres
HW	1536	10.1
		6.2 = predicted range
		(this is close to mean neap range)

from Figure 1.7.4 (see constructed lines) a height of tide will occur at 3h 35m before high water.

time of high water	1536
interval from HW	0335
time required	1201

A recent innovation in Volume 1 is the inclusion of a Part Ia. This consists of a tabulated solution of the height of tide for each hour throughout the year for a number of Standard Ports. There is one table for each month and the arguments hour of the day and date are used to cross reference the height of tide. The amount of data means that it can only be included for a few ports and only Plymouth, Poole, Southampton, Portsmouth, Rosyth, Liverpool, Avonmouth and St. Helier are given.

Standard Ports between Poole and Chichester Harbour

In this area the tidal curves exhibit such characteristics that the time of high water is largely indeterminate, while the time of low water is clearly defined. For the Standard Ports Poole, Cowes, Southampton and Chichester Harbour the curves are drawn relative to low water and when finding intermediate times the 'interval from low water' should be taken and not the 'interval from high water'. Apart from this the procedures are identical to those explained above.

Secondary Ports

Part II of ATT Vol 1 gives tidal information for a large number of secondary ports. This information is given as time and height differences to be applied to those at some Standard Port which has tides of a similar nature to the secondary port. The same geographical pattern as the Standard Ports is followed in the layout but the index gives a consecutive number as an alternative means of locating the port.

High and low water time differences

The time differences between high and low water at a secondary port usually vary between springs and neaps. There are two values given which should be taken as the maximum and minimum differences. These are tabulated against the time of high and low water at the Standard Port which will depend mainly on the spring-neap cycle. If the time of high or low water falls between the tabulated times then the time differences must be interpolated if accuracy is required. Figure 1.7.5 shows an extract from Part II of ATT Vol 1. The Standard Port is River Foyle indicated by the upper case title and the headings against the port name. For Ballycastle Bay a time difference of +0126 would be applied to a high water at River Foyle which occurs at 0100 or 1300. A time difference of −0112 would be applied to a high water at River Foyle which occurs at 0800 or at 2000. If the high water at River Foyle is between any of these values then interpolation is required for accuracy.

Example

Find the time of high and low water at Ballycastle Bay if the times at River Foyle are HW 0330 LW 1000.

The high water time required lies between the two tabulated times for high water of 0100 and 0800, a difference of 7 hours.

Time difference for	0100	+0126
Time required	0330	
Time difference for	0800	−0112
difference		0238 = 158 minutes

Thus the correction to be applied to 0100 will be

$$\frac{2.5 \times 158}{7} = 56 \text{ minutes}$$

Thus the time difference which must lie between +0126 and −0112 is:

$$+0126 - 56 = +0030$$

Time of HW at River Foyle	0330
Time difference	+0030
Time of HW at Ballycastle Bay	0400

Note: The 56 minutes must be applied to +0126 because the interval of 2.5 hours was measured from 0100 and the final time difference must lie between the two values +0126 and −0112.

No.	PLACE	Lat. N	Long. W	TIME DIFFERENCES High Water		Low Water		HEIGHT DIFFERENCES (IN METRES) MHWS	MHWN	MLWN	MLWS	ML Z_0 m
				Zone UT(GMT)								
658	**RIVER FOYLE (LISAHALLY)**	(see page 178)		**0100 and 1300**	**0800 and 2000**	**0200 and 1400**	**0700 and 1900**	**2.6**	**1.9**	**0.9**	**0.4**	
651	Ballycastle Bay	55 12	6 14	+0126	-0112	-0053	+0128	-1.3	-0.9	-0.2	0.0	0.85
652	Portrush	55 12	6 40	-0039	-0041	-0104	-0050	-0.7	-0.5	-0.1	0.0	1.13
653	Coleraine	55 08	6 40	-0004	-0106	-0109	-0005	-0.4	-0.1	0.0	0.0	⊙
	Lough Foyle											
654	Warren Point	55 13	6 57	-0055	-0115	-0155	-0117	-0.3	0.0	⊙	⊙	⊙
655	Moville	55 11	7 03	-0042	-0057	-0127	-0058	-0.3	0.0	+0.1	0.0	⊙
656	Quigley's Point	55 07	7 11	-0020	-0027	-0040	-0027	-0.3	-0.1	0.0	-0.1	⊙
657	Culmore Point	55 03	7 15	-0002	-0003	-0003	-0002	-0.1	-0.1	+0.1	0.0	⊙
	River Foyle											
658	RIVER FOYLE (LISAHALLY)	55 03	7 16	STANDARD PORT					See Table V			1.42
659	Londonderry	55 00	7 19	+0033	+0035	+0032	+0032	+0.1	+0.2	+0.3	+0.2	1.64
	Ireland											
660	Culdaff Bay	55 18	7 09	-0103	-0121	-0134	-0114	+0.2	+0.4	⊙	⊙	⊙

FIG. 1.7.5

The low water time required lies between the two tabulated times for low water of 0700 and 1400, a difference of 7 hours.

Time difference for	0700	+0128
Time required	1000	
Time difference for	1400	–0053
difference		0221 = 141 minutes

Thus the correction to be applied to 0700 will be

$$\frac{3 \times 141}{7} = 60 \text{ minutes}$$

Thus the time difference which must lie between +0128 and –0053 is:

$$+0128 - 60 = +0028$$

Time of LW at River Foyle	1000
Time difference	+0028
Time of LW at Ballycastle Bay	1028

Notes: The 60 minutes must be applied to +0128 because the interval of 3 hours was measured from 0700 and the final time difference must lie between the two values +0128 and –0053.

These calculations have produced answers accurate to the nearest minute. If this accuracy is not required then an adequate result could have been produced by visual inspection. In this case the values for the time differences were over 2.5 and 3 hours apart for HW and LW respectively. This would make visual interpolation difficult. This in fact is not the case for most secondary ports and visual interpolation to acceptable practical accuracy is easy, and with practice the above procedure becomes a quick and easy task for most secondary ports. There is also provided, in the introduction and explanation in the front of the tide tables a graphic method of achieving an accurate result. On graph paper, or using the diagrams provided the time differences are plotted on the y axis and the times for high or low water at the Standard Port plotted along the x axis. The two time differences given against the secondary port are then plotted against the tabulated high or low water times and joined with a straight line. The required time

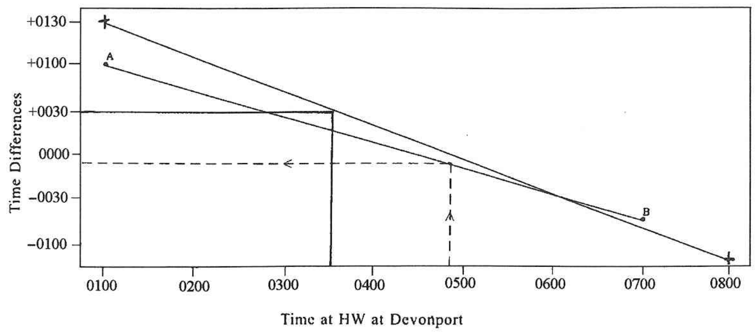


FIG. 1.7.6

difference is then read by entering the graph at the actual time of high or low water along the x axis, going vertically up to the line and then across to read the time difference from the y axis. The above solution for high water is shown worked in this manner in Figure 1.7.6 which shows how the example in the explanations at the front of the tide tables can be modified for a particular secondary port.

Height differences

Differences in tidal height between a secondary port and its Standard Port are tabulated for MHWS, MHWN, MLWS and MLWN. The differences between the spring and neap levels should be assumed to vary linearly and can be found by interpolation. For heights outside the mean range the height differences are found by extrapolation. The level of the tide in question at the Standard Port should be compared with the mean spring and neap levels for the Standard Port for interpolation purposes. The interpolation is, for the large majority of secondary ports, easily done visually if extreme accuracy is not required. Any variations in the level of chart datum between the Standard Port and the secondary port are allowed for in the height differences and the predicted height for the secondary port will be above chart datum for the secondary port.

Example (refer to Figure 1.7.5)

The height of high water at River Foyle is 2.1 metres and low water 0.6 metres. Find the height of high and low water at Ballycastle Bay.

MHWS level	2.6	difference	−1.3
Tide in question	2.1		
MHWN level	1.9	difference	−0.9
			0.4

Thus a correction should be applied to −1.3 which is

$$\frac{0.5 \times 0.4}{0.7} = 0.3$$

Thus the difference is $-1.3 + 0.3 = -1.0$

High water at River Foyle	2.1 metres
Difference	−1.0
High water at Ballycastle Bay	1.1 metres

By visual inspection the height difference for low water will be −0.1.

Low water at River Foyle	0.6 metres
Difference	−0.1
Low water at Ballycastle Bay	0.5 metres

Note again that if the height differences given are close and if extreme accuracy is not required this process can usually be done very quickly by inspection. There is also a graphic method described in the explanations in the front of the tide tables similar to that described for time differences.

Seasonal changes in mean level

At the foot of the pages containing secondary port data is given information on seasonal changes in mean sea level throughout the year. For this purpose ports are grouped together in blocks according to their ATT number, each block having the same values of seasonal change.

The seasonal variations are included in the heights given for Standard Ports but these may not be the same as the variations at the secondary port. Thus the seasonal variations at the Standard Port should be extracted from the Standard Port heights before the secondary port height differences are calculated. The season variation for the secondary port should then be applied to the calculated heights for that port.

This is done by correcting the height for the Standard Port by subtracting algebraically its seasonal variation for the month. This means that if the seasonal correction given is negative then it must be added and vice versa. The height differences can then be calculated and applied. Finally the secondary port seasonal variation for the month can be added algebraically. This means that if the correction is positive it is added and vice versa.

An inspection of Part II of the tide tables will show that in most cases the seasonal variations are very small. For the year 2002 there are only a few cases for UK ports when the value goes to as much as 0.2 metre. If the value for the Standard port is +0.2 and the value for the secondary port is −0.2, then neglecting the corrections would result in an error of 0.4 metre in the heights predicted. This is an extreme case however but the navigator should look at the values given and decide for himself if they are significant to his own circumstances.

Example (use extracts from the Admiralty Tide Tables)
Find the times and heights of high and low waters at Watchet (ATT 531) on 8th April.

Standard Port	HW	LW	HW	LW
Avonmouth	0408	1042	1639	2312
differences	−0040	−0105	−0042	−0110
Watchet	0328	0937	1557	2202
Avonmouth	10.1	3.5	10.4	3.2
seasonal corr.	+0.1	+0.1	+0.1	+0.1
Avon. corrected	10.2	3.6	10.5	3.3
Differences	−1.5	+0.1	−1.6	+0.1
Watchet	8.7	3.7	8.9	3.4
seasonal corr	−0.1	−0.1	−0.1	−0.1
Wat. corrected	8.6	3.6	8.8	3.3

Notes: the two HW time differences given for Watchet are −0035 and −0050 that is 15 minutes apart. The first Avonmouth high water lies 2.1 hours in a six hour period after the tabulated time of 0800. The time difference will be given by:

$$-0035 - \frac{2.1 \times 15}{6}$$
$$= -0040$$

The second high water lies 2.65 hrs after the tabulated time of 1400 in a six hour period and the difference will be:

$$-0035 - \frac{2.65 \times 15}{6}$$
$$= -0042$$

The low water differences are more separated (−0145 and −0040) and a little more care is required. Also the period between tabulated times of high water is either five hours or seven hours. The first low water difference will therefore be:

$$-0040 - \frac{2.7 \times 65}{7}$$
$$= -0105$$

In this case the seasonal corrections are small, are the same for both ports and have had no effect on the result.

To find the height of tide at a secondary port at a time between high and low water
Unless indicated otherwise in the tide tables the tidal rise and fall at a secondary port are similar enough to those at its Standard Port for the Standard Port tidal curves to be used for the secondary port. After finding the times and heights of the high and low waters at the secondary port the problem is then identical to that for the Standard Port. The spring and neap ranges at the secondary port must be found for comparison with the predicted range to determine which of the two curves to use.

Procedure

1. Extract the times of the high and low waters which bracket the time in question, for the Standard Port together with the heights.
2. Apply the time differences to obtain the times of high and low water at the secondary port. Be careful that after application of the time differences the times do not bracket the time in question. If this happens then new times will have to be taken for the Standard Port. Apply the height differences to find the heights at the secondary port.
3. Subtract the height of low water at the secondary port from that of high water to obtain the predicted range.

4. Take the difference between the time in question and the time of high water to find the interval from high water.

5. Apply the secondary port spring and neap height differences to the heights of MHWS, MHWN, MLWS and MLWN for the Standard Port to obtain these values for the secondary port.

6. Subtract the height of MLWN from the height of MHWN to obtain the neap range and the height of MLWS from MHWS to obtain the spring range. Compare these with the predicted range to determine whether to use the spring or the neap curve or if interpolation is necessary between the two.

7. Enter the tidal curve for the Standard Port with the interval from high water and extract the height of tide in the same manner as described for the Standard Port problem.

Example

Find the height of tide at Clevedon (ATT 525) at 1000 UT on 16th March. What will be the under keel clearance of a vessel of draft 8.1 metres, when passing over a shoal of charted depth 3.4 metres?

	HW	Ht	LW	Ht	
Avonmouth	0842	12.6	1533	1.7	1
Differences	−0019	−0.4	−0024	−0.0	2
Clevedon	0823	12.2	1509	1.7	
		1.7			
		10.5 = predicted range			3

seasonal correction is zero for both ports.

High water Clevedon	0823	
Time in question	1000	
Interval from HW	+0137	4

	MHWS	MHWN	MLWN	MLWS	
Avonmouth	13.2	9.8	3.8	1.0	
differences	−0.4	−0.2	+0.2	0.0	
Clevedon	12.8	9.6	4.0	1.0	5
	1.0	4.0			
spring range	= 11.8	5.6 = neap range			6

predicted range = 10.5 use spring curve

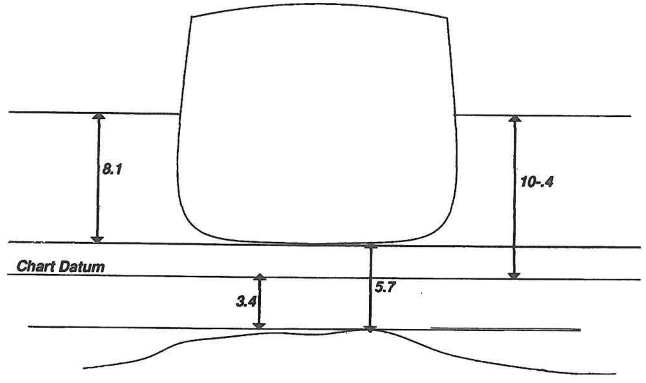


FIG. 1.7.6

from spring curve	height of tide	= 10.0	7
	charted sounding	= 3.4	
	depth of water	= 13.4	
	draft	= 8.1	
	clearance	= 5.3 metres	

To find the time between high and low water at a secondary port when there will be a given depth of water on a given tide

Procedure

1. Extract the times and heights of the high and low water which bracket the time in question for the Standard Port. Remember to take the time differences into account when deciding which high and low water to take.

2. Apply the time and height differences to obtain the times and heights of the high and low water at the secondary port. Subtract the low water height from the high water height to find the predicted range.

3. Apply the secondary port spring and neap height differences to the heights of MHWS, MHWN, MLWN and MLWS for the Standard Port to obtain these values for the secondary port.

4. Subtract the height of MLWN from that of MHWN to obtain the mean neap range and subtract the height of MLWS from that of MHWS to obtain the mean spring range for the secondary port. Compare these with the predicted range to determine which tidal curve to use or if interpolation is necessary.

5. Ascertain the required height of tide.

6. Enter the tidal curve and diagram to extract the interval from HW as described for a Standard Port. Apply the interval from HW to the time of high water to obtain the required time.

Drying heights

A drying height is a sounding on a chart for a point which lies above chart datum. This will therefore be a negative sounding and will be the height of tide when the point dries on a falling tide or covers on a rising tide.

Example

A vessel is berthed at Watchet alongside a quay with a drying height of 1.5 metres. Find the time when the vessel will take the ground on the falling PM tide on 10th January if the vessel's draft is 3.8 metres.

	HW	Ht	LW	Ht	
Avonmouth	1650	11.5	2329	2.7	 1
differences	−0042	−1.7	−0112	+0.1	
Watchet	1608	9.8	2217	2.8	 2
		2.8			
		7.0 = predicted range			 2

(seasonal corrections were zero for both ports).

	MHWS	MHWN	MLWN	MLWS	
Avonmouth	13.2	9.8	3.8	1.0	
differences	−1.9	−1.5	+0.1	+0.1	
Watchet	11.3	8.3	3.9	1.1	 3
	1.1	3.9			
spring range	= 10.2	4.4 = neap range			
predicted range	= 7.0	(approx halfway between spring and neap curves)			 4

Vessel's draft	3.8	(depth of water when taking the ground)
drying height	1.5	
Ht of tide	5.3	

From tidal curves interpolating between springs and neaps

interval	= +3h 50m	 6
time HW	= 16 08	
Time required	= 19h 58m	

Secondary ports between Swanage and Selsey

In this area secondary ports exhibit very individual tidal curves and it would not be appropriate to use the tidal curve for a Standard Port. Tidal curves are therefore given for secondary ports in this area and these may be found in the explanations pages preceding Part I. These curves should be used rather than the curve for the Standard Port as described above. Furthermore these ports have tidal characteristics which make the time of high water largely indeterminate while the time of low water is clearly defined. The curves are therefore drawn relative to low water and the 'interval from low water' should be taken and not the 'interval from high water', when finding intermediate times. In some cases there are three curves given and not the usual two spring and neap curves. The curve to be used should be selected by inspecting the range at the Standard Port which is always Portsmouth. The third curve is drawn for a range between springs and neaps when the two high waters at the secondary port are equal, and interpolation between the curves should be made between this curve and either the spring curve or the neap curve. At these ports there is a very large difference between the tidal characteristics at springs and neaps and this interpolation therefore becomes critical if a result with any degree of accuracy is expected.

Charted heights

Heights of terrestrial objects such as lighthouses and topographical features are expressed above MHWS. If these heights are required accurately above the water level, for example for calculating clearances when passing under bridges, a correction must be applied equal to the height of MHWS above or below the water level.

Example

Find the correction to apply to the charted height of a lighthouse at a place where the level of MHWS is 13.2 metres and the height of tide is found to be 9.5 metres.

From Figure 1.7.8 the correction may be seen to be the height of MHWS minus the height of tide.

Thus actual height = charted height + MHWS − height of tide

correction = +(13.2 − 9.5)

= +3.7 metres

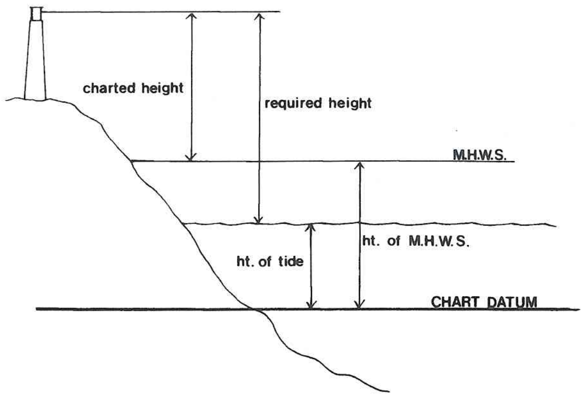


FIG. 1.7.8

Corrections to soundings in open water

The tidal information including the tidal curves given in Admiralty Tide Tables are for use only in the vicinity of the ports for which they are given. In offshore areas and inshore waters between secondary ports corrections to soundings may be obtained from co-tidal charts. The method is fully explained on each of these charts but is not included here as this is rarely done in the course of general navigation where any tidal height is normally regarded as a safety margin and navigation is conducted using charted soundings only. If underkeel clearances are in any way critical, such as for deep draught vessels in the southern North Sea and Dover Straits, co-tidal charts should be consulted to find when low waters occur in the open water. The mariner should also take heed of warnings given in Admiralty Tide Tables and in the Mariner's Handbook concerning negative storm surges, which occur regularly in these areas and cause less water than predicted, possibly by considerable amounts.

Admiralty Tide Tables Volume 2

Volume 2 covers Europe (excluding UK and Ireland), the Mediterranean and both sides of the Atlantic Ocean. The European ports are used in exactly the same way as described for Volume 1 but there are variations in the method used for other ports. There is no tidal curve associated with Standard Port predictions. Instead there is given in the explanatory section of the tables a diagram

showing Standard Curves, which are based upon an assumption of a sinusoidal tidal pattern. They are used in the same way as described for the tidal curves in Volume 1, but there are three curves given, instead of the two springs and neaps curves. These are for a 10, 12 or 14 hour tidal cycle from low water to the following low water. The use of the curves is therefore limited to tides which have these durations. Furthermore they can only be used for ports for which there is no shallow water correction given in Part III of the tables which gives harmonic constants. Within these limitations acceptable accuracy is achieved. These methods are also used for ports in Volumes 3 and 4 but it should be noted that in many parts of the world the semi-diurnal pattern of tides that we are familiar with is less evident sometimes becoming diurnal. The durations of these tides will not be within the limits stipulated for the use of these standard curves. In such cases the alternative method of using harmonic constants must be used.

EXERCISE 1.7.1

1. Find the times (UT) and heights of high and low waters at Avonmouth on 27th February.

2. Find the height of tide at a place off Avonmouth at 1715 UT on 31st March.

3. Find the depth of water beneath the keel of a vessel of draft 5.8 metres when passing over a shoal of charted sounding 2 metres at 1124 UT on 29th April off Avonmouth.

4. Find the depth of water over a rock of drying height 1.5 metres at the PM high water on 26th January off Avonmouth. Will this rock dry during the following tide?

5. Find the earliest time (UT) that a vessel of draft 6.5 metres can pass over a shoal of charted depth 2.5 metres off Avonmouth, with a clearance of 2.0 metres on the rising tide of the morning of 20th April.

6. A vessel is aground off Avonmouth with her forward section on a sandbank of charted drying height 1.0 metre. At what time UT can she expect to re-float on the midday rising tide of 9th March if the forward draft is 8.0 metres?

7. Find the height of a lighthouse near Avonmouth, above the water level at 0800 UT on 11th January, if the charted height of the lighthouse is 48 metres.

EXERCISE 1.7.2

1. Find the times and heights of all high and low waters at Sharpness Dock on 29th January.

2. Find the depth of water at a place off Sharpness where the charted depth is 2.5 metres, at the high water on the afternoon of 12th March, and the UT at this time.

3. Find the clearance under the keel of a vessel at anchor off Watchet at 1030 UT on 27th April if the charted depth is 3.2 metres and the vessel's draft is 5.5 metres.

4. Find the correction to charted soundings at a place off Weston-super-Mare at 1200 UT on 1st February.

5. Find the time UT when there will be 13.0 metres of water over a place where the charted depth is 5 metres, at Beachley on the rising tide of the morning of 19th April.

6. Find the latest time that a vessel can pass over a shoal off Watchet if the charted depth is 1.0 metre, on the falling tide of the evening of 1st January, if the vessel's draft is 6 metres and a clearance of 0.5 metre is required.

7. Find the height of tide at Bristol (Sea Mills) at 1500 UT on 15th January. What would be the charted sounding at a place which was just drying at this time?

MODULE 1.8

Radar Navigation

Radar has become an essential item of bridge equipment in merchant vessels and it is without doubt the most important of the electronic aids to navigation. Its role at sea is of a dual nature. The original purpose of providing merchant ships with radar was the detection of other vessels as an aid to successful avoidance of collision, and this perhaps is still its most important role. The quality and reliability of modern radars however has meant that it has become an important navigational instrument and it is with this role that we are mainly concerned here. In a book on navigation it is now impossible to ignore the role that radar plays.

We should not be too concerned in a work on practical navigation, with the operational theory of the radar, but some theoretical considerations which have a bearing on the practical use of the radar should be remembered first.

Basic Radar Theory

The purpose of the radar equipment is to detect the presence of objects in the vicinity and to present range and bearing information of those targets to the radar user. The principle of range measurement is the timing of a pulse of electro-magnetic energy from transmission to reception of a reflection arriving back at the aerial. The principle of bearing measurement is to sample each direction in turn by slowly rotating the aerial through 360 degrees. All other data presented by the radar concerning those targets are deduced from this information and from the way in which this information changes with time. For the present purpose, the radar equipment may be described by relatively simple functional blocks, without looking at the details of operation of components within those blocks. Figure 1.8.1 shows a diagram of such a simplified view of the radar equipment.

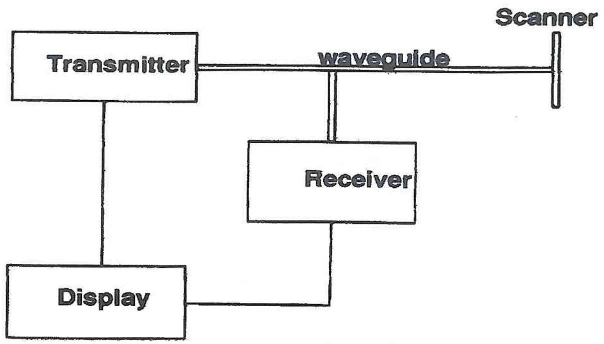


FIG. 1.8.1

The Transmitter

The transmitter unit generates short powerful pulses of radio energy. Such energy, of suitable frequency, on striking a target will be partially reflected or re-radiated. Some of this reflected energy will be directed back to the radar scanner. All civil marine radar use 3 cm or 10 cm wavelengths and in the UK the choice for a single radar unit on a ship will normally be 3 cm (approximately 10000 MHz). The number of pulses transmitted per second is referred to as the pulse repetition frequency (PRF) and typical values used are 1000 or 2000. The choice is determined partially by the need to wait until all reflections resulting from the transmission of one pulse to return to the scanner before the next pulse is transmitted. This is because if the radar is timing the interval between transmission and reception of any reflections, those reflections must be positively identified with the transmission with which they are associated. On the 12 mile range for example it will take about 500 microseconds for returns to arrive back at the aerial from the maximum range. This means that there must be at least 500 microseconds between transmitted pulses. A PRF of 1000 would probably be used for this range which corresponds to an interval of 1000 microseconds between pulses transmitted. On shorter ranges it will be possible to increase the PRF and this is normally done as this will increase the number of strikes on an object as the scanner rotates through the object's bearing and provide stronger radar echoes.

The duration of each transmitted pulse is important to the quality of the radar display. To some degree the radar operator has some choice of pulse duration, and he should understand the

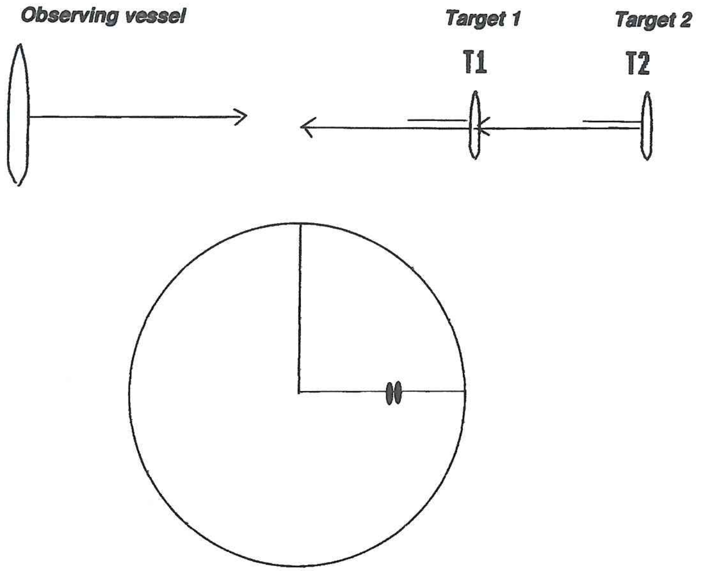


FIG. 1.8.2

implications of a selection of long, short or medium pulse lengths, which is usually provided by the radar controls. The pulse lengths (PL) used are described in microseconds of duration at the transmitter and typically are between 1 microsecond, which would be described as a long pulse, and 0.1 microsecond, which would be described as a short pulse. The pulse length determines the range discrimination of the radar equipment, which is its inherent ability to distinguish between the echoes of two separate radar targets which are on the same bearing. This can be understood by reference to Figure 1.8.2

Radio energy travels at approximately 3×10^8 metres per second or more conveniently 300 metres per microsecond. The leading edge of a pulse of duration 1 microsecond will therefore be 300 metres advanced on the trailing edge. If two objects on the same bearing are separated by less than half this distance then the leading edge of the return from the second target will overlap with the trailing edge of the return from the first target and the radar will receive back one lengthened pulse which will paint on the display as one elongated echo as shown in Figure 1.8.2. The range discrimination is therefore said to be half the pulse length, in the case of a 1 microsecond pulse 150 metres. On short ranges where power within a pulse is not such a great consideration then the pulse length is shortened perhaps to 0.1 microsecond to give a range discrimination of 15 metres. This will be done automatically in changing range but on medium ranges there is often a choice offered to the operator by a long/short pulse switch. From the above it should be evident that a short pulse will give a better defined and clearer display with smaller echoes representing the targets, and a long pulse will give a more powerful echo but the picture will be less well defined because of the larger individual echoes merging with one another. The short pulse will be the choice for normal operating conditions, with the long pulse being used to search for weaker echoes.

The Scanner or Aerial

The design of a radar scanner is such as to concentrate the radio energy into a beam which is narrow in the horizontal plane. This is essential to the measurement of accurate bearings as a reflection may have been directed back from anywhere within the beams horizontal dimension. The vertical dimension must be great enough to maintain energy radiation in the horizontal direction when the vessel is rolling and pitching. Typical vertical beamwidths will be 20 degrees.

The slotted waveguide scanner consists of the end section of waveguide with slots cut into it through which energy radiates. The size and spacing of the slots are dependent on the wavelength. Obviously there must be a rotating joint incorporated into the waveguide at the scanner. The horizontal beamwidth so critical to accurate bearing measurement is dependent upon the frequency and the length of the radiating aerial, or more accurately on the number of slots in the slotted waveguide aerial. As the radar is using a fixed frequency then the only factor to be considered is the aerial length. There is a practical limit on aerial size which usually decreases as the size of the vessel decreases. The horizontal beamwidth is typically 0.5 degrees to 2.0 degrees, but on small vessels when the size of the scanner is severely limited the horizontal beamwidth may be as much as 4 degrees.

Figure 1.8.3 illustrates the relative power transmitted in the horizontal plane against direction from the scanner. The beamwidth is measured between the half power points. As any one target will receive and return energy for as long as the beam is passing over it, echoes are distorted by being elongated around the display so that a point target will show as a roughly elliptical

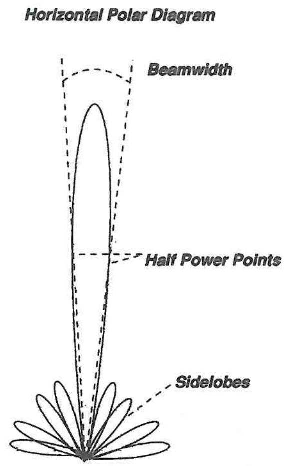


FIG. 1.8.3

shape. This is known as beamwidth distortion. Figure 1.8.4 also shows clearly the effect of the beamwidth on the radar display in that if two objects are at the same range on closely adjacent bearings then the beamwidth will determine whether the beam rotates completely over the first target before it reaches the second or not. This, and the beamwidth distortion, will determine whether the two objects produce two separated echoes on the display or paint as one elongated echo. This ability to distinguish between two such objects is called the bearing discrimination. It is less easy

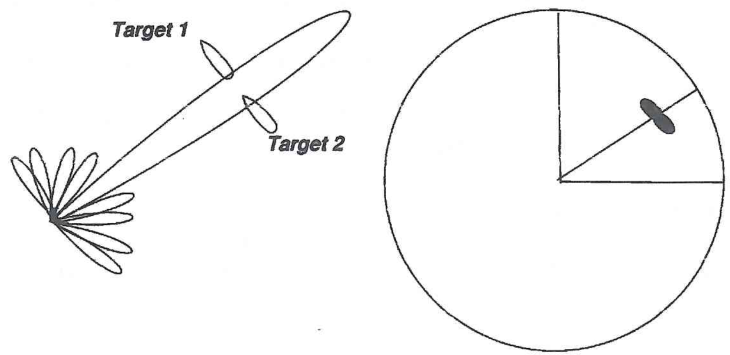


FIG. 1.8.4

to quantify than range discrimination but it should be remembered that a radar using a small scanner will give a less well defined picture as the echoes merge into those on adjacent bearings. A wide scanner will give a very much better defined picture. There is obviously nothing that the operator can do to improve the beam-width, but he should remember the implications. The beamwidth distortion will also cause exact position of coastal features such as headlands to be difficult to define, and the apparent closing up of small openings such as harbour entrances. These effects, amongst others, limit the accuracy of radar bearing information, which usually cannot compare with that which can be obtained visually.

The receiver and display

The design of marine radar receivers has changed radically over the past few decades. All marine radars display information on a raster scan display. Formerly radars presented the range and bearing information by means of a trace or timebase, which rotated around the display in synchronism with the scanner, the detected analogue pulse being used to brighten the trace to produce the visible echo. The polar coordinates of range and bearing are now converted to rectangular coordinates for display on a rectangular raster. This requires digitisation of the information received from each transmitted pulse, so that each trace or timebase is held as a series of 0 or 1 digits each representing a small cell within the trace and indicating the presence or absence of a returned echo. This process formed the key to further improvement in radar technology allowing the development of ARPA units, improving interference rejection techniques, and providing the means to a display which could be viewed in daylight. The display to the modern navigator appears very different to that which his counterpart of a few decades ago was used to. It could be argued that these changes as far as the navigator is concerned are largely superficial. The facilities which a modern sophisticated radar can offer may make his task a little easier but the basic navigational techniques remain largely unchanged.

Display Orientation

The navigator must be completely familiar with the various styles of radar display which are available to him, and their advantages and disadvantages.

A stand alone radar display is capable of providing:

Relative motion unstabilised head up display.

If a compass, usually a gyro compass, is interfaced with the radar the additional displays are available:

Relative motion stabilised north up.
Relative motion stabilised head up (Course up).

If a true motion unit is fitted to the radar the additional displays are available:

True motion sea stabilised.
True motion ground stabilised.

Relative Motion Unstabilised Head Up

The term relative motion refers to the fact that if the ship's position is represented always on the radar display as a fixed point usually at the centre of the display, then the movement of the echoes of other targets will be relative to this centre. This means that the movements of fixed targets such as land will be reciprocal to the forward movement of the navigator's own vessel. The movement of ship targets on the radar display will be the result of both the own ship movement and the target movement. Problems sometimes arise from the interpretation of such movements because when looking down on the plan view offered by the radar display it is logical to expect such a presentation to show true movements as they would be plotted on a chart. It is difficult to relate the blips representing the target ships to the vessels as they would appear from the wheelhouse window, in terms of aspect. However it should be remembered that the relative movement observed on the radar screen is in fact the relative movement that is always observed from the wheelhouse windows of the observing vessel in clear weather and as such may be regarded as the most desirable presentation. All that is lacking is a clear indication of the targets heading or fore and aft line, and we can now have that from an ARPA unit.

The word unstabilised refers to the fact that there is no input of compass information and all directions and bearings taken from the display are referred to the only reference available, that is the own ship's heading. True information must be obtained by the operator by applying the ship's heading. If this is the case then it is usual to orientate the picture with the heading marker upwards at 000 on the bearing scale, hence the term head up.

There is little difficulty arising from the fact that all bearing information is relative. This is what is required anyway when considering anti collision, and it is a trivial task to convert to true

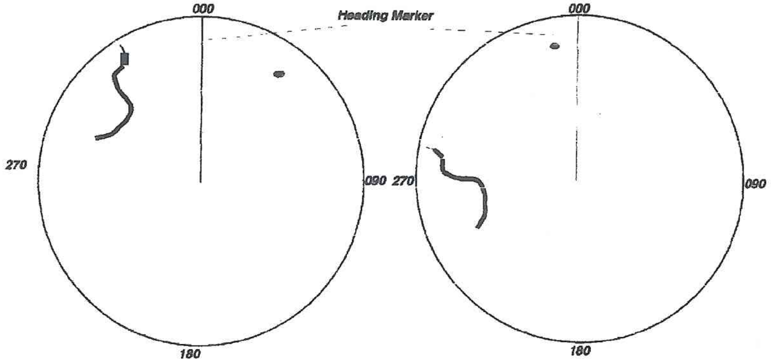


FIG. 1.8.5

for navigational uses. The main problems arise in the interpretation of the display for anti collision purposes when manoeuvring. In fact it is almost impossible. Any alteration of heading of the own ship will result in a corresponding and opposing movement of the target echoes around the display. No information can be taken from the display while own ship is actually swinging, land areas and target echoes will smear around the screen, and any information built up on the display regarding the movement of targets is really lost and the observer must start again. Moreover it is almost impossible to detect the fact that another target ship alters course or speed during the own ship manoeuvre or shortly afterwards and there must be a further period of observation after the ship's head has steadied until the situation is clear again.

If the own ship is yawing all targets will tend to move in sympathy with the yaw causing a blurring of the targets and trails. It may be noted here that while the choice of display is largely a matter of personal preference, the unstabilised display should never be used if any stabilised display is available.

Figure 1.8.5 shows the result of an alteration to starboard of about 60 degrees on the unstabilised display.

Relative motion stabilised north up

Arguably the most important advance in marine radar is the provision of an input from a compass, usually the gyro compass. The ship's head is transmitted to the radar and the heading marker can be displayed in the direction of the ship's head. In effect the outer bearing scale on the display becomes a compass

repeater. This means that the 000 degree mark on the bearing scale now represents north, hence the term 'north up', and the bearing scale is a scale of true bearing (in the absence of any compass error).

The features of a stabilised display are:

1. Any changes in the heading of the own ship will result in a movement of the heading marker and not the land or ship target echoes. This prevents blurring of the echoes during alterations or yaw, but the most important point is that it makes it far easier to detect changes in course or speed of other vessels while the own ship is manoeuvring. Measurements of range and bearing can be made while swinging and any plotting that is done on screen can be continued. It is far easier to predict the future movements of target echoes on the display, resulting from the own ship's manoeuvre.

2. Bearings measured from the radar display are true bearings and can be plotted on a chart after any compass error is allowed for.

The fact that north is shown at the top of the display is largely irrelevant. The display has the same orientation as a chart but this is a minor advantage. In fact for anti collision work it is an advantage to have the head up to correspond with the view from a vessel's wheelhouse window.

Figure 1.8.6 shows the result of an alteration to starboard of about 60 degrees on the north up stabilised display.

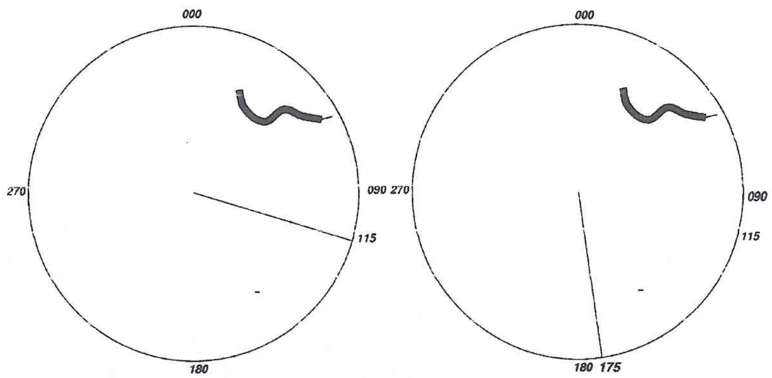


FIG. 1.8.6

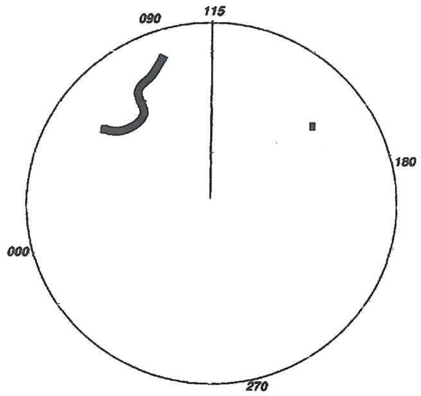


FIG. 1.8.7

Course Up (Relative motion stabilised head up)
The stabilised display is presented with the heading marker in the head up position on the outer bearing scale. The bearing scale will indicate the course at the head up position. The features of this display are exactly the same as the north up stabilised display except that the whole display has been rotated to provide a head up orientation. However if the vessel alters course (or yaws) the heading marker will move from the head up position because of the stabilisation. In other words the head up position represents the ship's course rather than the head. During the swing of an alteration the head up orientation is temporarily lost but when the vessel is steady on a new course the course up may be reset to make the vessel's new heading the new course up. This becomes the ideal orientation for use for anti collision purposes.

True Motion
The true motion display is a relative motion display which moves across the screen. True motion is an illusion created by making the electronic centre of the display, that is the own ship's position, track across the screen at a rate which represents the ship's speed on the display, and in the direction of the ship's course. This forwards motion of the centre offsets the relative movement of targets towards the own ship that is caused by the own ship's advance. The result is that the direction and speed of the target echoes across the display are representative of that target's true course and speed and fixed targets remain stationary on the display. The user gets a true plan view of the situation with the own ship and the targets progressing through the area displayed on the screen.

True motion must be carefully set up by making sure that the correct inputs are provided for the tracking. It requires an input from the compass hence it must be a stabilised display and it requires an input of ship's speed. This may be from a log or it may be a manual input.

True motion may be used in either of two modes.

Sea Stabilised True Motion
This mode results if the own ship's course and speed inputs provided to the radar are course steered, and log speed, that is course and speed through the water. Any movements of target vessel's observed from such a display will also be relative to the water, and it is this mode therefore that is required when using the radar for anti collision purposes, on the assumption that any tidal or current motion will be the same for both observing vessel and target vessels. It should be noted however that this need not be the case and misleading information may result in areas where strong and variable tidal conditions exist. Land targets and fixed marks will exhibit a movement on the display which is the reciprocal of the tide.

Ground Stabilised True Motion
This mode results if the course and speed inputs are relative to the ground, that is course made good or ground track, and ground speed. These inputs are achieved by inputting the course steered and the log speed through the water and also the correct tidal values on the tide controls. Note that a doppler log in bottom tracking mode will also give the speed over the ground. Knowledge of the exact tidal values is not always available, and ground stabilisation is often achieved by adjusting the course and speed controls such that a fixed target remains stationary on the display. This can be done by making a small mark over a suitable target and adjusting the course and speed controls until it remains under the mark. Any land target, buoy or vessel at anchor will then have zero motion across the display. All information about target echoes observed will be relative the ground, and this could give false information about target aspect.

This is the mode which should be used when using the true motion radar for navigational purposes.

Radar as a navigational aid

Radar is the one electronic aid which provides information of the vessel's position not as latitude and longitude but as range and bearing information from surrounding dangers, that is the land. In this respect the radar may be considered as an interface between the compass and the navigator or perhaps as an electronic azimuth mirror. In other words it will provide the navigator with the information that he would visually observe directly from the compass. As a bonus it also provides the invaluable range information that is normally unavailable visually.

One skill that the radar navigator must learn is the interpretation of the radar display of land targets. If any land feature is going to be used for navigation, it must be correctly identified with its corresponding charted image. The objects usually used for visual observation such as lighthouses and navigation marks do not show up well on a radar and it is usually prominent land features such as headlands and islands which can be identified. It must be said that the only useful land information on the radar display as far as navigation is concerned is the outline of the coastline. Inland features can never be identified with any amount of certainty. We therefore look for features such as headlands, islands, breakwater ends, and large prominent isolated rocks.

It must also be remembered that the coastline that is displayed on a radar may not be the coastline that is shown on a chart. The radar will not see around corners and will only display the facing coastlines. At long range the radar coastline may consist of inland contours, the coastline proper being below the radar horizon. The charted coastline is the level of MHWS and as such some of the charted land will at times be covered. At low water where there is a large tidal range coupled with a gradually sloping foreshore the radar coastline may be very different from the charted coastline and charted features will be difficult to identify. The navigator must therefore carefully select the elements of the radar display that he is going to use for fixing his ship's position.

There are two basic techniques of navigation that the radar provides. Which of these is used will depend upon the nature of the navigational task which is being performed. As part of his preparations for a passage the navigator must decide before the voyage begins which navigational technique is going to be his primary method of monitoring his position and what methods he has of checking or verifying the results. When navigating well clear of any dangers on a coastwise passage the radar can be used to gain range and bearing information which is used to plot the

vessels position on the chart. When navigating in what may be termed pilotage waters when the vessel may be close to dangers then a technique referred to as parallel indexing should be used. This provides immediate and continuous indication of where the vessel is in relation to the required ground track in the same way that leading marks do in clear weather.

Ranges and Bearings

A vessel's position may be plotted on a chart from observations of range and or bearings of identified charted objects on the radar display. Such radar information should not be used to the exclusion of visual information except in poor visibility. The use of radar in clear weather to verify visual position fixes will increase the navigator's confidence in its use in poor visibility.

It is essential that an electronic bearing indicator (EBI) is provided. This is a radial line drawn from the electronic centre which may be rotated by an operator control and which is usually linked to a numeric readout of its direction from the observer. It is a simple matter to rotate the EBI onto a target and read off its bearing and a series of bearings can be taken quickly. If the object being observed is an isolated target then the EBI should be adjusted to pass through the centre of the target. Often the best target identification is gained by laying the EBI to touch the left hand or right hand extremity of the land image. When laying such a bearing onto a chart the parallel rulers may be similarly lined up on the left or right hand edge of the land as seen from the observing vessel. The exact position of the headland on the radar, which may be masked by the effect of beamwidth distortion may be better seen by a temporary reduction in gain.

The accuracy of bearings obtained from radar should be checked regularly. On an unstabilised display the EBI should be laid against the heading marker. Any numeric readout should read zero, and the EBI from a centred display should be against the zero mark on the outer bearing scale. On a stabilised display the EBI laid along the heading marker should produce the same reading as the master compass heading, remembering that any compass error will also be present in the EBI reading. The absolute accuracy can then be checked by observing a target right ahead in clear weather and verifying that the radar image appears under the heading marker.

Ranges can be measured using a variable range marker which consists of a range ring whose range is varied by an operator control and is linked to a numeric readout. The accuracy of the

VRM should be checked from time to time against the fixed range rings and also by observation of known distances. The sum of the two ranges to two known objects on reciprocal bearings is a useful way of checking the accuracy without having to know the exact ship's position. Many modern radars provide a facility whereby a movable cursor can be placed over a target and both range and bearing read instantaneously from a numeric display of cursor range and bearing. Radars which are interfaced with GPS receivers may also be capable of providing a latitude and longitude of the cursor position together with the ship's position. This is a useful aid to identifying radar targets but should be used only for identification and not as an absolute indication of target position.

Bearings on the unstabilised display

When using an unstabilised display for taking bearings it is necessary to convert the relative bearing to a true bearing by adding the ship's head. The ship's head must be noted at the exact time of observation to minimise any inaccuracy due to yaw. It is not enough to use the course being steered. Any error of the compass must then be applied before laying the bearing off on the chart.

Example

The relative bearing of a lightship was measured from a radar display as 248°R. The ship's head at the time was 176°C. Variation was 3°E and deviation 2°E. Find the true bearing.

Ship's head	176°C
Compass error	5°E
Ship's head	181°T
Relative bearing	248°R
True bearing	429°T
	360°
True bearing	069°T

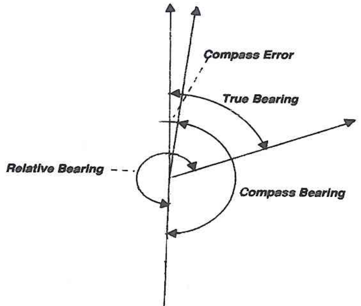


FIG. 1.8.8

Bearings on stabilised displays

These required only the compass error to be applied as described in Module 1.3.

Ranges

The range accuracy of a marine radar is inherently good. As long as the position of the object used is accurately identified on the chart, an accurate position can be obtained quickly by crossing two or more range circles with no corrections to apply to the raw readings from the radar. As always at least three position lines should be used to fix, usually a mixture of ranges and bearings.

Parallel Indexing

Parallel indexing is a means of continuously monitoring the progress of the vessel along pre-prepared tracks without resorting to fixing the position on a chart. It is used primarily in navigational situations which require a pilotage technique when instant cues are necessary to prompt the pilot when action is required. It can also be used to good effect in more open water coastal navigational situations. The disadvantage of using it in more open coastwise passages is that it is not conducive to good chartwork as a record of the vessel's progress, and it should be used as back up to the position fixing technique described. If this is done the navigator's skill in using the parallel index is increased and his confidence in using the technique in pilotage waters will be enhanced.

Parallel indexing is primarily used on stabilised radar displays. Once understood it is an easy and therefore safe technique. It can also be used with a properly ground referenced true motion display. Although possible on an unstabilised display the navigator should be very confident in his ability to do so as the lack of stabilisation introduces difficulties which if not mastered can lead to disaster.

Use is made of engraved lines inscribed on a perspex screen over the radar display, this screen being rotatable. Such fittings are rarely found on radars now that raster scan sets are the norm. There should however be means of displaying electronically parallel index lines on modern radars. The minimum provision needs to be a cross index line similar to an EBI whose azimuth is controllable, and a parallel index line at right angles to the cross index line at a range which is controllable, (the cross index range). Some modern radars actually produce the multiple fixed index lines at ranges corresponding to the range rings as in Figure 1.8.9.

Making good a course line to pass a given distance off a point of danger (Stabilised displays)

If a specific ground track is required to pass a given distance off a radar conspicuous danger then the parallel index line should be

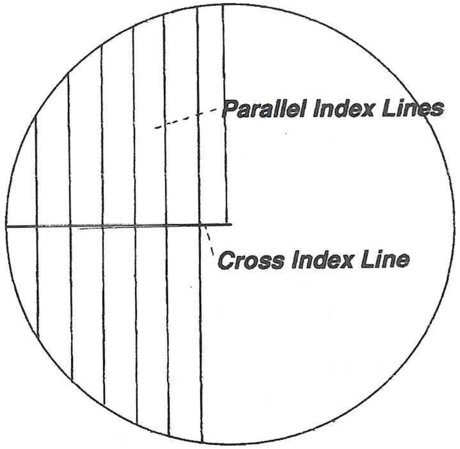


FIG. 1.8.9

rotated to represent the direction of that track. The cross index range should be set to the distance to pass off. If the vessel is on the correct track then the radar image of the point should be on the parallel index line. If not then the vessel is either inside or outside the course line and a course correction should be applied until the radar image of the point is on the parallel index line. When this occurs the course should be steered with an allowance for set to maintain the radar image of the point on the parallel index line. Once set the parallel index line should not be altered. This is illustrated in Figure 1.8.10.

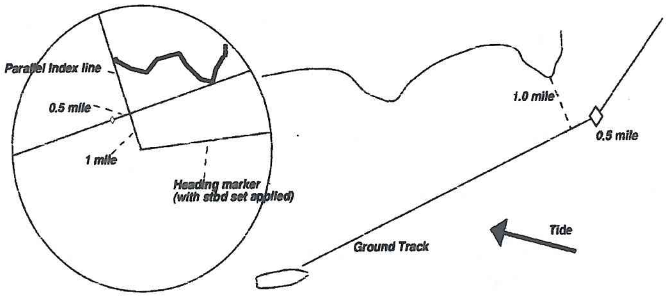


FIG. 1.8.10

To find what course to steer to pass a given distance off a danger (Stabilised displays)

To shape a course from wherever the vessel is, to pass a distance off a radar conspicuous danger then the cross index range should be set to the distance to pass off and the parallel index line rotated until the radar image of the point is on the parallel index line. The direction of the parallel index line then represents the course to make good, and this course should be steered with an allowance for set in order to maintain the radar image of the point on the parallel index line, remembering to check that the ground track selected in this way must be checked on the chart for hazards or for sufficient under keel clearance.

Alterations of course at waypoints while parallel indexing

An indication of when the vessel arrives at a waypoint position can be obtained by marking on the parallel index line the point where the radar image will be when the vessel reaches the waypoint position. Thus in Figure 1.8.10 the vessel will be at its waypoint when the point of land being monitored is 5 cables abaft the beam. This point is indicated on the parallel index line on the radar display. The navigator then monitors the progress of the radar image of the point of land until it reaches his marked point.

When in pilotage waters the advance of the vessel in making a turn may be significant compared with the width of the available channel. If this is the case then the turn should be initiated some distance ahead of the waypoint and marked on the chart as a wheel over position. The position marked on the parallel index line should then be the wheel over position rather than the waypoint. The amount of advance to allow for is a matter of the navigator knowing the manoeuvring characteristics of his ship.

When the alteration of course is made the navigator should be ready to readjust his parallel index line and cross index range to ensure that the turn has resulted in the vessel being on the next ground track and to monitor the vessel along the next leg of the track.

Parallel indexing on an unstabilised radar

The technique is uncomplicated on the stabilised display because once set the parallel index lines are not altered. The stabilisation maintains them representing the direction of the ground track. On an unstabilised display each time the vessel alters course the parallel index line must be reset, always remembering that it must represent accurately the direction in which the vessel is required to move. It is easy to make an error, or to forget to do this and therefore the technique should be used with caution. The navigator must be very confident in his ability to use it in critical situations. It is however, as always, a matter of experience and practice in non critical situations.

Preparations for parallel indexing

If it is decided during the production of the passage plan that parallel indexing is to be used as a means of monitoring the vessel's progress then some preparation is advisable. Parallel indexing should only be used as the primary method of monitoring in pilotage waters, and very often by the officer of the watch as a check on the pilot in clear visibility. In such circumstances the rapidity with which the vessel is moving through confined waters means that the more pre planning that can be done the easier will be the navigator's task when the passage is made. Ground tracks should be decided upon, radar conspicuous targets to be used should be chosen, cross index ranges should be measured and this information clearly marked on the chart. It is also a good idea to put it into note form that can be kept by the radar for reference. For selected parts of the track it may be advisable to note the margins of cross track error that are allowable.

Extension of the parallel indexing technique

The techniques described above are all possible with relatively simple provision on the display of easily manipulated parallel index lines. Many modern radars have provision to further enhance these techniques and if the navigator takes the time to thoroughly familiarise himself with their use he can make the monitoring task easier. It must be stressed however that the more sophisticated are the facilities provided the more care must be taken in their use and in the preparation for their use. Provisions vary between different manufacturers and here it is only necessary to give an outline of what the navigator may find on his radar. It is his responsibility to explore the possibilities provided for him. But in any case to do this requires a thorough understanding of basic techniques.

Multiple parallel index lines

Sophisticated radars may provide a facility to draw multiple lines on the display electronically. Such facilities are sometimes called mapping facilities and can be used in various ways. The technique to be described requires the facility to display a series of lines

which are maintained on the display in the same position relative to the electronic centre (own ship's position). This is the modern equivalent to drawing lines on a relative motion screen with a chinagraph pencil.

The parallel index line is used to represent the track across the radar display that a radar conspicuous target will make if the vessel makes good her intended ground track. If a number of lines can be drawn then it is possible to create a series of such lines to represent the track of the radar target across the display during a whole series of tracks through an area of close navigation. When the lines are displayed on commencing the passage there is a minimum of manipulation to be done by the navigator whose task is now to con his vessel so that the radar target follows the displayed lines. The nature of the preparation of these lines means that it must be done at the passage planning stage and therefore there must be provision to store the results of the preparation for instant recall at the time. Having been done however it is there for use for as many times as the navigator finds himself making that same passage.

Example

Figure 1.8.11 shows a series of ground tracks of a vessel making a passage into Plymouth Sound and coming to an anchorage. The illustrated radar display shows the parallel index lines that could be produced to monitor the vessel's progress from the port limits to the anchorage.

The radar conspicuous target chosen was the western end of Plymouth breakwater, as this will be instantly recognisable and will be on the radar display at all times during the passage. The lines were prepared from the chart by taking the range and bearing of the breakwater at each waypoint. The positions of the ends of each index line corresponding to the waypoints will be positioned on the radar display using these ranges and bearings, and they will remain in the same position on the display relative to the own vessel's position. Note that the resulting pattern of index lines is a mirror image of the course lines on the chart. (In defining the start and end positions of the lines on the radar the navigator should ask himself where will the radar image of the end of the break-water be if my vessel is at a waypoint).

Navigation Lines

On sophisticated marine radars particularly those with ARPA facilities the technique is further enhanced by the ability to display

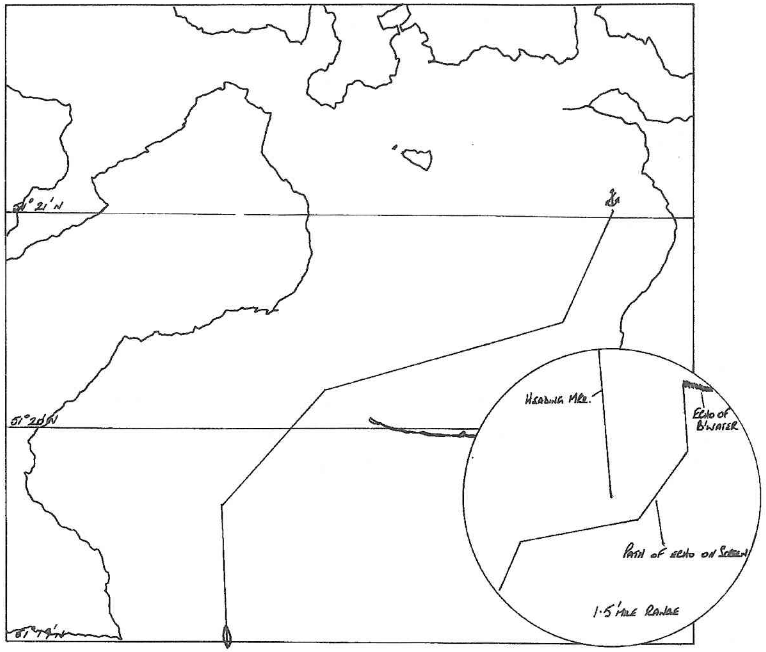


Fig. 1.8.11

lines and maintain them ground referenced rather than referenced to the electronic centre of the display. If this is the case then it is possible to define a set of lines on the display to represent lines that are drawn on the chart. They could be used to mark the vessel's ground track, they could be used to mark the limits of a channel through which the track passes, or they could be used to mark coastlines or the limits of traffic separation schemes through which the vessel passes. Once prepared and stored they can be retrieved, displayed and used as many times as is necessary. The navigator's task is then to make sure his vessel is in the correct place relative to those lines. If the lines are used to represent his ground tracks he must con his vessel to stay on the lines. If they are used to mark the limits of a channel he must make sure his vessel stays between the marked lines. Figure 1.8.12 shows the same passage through Plymouth Sound but using navigation lines to mark the vessel's ground tracks. This time they should look exactly like the tracks on the chart.

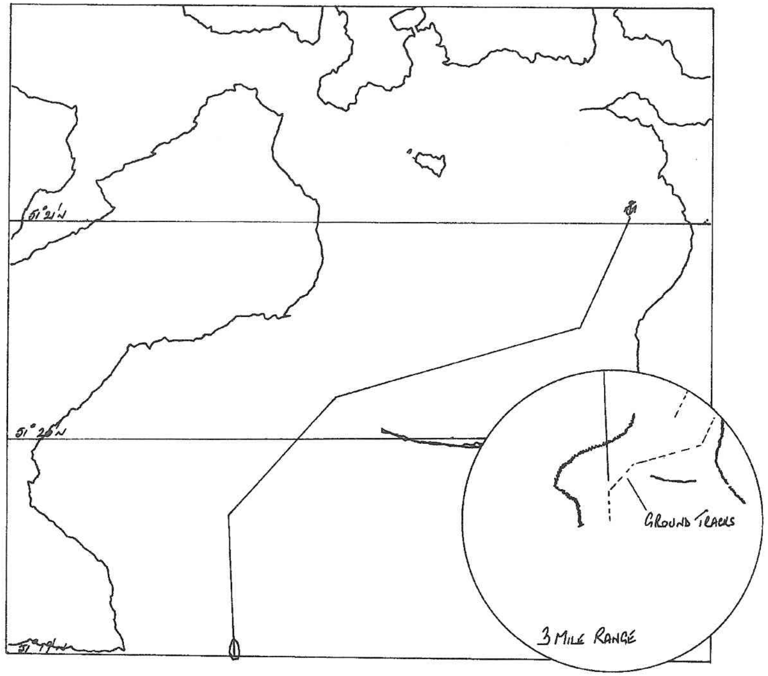


Fig. 1.8.12

These lines must be kept in the correct position relative to the fixed dangers amongst which the vessel is navigating, that is they must be ground stabilised. This may be achieved in one of two ways. The radar itself if it has full ARPA facilities may track a designated fixed target to monitor the set and drift of the tide. This can then be used to shift the lines in the reciprocal direction. Alternatively the radar may have the facility of taking a set of waypoints from a GPS receiver together with the vessel's own latitude and longitude and displaying the waypoints at the correct range and bearing from the vessel's own position. The waypoints may then be linked by navigation lines. Either way great danger exists if the navigation lines are not being displayed correctly and it is essential that the navigator has some check on their validity. As with all technology the more complex it becomes the greater the possibility of malfunction or misinterpretation and more vigilant the user must be to ensure correct operation.

One way of monitoring the validity of the navigation lines is to use some lines or points to represent fixed features that can be easily recognised on the radar such as breakwaters, lengths of coastline, or just buoys or light vessels. Instant recognition as to whether the computer generated line is being displayed over the radar image of the feature can then be made.

SECTION 2

Celestial Navigation

Celestial marine navigation means the monitoring of the navigation plan by means of observations of the heavenly bodies. This includes the fixing of the vessel's position and the monitoring of the compass error.

Modules included in this section are:

2.1 The Celestial Sphere and the Nautical Almanac.

2.2 Time.

2.3 Compass Error by Astronomical Observation.

2.4 Altitude and Azimuth. Correction of Altitudes.

2.5 Position Lines from Astronomical Observation.

2.6 Latitude by Meridian Altitude.

2.7 Position Lines from Bodies out of the Meridian.

2.8 Latitude by Pole Star.

MODULE 2.1

The Celestial Sphere and the Nautical Almanac

It is easy to imagine, when looking at the night sky, that the stars are on the inside of a huge dome which could be part of a sphere surrounding the earth with the earth at the centre of the sphere. This concept is useful to the navigator and it can be used because he is primarily concerned with angular measurement and although the heavenly bodies are at different distances from the earth this does not affect the measurements that the navigator is making. He is concerned with angles subtended at the centre of the earth between heavenly bodies and his planes of reference.

Thus the concept of the celestial sphere is a sphere of infinite radius centred upon the centre of a spherical earth. Upon the inside of this sphere are projected from the centre of the earth, the apparent positions of all the celestial bodies. Although the closer bodies such as the sun and planets move against the stars due to the movement of the earth around the sun, their angular position at any time is not affected by our concept of the earth as the centre of the universe and we are free to imagine that the sun and planets are in fact orbiting the earth.

In order to measure and express the positions of heavenly bodies on the celestial sphere we require a coordinate system similar to latitude and longitude on the earth.

The Equinoctial

The Equinoctial is the projection of the earth's equator out onto the surface of the celestial sphere. It is therefore a great circle on the sphere with its plane in the same plane as the equator. It is sometimes called the Celestial Equator.

Celestial Poles

The Celestial Poles are the projections of the earth's poles out onto the celestial sphere or the points on the sphere where the earth's axis of rotation, if produced in both directions would meet the sphere. The whole celestial sphere appears to rotate around the celestial poles due to the rotation of the earth.

Parallels of Declination
These are small circles on the celestial sphere the plane of which are parallel to the equinoctial.

Celestial Meridians
These are semi great circles which join the two celestial poles cutting the equinoctial in a right angle.

The Ecliptic
The sun appears to move against the background of the stars due to the movement of the earth around the sun. Again the concept of the earth at the centre of the celestial sphere with the sun moving around the earth may be useful. The Ecliptic is the great circle produced by the projection of the sun onto the celestial sphere as the sun moves around the sphere once in one year. It will appear to move eastwards against the stars. The ecliptic will be inclined to the equinoctial by approximately 23° 27′ which is the inclination of the earth's axis of rotation to the plane of its orbit around the sun. The ecliptic will cross the equinoctial at the points occupied by the sun at the spring equinox and autumnal equinox. These points are referred to as the First Point of Aries and the First Point of Libra respectively.

The coordinates used to measure a position on the celestial sphere are Declination and Sidereal Hour Angle (SHA).

Declination
Declination may be thought of as the celestial equivalent of latitude. (We must not use the expression celestial latitude as this has another meaning.)

It is defined as the arc of the celestial meridian which passes through the body concerned contained between the equinoctial and body itself. It is measured north or south of the equinoctial between 0° on the equinoctial and 90° at the celestial poles.

In fact the declination of a celestial body will be the latitude of the point on the earth's surface directly under the body, in other words if an observer has a star directly overhead, its declination will be the same as the observer's latitude. Figure 2.1.1 shows the declination of a point in the northern hemisphere.

(Note: The term declination is used by non seafarers to mean the magnetic variation.)

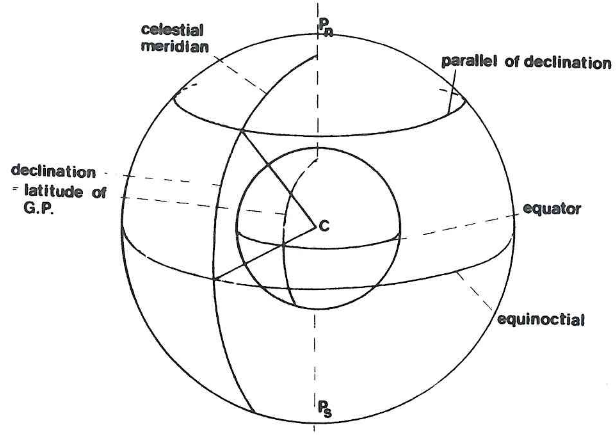


FIG. 2.1.1

Hour Angle
In general the term hour angle means an angular measurement between two celestial meridians. Celestial meridians are also referred to as hour circles.

Sidereal Hour Angle (SHA)
SHA is the angle at the celestial poles between the celestial meridian through the position concerned and the celestial meridian through the First Point of Aries. The meridian through the First Point of Aries is chosen arbitrarily as a reference meridian on the celestial sphere in the same way as the Greenwich meridian is chosen from which to measure longitude on the earth. SHA, (unlike longitude), is measured from 0° to 360° westwards from Aries.

Note: For convenience the First Point of Aries is usually denoted by a symbol representing the horns of the ram (♈).

Figure 2.1.2 shows the SHA of two points, one to the west of Aries (SHA less than 180°) and one to the east of Aries. (SHA greater than 180°.)

Right Ascension (RA)
Right Ascension (RA) is (360° – Sidereal Hour angle (SHA)) and is measured in units of time. RA is not normally used in marine navigation but may sometimes be referred to.

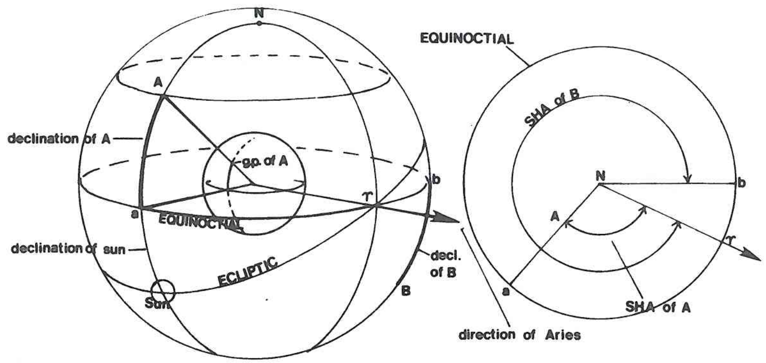


FIG. 2.1.2

Movement of celestial bodies on the celestial sphere

The celestial navigator uses the sun, moon, the four brightest planets, and a selection of the brighter stars in his observations. His choice is dictated by the fact that in order to use a body he must be provided with that bodies position in terms of declination and SHA. This information is obtained from the Nautical Almanac which will be discussed in detail later. Here we will consider in general terms how the various bodies move on the celestial sphere.

We will consider that the Declination and SHA of a fixed point on the celestial sphere remains constant. This is valid over a short period of time although there are movements of the Equinoctial and of the First Point of Aries, which will cause changes over a long period of time.

The declination and SHA of a star are fairly constant. Stars, although they have extremely large velocities through space are vast distances from the earth and appear to be stationary with respect to each other and with respect to the equinoctial and the First Point of Aries. This is why they always appear in the familiar shapes of the constellations. Their movements however will cause very slow changes in declination and SHA but these will be extremely small. For many purposes a star is considered to be a fixed point in space.

The declination and SHA of the sun changes relatively rapidly. The SHA will change through approximately 360° in one year decreasing from 0° or 360° at the spring equinox, to 270° at the summer solstice, 180° at the autumnal equinox, 90° at the winter

solstice and back to 0° at the following spring equinox. The change in SHA is a result of the earth's motion in orbit around the sun. The declination will increase from 0° at the spring equinox when the sun passes the First Point of Aries to approximately 23° 27′ N at the summer solstice, back to zero at the autumnal equinox, to 23° 27′ S at the winter solstice and back to zero at the next spring equinox. The declination change is a result of the inclination of the earth's spin axis, which points to an almost constant direction in space, to the plane of the earth's orbit around the sun.

The declination and SHA of the planets change due to the earth's movement around the sun and also as a result of the planets' own movement around the sun. These changes are therefore more complex than that of the sun, but in general the SHA decreases as the planet moves eastwards as a result of the earth's orbital motion. The inferior planets however exhibit a westerly movement as they pass between the earth and the sun. The declinations depend upon the inclination of the planets' orbits to the equinoctial.

The moon shows the most rapid changes in declination and SHA because of its proximity to the earth and its monthly orbital motion around the earth. The plane of its orbit around the earth is at an angle of about 5 degrees to the plane of the ecliptic. Its maximum range of declination will therefore be $(23° \ 27' + 5°)$ N and S. The SHA will decrease by 360 in each sidereal month.

The Geographical Position (GP)

This is the point on the earth's surface directly under a celestial body. An observer at the GP will have the body directly overhead. The calculations involved in celestial navigation require the latitude and longitude of the GP at the moment of observation.

The position of the GP is defined by the declination and the Greenwich Hour Angle (GHA).

The declination will be the latitude of the geographical position. The term declination is used rather than the term latitude in this respect. The longitude of the GP is expressed by the term Greenwich Hour Angle (GHA). These two terms really mean the same thing. However there is a basic difference in that whereas longitude is measured from 0° to 180° degrees east and west from Greenwich, GHA is always measured from 0° to 360° westwards from Greenwich. Perhaps it would simplify matters if this convention was adopted for longitude also.

The declination of a body is the latitude of the GP. The SHA however cannot give directly the GHA because the earth and the

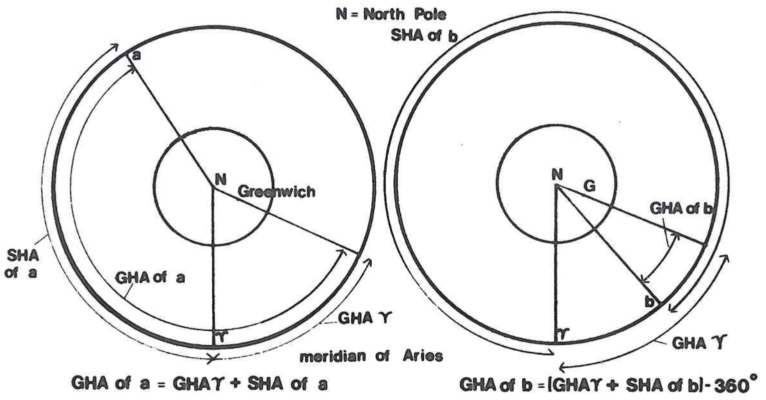


FIG. 2.1.3

Greenwich meridian with it, are rotating within the celestial sphere 360 every day, and so the GHA of every celestial body is changing approximately 360° every day as is evident in the daily motion of the heavens from east to west.

In order to convert from SHA to GHA the Nautical Almanac tabulates throughout the year the values of the longitude or GHA of the First Point of Aries. This is the angular distance between the meridian from which GHA is measured and the meridian from which SHA is measured. The First Point of Aries will, like every other point on the celestial sphere exhibit an east to west motion because of the earth's rotation and therefore its GHA will increase from 0° to 360° in each 360° rotation of the earth, which takes about 23 hours 56 minutes.

To Convert from SHA to GHA

(This will only be necessary when using stars for celestial observations as the Nautical Almanac tabulates the GHA of all other bodies directly rather than the SHA.)

The GHA of a celestial body such as a star is obtained by its SHA plus the GHA of the First Point of Aries. If this comes to more than 360° then 360° is subtracted.

Thus:

$$\text{GHA star} = \text{GHA Aries} + \text{SHA star}$$

Example

Find the GHA of the star Rigel if its SHA is 281° 41′ and the GHA is 126° 15′.

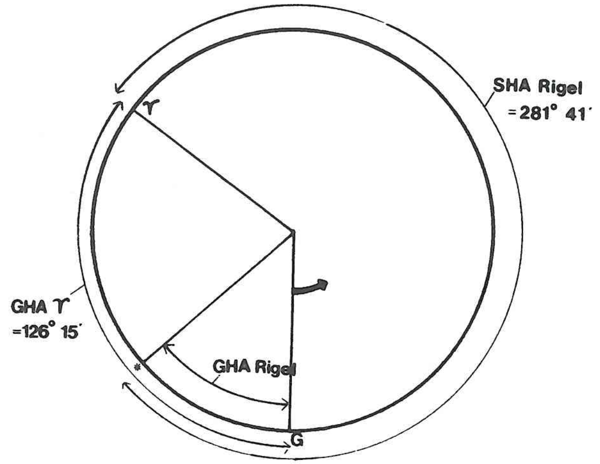


FIG. 2.1.4

Refer to Figure 2.1.4

$$\begin{aligned} \text{GHA } ♈ &= 126^\circ\ 15' \\ \text{SHA Rigel} &= \underline{281^\circ\ 41'} \\ \text{GHA Rigel} &= 407^\circ\ 56' \\ & \quad\ \underline{360^\circ} \\ \text{GHA Rigel} &= \ \ 47^\circ\ 56' \end{aligned}$$

The Nautical Almanac consists mainly of the 'daily pages', which have data for three days on each double facing page. These enable the navigator to extract the declinations and GHA's of the First Point of Aries, the sun, the moon and the four navigational planets for each hour throughout the year. The declination and SHA is given also for the stars but only one value is given for each three day page as the values are changing but slowly. The increment pages in the back of the almanac allow interpolation between the hourly values for each second in the hour.

The increment pages in the back of the almanac contain a table for each minute of the hour. Each table contains an increment for each second within the minute, which is, as the name implies, an increment to the hourly values in the daily pages. Different values of increment are given for the sun and planets, Aries, and Moon.

To extract GHA ♈ from the Nautical Almanac given the UT (Greenwich Mean Time)

Procedure

1. Locate the correct daily page for the date in question.
2. From the left hand column of the left hand page (headed ARIES), extract for the correct date the GHA for the hour of the UT.
3. Locate the correct increment page in the back of the almanac containing the column for the minute of the UT, and against the value of seconds in the UT extract the increment given in the middle column headed ARIES.
4. Add the increment to the hourly value in 2.

Example 1 (using the extracts from the Nautical Almanac)
Find the GHA ♈ for a UT of 22h 14m 46s on 5th January.

GHA ♈ 22h 5th	75° 04.8′
Increment 14m 46s	3° 42.1′
GHA ♈	78° 46.9′

Example 2
Find the GHA of the star Dubhe at UT 09h 18m 16s on 30th March.

GHA ♈ 09h 23rd	322° 20.4′
Increment for 18m 16s	4° 34.8′
GHA ♈	326° 55.2′
SHA Dubhe	194° 00.6′
GHA Dubhe	520° 55.8′
	360°
	160° 55.8′

The increment tables for Aries are based upon an hourly change of hour angle of 15° 02.46′ per hour, this being the rotation of the earth in one solar hour. Alternatively the increment therefore can be found using a calculator by:

$$\frac{15^\circ\ 02.46' \times \text{increment in time (expressed in minutes and decimals)}}{60}$$

The calculator should have a conversion function to express degrees, minutes and seconds to decimals of a degree, and vice versa, at the press of one button. This may also be used to convert hours, minutes and seconds into hours and decimals of an hour.

For bodies other than stars the GHA is tabulated directly and there is never any need to know the SHA. The process of extracting the GHA for the sun, moon or planets then is similar to that described for Aries. The GHA for the whole hour is taken from the appropriate column and an increment added. The increment must be taken from the correct column in the increment pages, the first column for the sun and planets, and the third column for the moon. Also there is an additional correction for planets and the moon called the ‘v’ correction. The value for ‘v’ for a planet must be taken from the bottom of the appropriate planet column in the daily pages and this value interpolated by the use of the table provided in the increment pages at the right hand side of each minute table. This is merely a matter of reading off the correction against the value of ‘v’ from the daily pages. This correction is applied along with the increment. The correction is usually positive but note that it is sometimes negative for Venus.

A value for ‘v’ is also given for the moon but for each hourly entry. This is interpolated in the same way as for the planets in the increment pages.

Note: the need for a ‘v’ correction is merely because the increment tables for planets are based upon a change of hour angle of 15° per hour. The ‘v’ correction allows for the fact that planets change their hour angles at slightly variable rates. Jupiter, Saturn and Mars always have a change of greater than 15° per hour while Venus changes hour angle sometimes greater and sometimes less than 15° per hour. The correction for the moon is because a minimum value of 14° 19′ is used to compile the moon's increments and the change of hour angle may be as much as 14° 37′. The moon's ‘v’ correction is always positive.

The increments for the sun can be obtained from a calculator by:

$$\frac{15 \times \text{increment in minutes and decimals of a minute}}{60}$$

for a planet, for accuracy the ‘v’ correction must be incorporated and the formula must be modified by using (15° + v (expressed in degrees)).

For the moon the formula would be:

$$\frac{(14^\circ\ 19' + v) \times \text{increment in minutes and decimals}}{60}$$

Finding the Declination

For stars there is one value of declination given for each three day page in the daily pages. This value needs no corrections and is taken as constant for the three days.

For the sun moon and planets a declination is given for each hour of UT. This must be interpolated for the minutes of the UT. To aid this process a value of 'd' is given which is merely the hourly change in declination. For the sun and planets one mean value is sufficient for the three days. For the moon a value is given against each hourly entry. 'd' is interpolated for the minutes by using the same interpolation table as for 'v' in exactly the same way. The sign of the 'd' correction must be seen from the values of successive declination hourly entries.

Note that sometimes this interpolation can be done equally well by visual inspection except perhaps in the case of the moon.

Example 1

Find the GHA and declination of the sun at UT 13h 14m 16s on 4th January.

GHA 13h	13° 47.4′	decl.	22° 43.8′ S (d = 0.3)
Increment 14m 16s	3° 34.0′	d	−0.1′
GHA sun	17° 21.4′	decl.	22° 43.7′ S

Example 2

Find the GHA and declination of the moon at UT 18h 20m 12s on 5th January.

GHA 18h	49° 54.5′ (v = 11.0)	decl.	19° 11.8′ S (d = 9.8)
Increment 20m 12s	4° 49.2′	d	−3.3′
v	3.8′	decl.	19° 08.5′ S
GHA moon	54° 47.5′		

Example 3

Find the GHA and declination of Saturn at UT 13h 29m 59s on 5th January.

GHA 13h	216° 04.5′ (v = 2.7)	decl.	22° 02.2′ N
Increment 29m 59s	7° 29.8′		
v	1.3′		
GHA Saturn	223° 35.6′		

EXERCISE 2.1.1

Find the GHA and declination of the sun on:

1. 7th January at 10h 20m 00s UT.
2. 19th September at 15h 10m 00s UT.
3. 19th December at 11h 18m 25s UT.
4. 28th June at 17h 28m 34s UT.
5. 30th September at 04h 15m 47s UT.

EXERCISE 2.1.2

Find the GHA and declination of:

1. The moon on 8th January at 02h 00m 20s UT.
2. The moon on 19th September at 08h 20m 40s UT.
3. The moon on 19th December at 06h 25m 42s UT.
4. Venus on 19th September at 11h 15m 10s UT.
5. Mars on 21st September at 21h 21m 20s UT.
6. Jupiter on 8th January at 20h 11m 20s UT.

Local Hour Angle (LHA)

Whereas the Greenwich hour angle is the angular distance of a body westwards from the Greenwich meridian, the local hour angle is the angular distance of a body west of the local or observer's meridian.

LHA can be defined as the arc of the equinoctial contained between the observer's celestial meridian and that of the body, measured westwards from the observer from 0° to 360°.

Clearly the relationship between the GHA and the LHA is the longitude of the observer.

From Figure 2.1.5 it can be deduced:

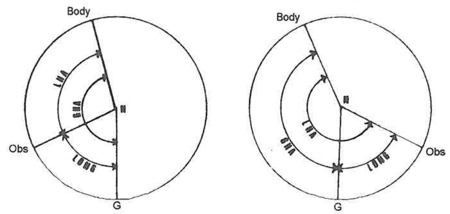
$$\text{LHA} = \text{GHA} - \text{westerly longitude}$$

and

$$\text{LHA} - \text{GHA} + \text{easterly longitude}$$

The value obtained should be adjusted by either adding or subtracting 360° so that it falls within the range 0° to 360°.

The mnemonic Longitude WEST Greenwich BEST, Longitude EAST Greenwich LEAST may be helpful.



Long. West Long. East

FIG. 2.1.5

Example 1
Find the LHA of the sun at 23h 10m 48s UT on 7th January if the longitude of the observer is 153° 20′ E.

GHA 23h	163° 24.7′
Increment	2° 42.0′
GHA	166° 06.7′
Long E (+ve)	153° 20.0′
LHA	319° 26.7′

Example 2
Find the LHA of the moon on 31st October at 15h 21m 29s UT for an observer in longitude 50° 42.0′ W.

GHA 15h	320° 21.3′	v = 6.7
Increment	5° 07.6′	
v corr	2.4′	
GHA	325° 31.3′	
Long W (−ve)	50° 42.0′	
LHA	274° 49.3′	

EXERCISE 2.1.3

Find the LHA in each of the following:

1. The sun on 19th December at 08h 25m 30s UT. Longitude 125° 10.0′ E.
2. The sun on 19th September at 21h 18m 57s UT. Longitude 72° 18.3′ W.
3. Aries on 21st September at 03h 10m 41s UT. Longitude 140° 10.2′ W.

4. Arcturus on 19th December at 20h 15m 40s UT. Longitude 164° 16.2′ E.
5. Kochab on 18th December at 21h 8m 14s UT. Longitude 38° 20.2′ W.
6. The sun on 19th September at 18h 20m 40s UT. Longitude 162° 20.0′ W.
7. Aries on 29th June at 20h 00m 12s UT. Longitude 17° 33.0′ W.
8. Betelgeuse on 30th September at 20h 21m 20s UT. Longitude 162° 00.0′ W.

In addition to the functions described the daily pages also provide data for:

(i) finding times of meridian passage of bodies,
(ii) finding times of sunset and sunrise,
(iii) correction of altitudes.

Explanation of the use of the almanac for these functions will be dealt with in the module 2.6 which covers ‘Latitude by Meridian Altitude’, Module 2.3 which covers ‘Compass Error by Amplitude’, and Module 2.4 which covers ‘Correction of Altitudes’.

MODULE 2.2

Time

The basis of our measurement of time must be the succession of day and night caused by the rotation of the earth about its polar axis. There is however a need to define units of time with high precision. The degree of precision required has become so high that no longer can the movement of any astronomical body provide a satisfactory definition of the basic unit of time, the second, but we cannot escape the fact that the sun governs out lives and that any system of time measurement must be regulated to the daily motion of the sun.

The solar day

This is defined as the interval of time between two successive transits of the sun across the same meridian.

This motion of course is due to the rotation of the earth and during this time the sun changes its LHA by 360°. It is convenient to assume the day to begin when the sun crosses the observer's anti-meridian, when the LHA is 180°. This is referred to midnight. The day ends at the next such transit. The interval is divided into 24 hours, midnight being 0000, and the next midnight 2400 or 0000 of the next day. Midday when the LHA is zero will therefore be 1200 hrs.

We can say therefore that the measurement of time is merely an expression of the position of the sun relative to the observer. Measurement of time and measurement of LHA are therefore synonymous.

As we are using the sun to measure time it is called solar time and as we are measuring the time with respect to the observer it is called local time and of course it follows that the local time will be different for each meridian. Greenwich time is the local time for the Greenwich meridian and as Greenwich is used as a reference from which to measure longitude Greenwich time is used as a universal reference time.

The solar day is longer than the rotation period of the earth by approximately 4 minutes. This is because of the movement of the

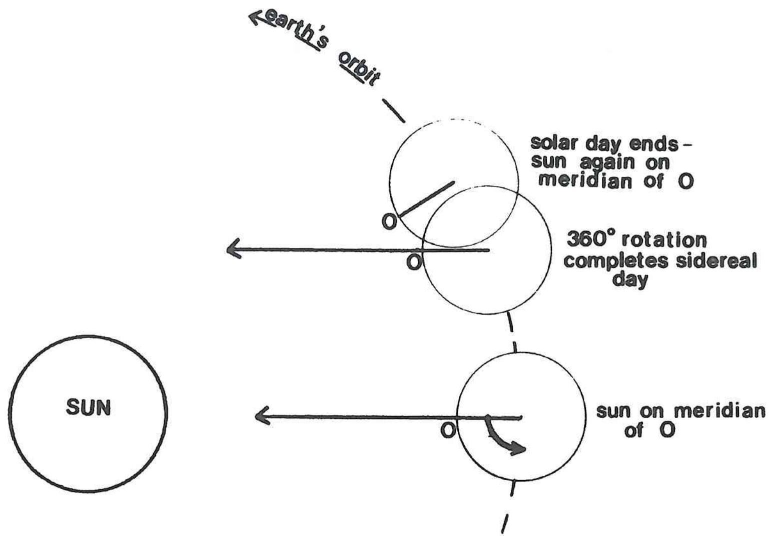


FIG. 2.2.1

earth around its orbit of about one degree during the period. This causes the sun to appear to move eastwards against the stars by about one degree, and the earth must therefore rotate by about 361° to cause two successive transits of the sun across any one meridian. This is illustrated in Figure 2.2.1.

If the rotation of the earth is measured against a fixed point in space such as the first point of Aries then the rotation of the earth will be 360 degrees within the period. Such a period is called a **sidereal day** which will be approximately 23h 56m of solar time. This is the reason why the meridian passage of Aries occurs about four minutes earlier each day.

The use of the apparent sun presents problems in the measurement of accurate time keeping because of the irregularities in its motion. During the solar day the sun changes its LHA by 360° and because the earth is rotating from west to east the LHA will be always increasing. The rate of increase is affected by irregularity in the movement of the earth around its orbit. This has already been stated as about one degree per day. This however changes throughout the year due to the ellipticity of the earth's orbit. It will be greater at perihelion than at aphelion. This is reflected in an irregular movement of the sun against the stars.

There are also an irregular change in the sun's SHA due to the obliquity of the ecliptic. This will cause the sun to move eastwards faster at the solstices than at the equinoxes when it is crossing the equinoctial at 23° 27′.

The irregularities of apparent time are smoothed by using an imaginary mean sun which moves around the equinoctial at a constant rate. The SHA of this Astronomical Mean Sun will differ from that of the true sun by an amount which is referred to as the equation of time (usually expressed in units of time). The component due to the ellipticity is assumed to be zero at perihelion which occurs in January. It will also be zero at aphelion, with two maximums throughout the year. The maximum value of this component is about 1.75° of SHA. The component due to the obliquity is assumed to be zero at the spring equinox and will also be zero at the vernal equinox and the two solstices thus having four maximums throughout the year. This component amounts to about 2.5° of SHA. The algebraic summation of the two components gives the equation of time which goes to zero at four times throughout the year with zero values on April 16th, June 13th, September 2nd and December 25th as shown in Figure 2.2.2. The value is given twice for each day throughout the year in the daily pages of the Nautical Almanac. This causes the LMT of meridian passage of the true sun to differ from 1200 hrs by the amount of the equation of time.

Time measured by the movement of the imaginary Astronomical Mean Sun with reference to the local meridian is called

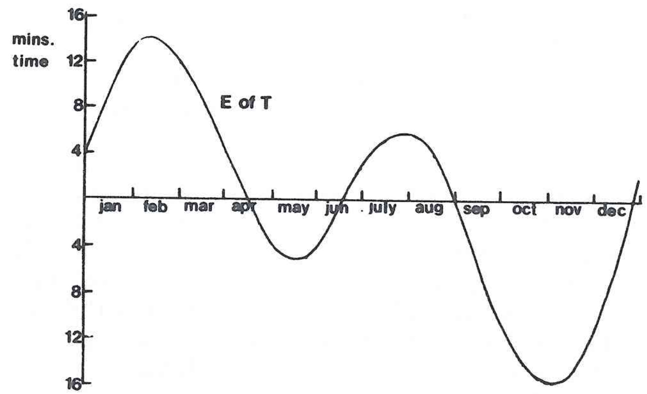


FIG. 2.2.2

Local Mean Time (LMT). Again it will differ from Local Apparent Time (LAT) by the amount of the equation of time.

The LMT of the Greenwich meridian is called Greenwich Mean Time (GMT). It will differ from Greenwich Apparent Time (GAT) by the Equation of Time. In fact GMT is now referred to as **Universal Time** or **UT**.

Relationship between hour angle and time

From the foregoing we can summarise by saying:

Local Mean Time (LMT) = LHA of the mean sun ± 12 hours
Local Apparent Time (LAT) = LHA of the true sun ± 12 hours

and

Greenwich Mean Time (GMT) = GHA of the mean sun ± 12 hours
Greenwich Apparent Time (GAT) = GHA of the true sun ± 12 hours

Relationship between local and Greenwich times

Just as the relationship between GHA and LHA is the longitude, the relationship between Greenwich time and local time will be the longitude in time.

LMT = GMT + E'ly longitude
LAT = GAT + E'ly longitude
LMT = GMT − W'ly longitude
LAT = GAT − W'ly longitude

LMT = LAT ± equation of time
GMT = GAT ± equation of time

Example 1

Given LAT = 18h 06m 14s 4th January in longitude 94° 15′ W find the GMT.

LAT	4th 18h 06m 14s
long	06h 17m
GAT	4th 24h 23m 14s
E of T	05m 02s
GMT	4th 24h 28m 16s
=	5th 00h 28m 16s

Example 2
Given GAT = 07h 14m 30s 1st November in longitude 178° 14′ E find the LMT.

GAT	1st 07h 14m 30s
E of T	16m 24s
GMT	1st 06h 58m 06s
Long	11h 52m 56s
LMT	1st 18h 51m 02s

Universal Time (UT)
Universal Time is mean solar time on the Greenwich meridian, in other words Greenwich Mean Time. UT0 is GMT as observed. UT1 is UT0 corrected for small variations in the position of the pole relative to the earth's surface. This is still an irregular time scale due in part to seasonal variations in the rate of rotation of the earth. It is however the time scale that the navigator needs to rate his chronometer, and it is the time argument which is used in the daily pages of the Nautical Almanac. The time scale which is broadcast as time signals is not UT but UTC.

Coordinated Universal Time (UTC)
This is a time scale based upon atomic time standards and now used for international time signals. It is a precise and regular time scale and therefore will deviate from the irregular UT which the navigator requires. The differences however are small enough for the marine navigator to safely ignore and any inaccuracy in observed longitudes are likely to be negligible compared to other sources of error in observations. It was stated in the introduction to this module that any time scale must be regulated to the daily motion of the sun which regulates our lives, and the atomically generated UTC is not allowed to diverge from UT by more than 0.9 of a second. This is done by introducing as required step adjustments of exactly one second. These adjustments are known as Leap Seconds and are normally made if required at 2400 on 31st December and 30th June. The only way in which this will affect the navigator is that he may notice a jump of one second in his chronometer error.

If UT is required to an accuracy of greater than 1 second then a correction called DUT1 is coded into time signals. DUT1 is as the name implies the difference between the broadcast time signal and UT1. A more accurate UT will then be given by UT1 = UTC + DUT1. Alternatively the explanation in the Nautical Almanac give corrections to be applied to observed longitudes tabulated against possible values of DUT1. Inspection of the value of these corrections will show that no significant error will be caused by ignoring them.

Sidereal Time
Sidereal Time is time measured with reference to the first point of Aries by timing transits of stars across the meridians of time observatories. The sidereal day will be shorter than the solar day by approximately four minutes causing transits of Aries and the stars to get earlier each day by that amount. A conversion from sidereal time to mean solar time may be done by:

sidereal time = LHA mean sun + right ascension mean sun

or

sidereal time = LMT ± 12 hrs + right ascension mean sun

It may be evident from the foregoing that the marine navigator when making his observations is in fact measuring his local time by calculating from his observation of altitude the local hour angle of the observed body. If he observes the sun he is measuring his local apparent time. If he observes a star then he is measuring his local sidereal time. A comparison with Greenwich time would give the longitude. This simple principle is evident in the longitude by chronometer method of sight reduction (see Module 2.7). However matters are complicated by the fact that in practice due to the nature of his observations the same observed information

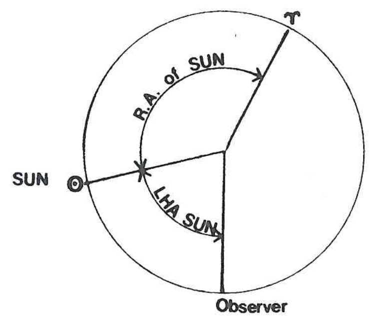


FIG. 2.2.3

would be obtained from many positions and these ambiguities must be resolved. In theory however a position could be obtained by one observation of a body as it crosses the observer's meridian. The altitude would give the latitude and the UT of transit would give the longitude. It is never quite so easy in practice however and to time meridian passage of a body with sufficient accuracy, from a moving platform such as a ship is not possible. Practical methods of obtaining the position from timed observations of altitudes are discussed in module 2.7.

Precession of the equinox

The precession of the equinox is a small westwards motion of the First Point of Aries along the ecliptic.

This is caused by the gravitational attraction of the sun on the earth's equatorial bulge which precesses the earth's spin axis. The result is a movement of the poles of the equinoctial as defined by the earth's spin axis, in a slow circle around the pole of the ecliptic. As the equinoctial moves around with the poles the points at which it cuts the ecliptic move westwards as shown in Figure 2.2.4.

The rate at which Aries is precessing is about 50 seconds of arc per year. One revolution around the ecliptic would take about 25,800 years. As a result of the precession the First Point of Aries no longer lies in the zodiac of Aries but has moved through Pisces and into Aquarius. Passage through one zodiacal sign takes just

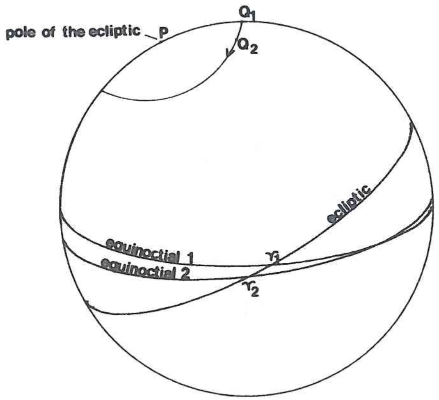


FIG. 2.2.4

over 2000 years. A more immediate effect of the precession is to cause a small annual change in values of declination and SHA of the stars. It also causes a movement of the pole of the equinoctial amongst the stars. At present it lies very close to a star which we know as Polaris (see Module 2.8). In the past other stars have been pole stars and at present the pole is moving away from Polaris.

The precession also means that the recurrence of the seasons is slightly less than the period that the earth takes to orbit 360 around the sun. This is called the sidereal year and is 365d 06h 09m 09s. The seasons actually recur in 365d 05h 48m 46s. This is called the tropical year and is the period in which the civil year of 365 days must be kept aligned in order to maintain our seasons at the same time of the year. This is done by the inclusion of an extra day in the civil year every four years which are then called leap years. This in fact is an over compensation and to allow for this three leap years are excluded every four hundred years. Thus at the turn of the century when the century number is not divisible by 4, the leap year is excluded. Although the year 2000 was a leap year the year 2100 will not. The residual error amounts to about one day in four thousand years but nobody seems to worry about this yet.

MODULE 2.3

Compass Error by Astronomical Observation

The gyro compass error or the magnetic compass may be found by observing the compass bearing of an astronomical body and comparing it with the true bearing calculated for the time of observation. The 'Azimuth Solution' is used for any body which is visible although high altitude observations of azimuth are inaccurate and should be avoided. The 'Amplitude Solution' is used if the compass bearing is observed at the moment of rising or setting.

Azimuth

The azimuth is the angle Z in a spherical triangle PZX, formed by the observer's meridian, the bodies' meridian, and the great circle which passes through the observer (or his zenith), and the body itself. The PZX triangle is discussed in detail in Module 2.7. For the present it suffices to say that the angle Z or the azimuth is the angle between the observer's celestial meridian and that great circle between the observer and the body. It is therefore the direction or bearing of the body from the observer, and sometimes the terms azimuth and bearing are used to mean the same thing. The azimuth however is measured from 0° to 180° from the elevated pole either clockwise or anticlockwise to the body so that in north latitude it is either named north and east or north and west, whilst in south latitude it is either named south and east or south and west.

Having obtained the azimuth it is usual to convert to bearing in three figure notation. If the azimuth is named north and east then the bearing will have the same value in three figure notation. If the azimuth is named north and west then the bearing will be (360 – azimuth). If the azimuth is named south and east then the bearing will be (180 – azimuth), and if named south and west it will be (180 + azimuth). This is illustrated in Figure 2.3.1.

When a body rises it will be to the east of the observer's meridian. The LHA will be between 180° and 360°. As it rises in the sky the LHA will increase until the body crosses the observer's

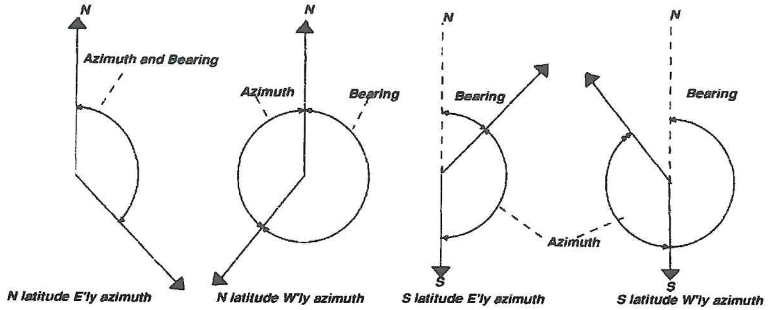


FIG. 2.3.1

meridian, when the LHA will be 360° or zero increasing further until the body sets to the west. Figure 2.3.2a shows the celestial sphere from directly above the observer's zenith (Z). P is the elevated north pole and angle P is the local hour angle. The observer's meridian is the vertical line NS. The equinoctial is shown as WQE. A body is shown rising at X and it will travel around the small circle of declination shown. This body has declination same name as the observer's latitude and less than the latitude and will pass to the south. For such a body the azimuth (angle Z) can be seen to increase from a minimum at rising to a maximum as it crosses the observer's meridian. It will then change sign from east to west and decrease to a minimum at setting. This would also be the case if the body was in south declination as in Figure 2.3.2b.

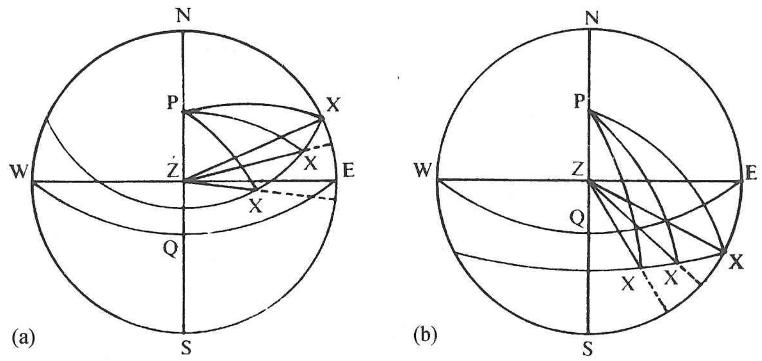


FIG. 2.3.2

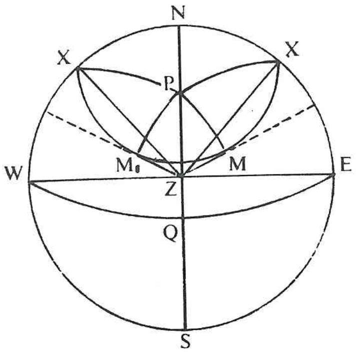


Fig. 2.3.3

Figure 2.3.3 shows the same observer in north latitude but a body with greater declination. It will pass to the north of the observer at meridian passage and the angle Z the azimuth will first increase as the body rises but will then decrease to zero as the body crosses the meridian, increasing and then decreasing to the west as the body sets.

It is the angle Z which we need to calculate to find the azimuth and thence the true bearing. This can be done by spherical trigonometry. However it is normal practice to produce a quick solution from precomputed tables which are contained in Nautical Tables. These tables are called ABC tables and in general are designed for the solution of spherical triangles when the three known parts and the unknown to be solved are all adjacent to each other.

The ABC Tables

It is useful here to give an explanation of the ABC tables although this is not necessary for the practical use of the tables. However it will be seen that the explanation will give us a quicker way to use the ABC method by using a calculator. This avoids the sometimes awkward interpolation associated with the tables, and usually speeds up the process.

The ABC tables are based upon a spherical trigonometry formula called the four part formula. This is used when three known parts and the unknown are all adjacent. The formula states:

(cot outer side × sin inner side) = (cot outer angle × sin inner angle) + (cos inner angle × cos outer angle). This is shown in Figure 2.3.4 which shows a PZX triangle.

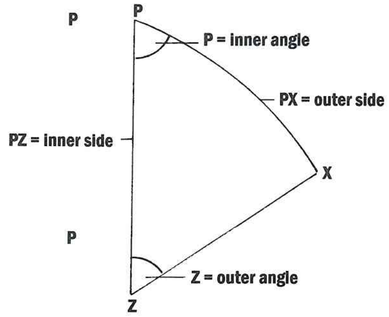


Fig. 2.3.4

The adjacent parts in the triangle which we know or can find are:

1. PX = 90 – declination. This is the outer side.
2. P = the LHA. This is the inner angle.
3. PZ = 90 – latitude. This is the inner side.

Z is the adjacent part which we need to solve for and is the outer angle, and therefore by the four part formula:

$$\cot PX \sin PZ = (\cot Z \sin P) + (\cos PZ \cos P)$$

or substituting complements:

$$(\tan \text{declination} \cos \text{latitude}) = (\cot \text{azimuth} \sin \text{LHA}) + (\tan \text{latitude} \cos \text{LHA})$$

Dividing by sin LHA cos lat gives

$$(\tan \text{dec} \operatorname{cosec} \text{LHA}) - (\cot \text{az} \sec \text{lat}) + (\tan \text{lat} \cot \text{LHA})$$

In the ABC tables the value of (cot az sec lat) is tabulated as quantity C. The value of (tan dec cosec LHA) is tabulated as quantity B, and the value of (tan lat cot LHA) is tabulated as quantity A, so that:

$$C = B - A$$

or

$$-C = A - B \qquad (1)$$

If B is greater than A then C becomes negative then

$$C = B - A \qquad (2)$$

If latitude and declination are of opposite names then the side PX is greater than 90 and its tangent is negative so that B becomes negative, and then:

$$-C = A + B \qquad (3)$$

Thus

$$-C = A \pm B$$

Under the conditions of (1) and (3) which make C negative then the quadrant in which the bearing lies is named opposite to the latitude. When C is positive as in (2) then the bearing is named the same as latitude. However to make things more complicated the negative sign in (1) is ignored in the tabulation of C and so these rules must be reversed.

Thus if C is positive the quadrant is named opposite to the latitude
and if C is negative the quadrant is named the same as the latitude.
The quadrant is named east when the body is rising and west when the body is setting.

These rules for applying the correct signs to A and B and for interpreting the quadrant of C are fairly complicated and difficult to remember. However they are given in a simple to follow way with the A, B and C tables. A practical approach to the problem therefore is to use the ABC tables and follow the rules associated with them but to obtain the values of A, B and C from a calculator. The equations for A, B and C can be found in the explanation of the tables and may be written in the pages of the ABC tables to help the memory. The advantage of doing this is that the interpolation sometimes necessary when getting the values from the tables is avoided. The method adopted is a matter of personal choice and will depend upon the relative skills of the navigator in interpolation and in using his calculator.

The Azimuth Problem

Procedure

1. Observe the compass bearing of a selected body, and note the UT.
2. Find using the Nautical Almanac the GHA of the body and the declination for the UT.
3. Apply the DR longitude to give the LHA.
4. Using the DR latitude, declination, and LHA, find the azimuth or bearing from ABC tables and/or calculator.
5. Compare the true bearing obtained in 4 with the observed compass bearing to give the compass error, and apply the variation to give the deviation for the ship's head.

Example 1 (refer to Figure 2.3.2a)
In DR position 52° 48′ N 42° 18′ W at UT 1858 hrs on the 1st November the compass bearing of the sun was observed to be 244°. Find the error of the compass and the deviation for the ship's head if the variation was 4° W.

UT	18h 58m	1st Aug
GHA	94° 06.0′	declination 14° 28.7′ S
Increment	14° 30.0′	
GHA	108° 36.0′	
longitude	42° 18.0′ W	
LHA (angle P)	66° 18.0′	

From Burton's ABC tables		From Norie's ABC tables	
A	0.578 +	A	0.58 S
B	0.282 +	B	0.28 S
C	0.860 +	C	0.86 S

bearing	S 62.5° W
true bearing	242.5°
compass brg	244°
compass error	1.5° W
variation	4.0° W
deviation	2.5° E

Example 2
On March 30th in DR position 25° 40′ S 175° 45′ W at LMT 0325 the compass bearing of the star Gacrux was observed to be 213.5° by gyro compass. Find the error of the gyro.

LMT	0325	
longitude	1143	
UT	1508 30th	

GHA ♈ 15h	52° 35.2′	declination 57° 07.9′ S
increment	2° 00.3′	
GHA ♈	54° 35.5′	
SHA star	172° 09.6′	
GHA star	226° 45.1′	
longitude	175° 45.0′ W	
LHA	51° 00.1′ (body setting)	

From Burton's ABC tables		From Norie's ABC tables	
A	0.389 +	A	0.39 N
B	1.991 −	B	1.99 S
C	1.602 −	C	1.60 S
bearing	S 34.7° W	bearing	S 34.7° W

true bearing	214.7°
compass brg	213.5°
gyro error	1.2° Low

EXERCISE 2.3.1

1. On 19th September in DR position 42° 50′ N 46° 10′ W at 11h 20m 19s UT the sun bore 124° by compass. Find the compass error and the deviation for the ship's head. Variation 6° W.

2. On 6th January in DR position 48° 20′ S 96° 30′ W at 20h 12m 30s UT the sun bore 311° by compass. Find the compass error and the deviation for the ship's head. Variation 3° W.

3. On 19th December in DR position 46° 15′ N 168° 35′ W the observed azimuth of the sun was 158° C at 2029 UT. Find the compass error and the deviation for the ship's head. Variation 4° E.

4. On 28th June at 0745 LMT at the ship in DR position 38° 10′ S 124° 10′ E at 2358 UT the compass bearing of the sun was observed to be 059°. Find the compass error and the deviation if the variation was 2° W.

5. On 19th September at 1520 LMT the sun bore 262.5° by compass to an observer in DR position 19° 20′ N 149° 50′ E. Find the compass error.

EXERCISE 2.3.2

1. On 19th December at 0600 ship's time in position 46° 40′ N 168° 20′ W the observed bearing of the star Rigel was 230° C at 1515 UT. If the variation was 4° W find the compass error and the deviation for the ship's head.

2. On 29th June at 0211 UT the star Rasalhague bore 247° C to an observer in position 38° 20′ N 5° 40′ E. If the variation was 6.5° E, find the compass error and the deviation for the ship's head.

3. On 1st November at 0310 ship's time in position 41° 15′ N 145° 26′ E when the chronometer which was correct on UT showed 05h 28m 19s, the star Procyon was observed to bear 135° C. If the variation was 1° W find the compass error and the deviation for the ship's head.

4. On 6th January at ship's time 2200 in position 46° 20′ N 47° 52′ W the star Schedar bore 316° C when the chronometer which was correct on UT indicated 01h 11m 28s. If the variation was 7° W find the deviation for the ship's head.

5. On 20th September at about 0330 at the ship in position 32° 24′ S 80° 15′ E the star Peacock was observed bearing 220° C when the chronometer showed 10h 14m 20s. If the variation was 4° W find the compass error.

The Amplitude Problem
The amplitude of a body is the angle between the direction of east when rising or west when setting and the direction of the body.

Thus the amplitude is just another way of expressing the bearing of the body when rising or setting. The bearing in quadrantal notation will be (90 − amplitude). Figure 2.3.5 shows, for an observer in north latitude, a body (X) of same name as latitude at rising. The amplitude will be arc EX and named E and N. A body (X′) of declination opposite to latitude is shown with amplitude arc EX′ and named E and S.

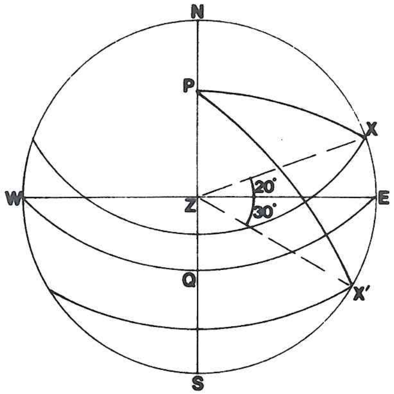


FIG. 2.3.5

If the bearing of the body is observed at the moment of rising or setting then the true amplitude can be very quickly found and thus the compass error. In practice the sun and the moon are the only suitable bodies to use for amplitudes as they are the only ones which are clearly visible when near the horizon. The true moment of rising or setting, that is when the true altitude is zero, occurs when the body is slightly above the visible horizon due to the effects of dip and refraction. As a general rule of thumb the compass bearing should be taken when the lower limb is about a semi diameter above the visible horizon. In lower and medium latitudes the bearing is changing fairly slowly and small errors in the time of observation will not affect the accuracy of the result. In very high latitudes however the bearing will be changing relatively quickly and more care must be taken.

The true amplitude can be found from amplitude tables which will be found in nautical tables. These are easy to use but a much quicker and easier solution is to use the amplitude formula with the aid of a calculator. The true amplitude can be found from the formula:

$$\text{sine amplitude} = \text{sine declination} \times \text{secant latitude.}$$

This is the formula upon which amplitude tables are based and it is a personal choice as to which method is used.

The amplitude is named east when the body is rising and west when it is setting and north or south according to the name of the declination.

If required the times of rising or setting may be found from the daily pages of the Nautical Almanac.

To find the times of sunrise or sunset

The time of sunrise and sunset is given on the right hand side of the right hand page in the daily pages. It is given once for the three days on the page, the figure referring to the middle day. In normal latitudes this will suffice for the three days. In high latitudes it will be necessary to interpolate between the middle days of the previous or following page. The argument latitude must be used against which to extract the time, interpolating between tabulated values. In fact the interpolation is not linear and for accuracy it should be done using the Table I on the page immediately following the increment tables in the Nautical Almanac. Full instructions are given with the table. If however there is little change in the tabulated values no significant error will be caused by interpolating visually assuming a linear change.

The times extracted from the daily pages are local mean times (LMT) and for the sun may be used to apply to any meridian. All that needs to be done is to apply the longitude expressed in time to obtain the UT which is used for taking the declination.

Example 1

Find the UT of sunrise on 20th September to an observer in position 50° N 165° 24′ W.

LMT sunrise in 50 N for 19th	05h 42m
longitude in time	11h 02m
UT	16h 44m 20th

Example 2

Find the time of sunset on 6th January to an observer in position 55° N 172° 30′ E

LMT sunset 55 N 5th	15h 47m
LMT sunset 55 N 8th	15h 51m
LMT sunset for 6th	15h 48m
longitude in time	11h 30m
UT	04h 18m 6th

To find times of moonrise and moonset

The times of moonrise and moonset are tabulated against latitude in the same way as for the sun. The times are given however for each of the three days, because of the rapid changes. Obviously the time should be taken for the day in question, but again because